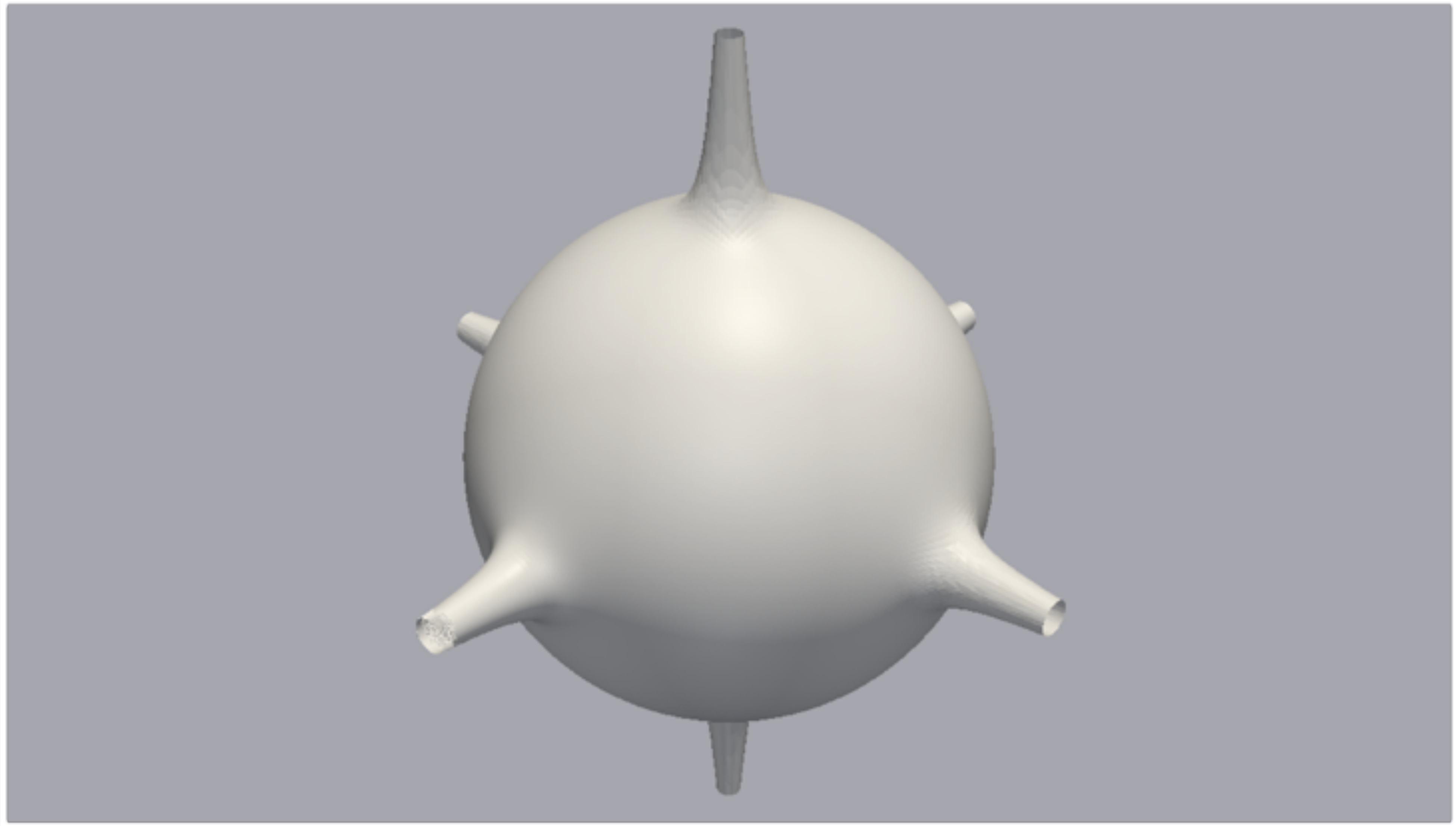


Eloisa Bentivegna, Università degli Studi di Catania & INFN

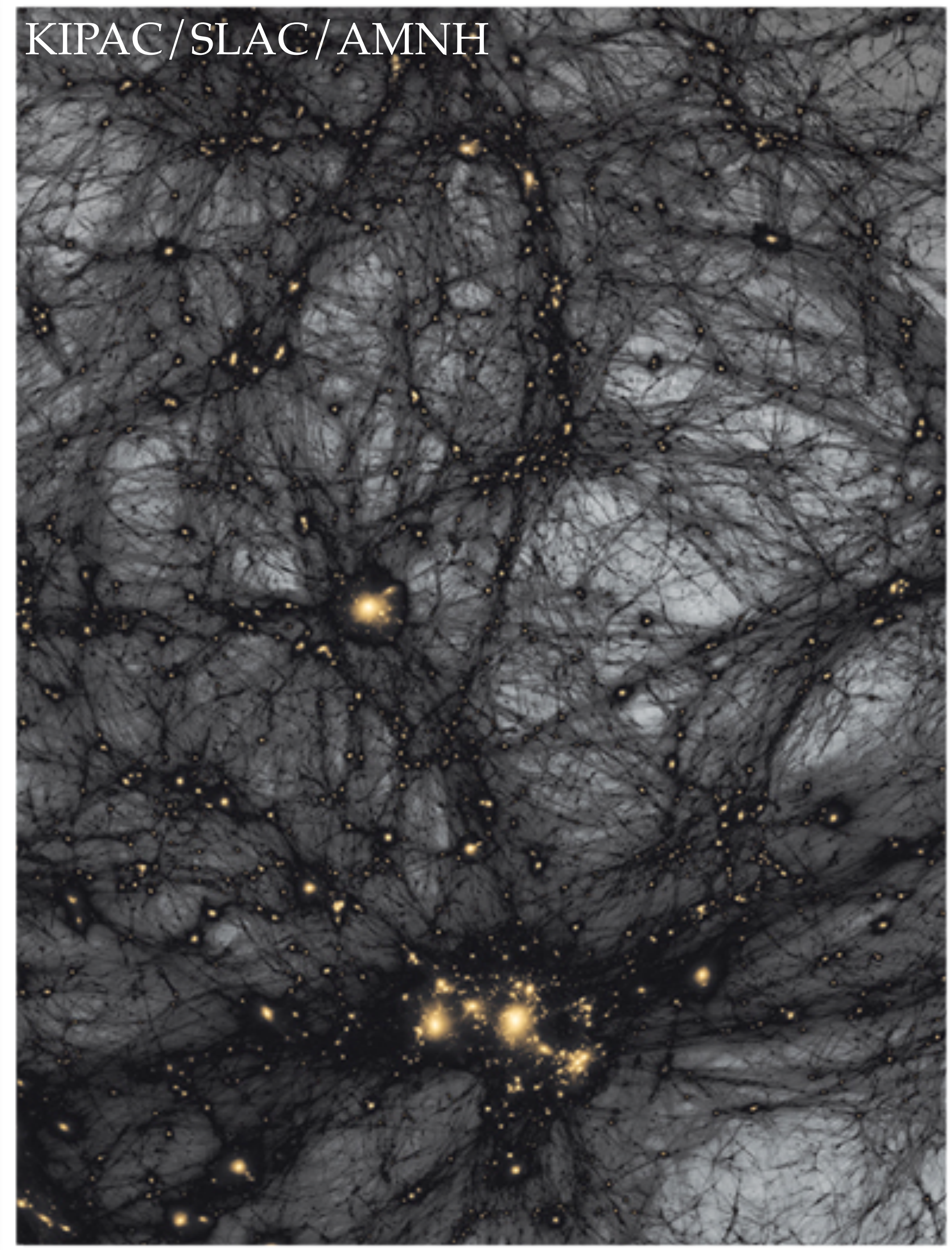
Numerical Relativity and Inhomogeneous Cosmological Models

INHOMOGENEOUS COSMOLOGIES, TORUŃ

July 3rd, 2017

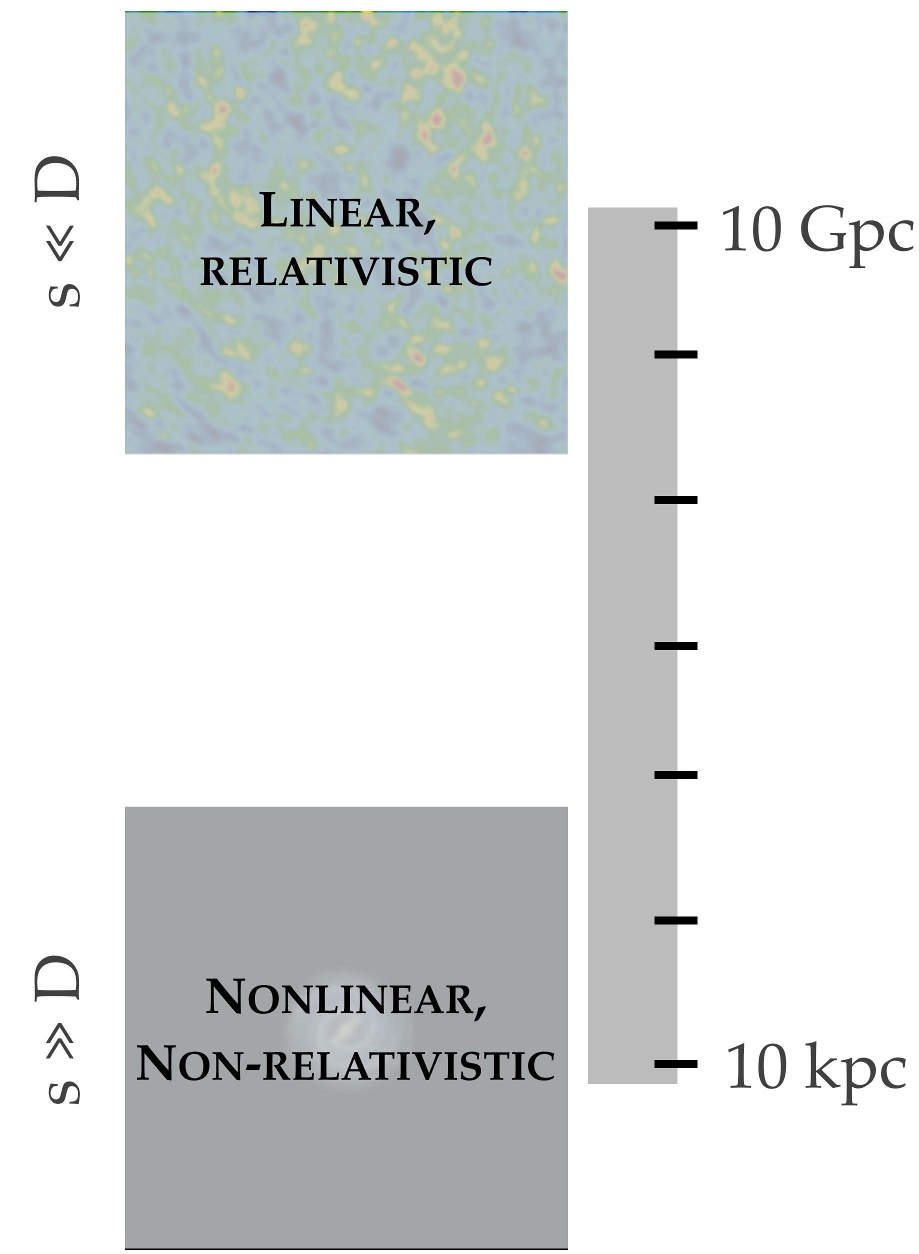
Open questions

- Are there important **relativistic effects** in cosmology which our models currently neglect (e.g. Concordance)?
  - Conceptual issues
  - Observational tensions
- How do structures **assemble** in GR?
- What is the relationship between relativistic **isolated-object and FLRW solutions**?



Concordance Model

- Cosmological Principle: *On large scales, the Universe is homogeneous and isotropic.*
- Standard model based on three ingredients:
  - FLRW background
  - Linear perturbations of FLRW on the larger scales
  - Newtonian gravity on the smaller scales



Concordance Model

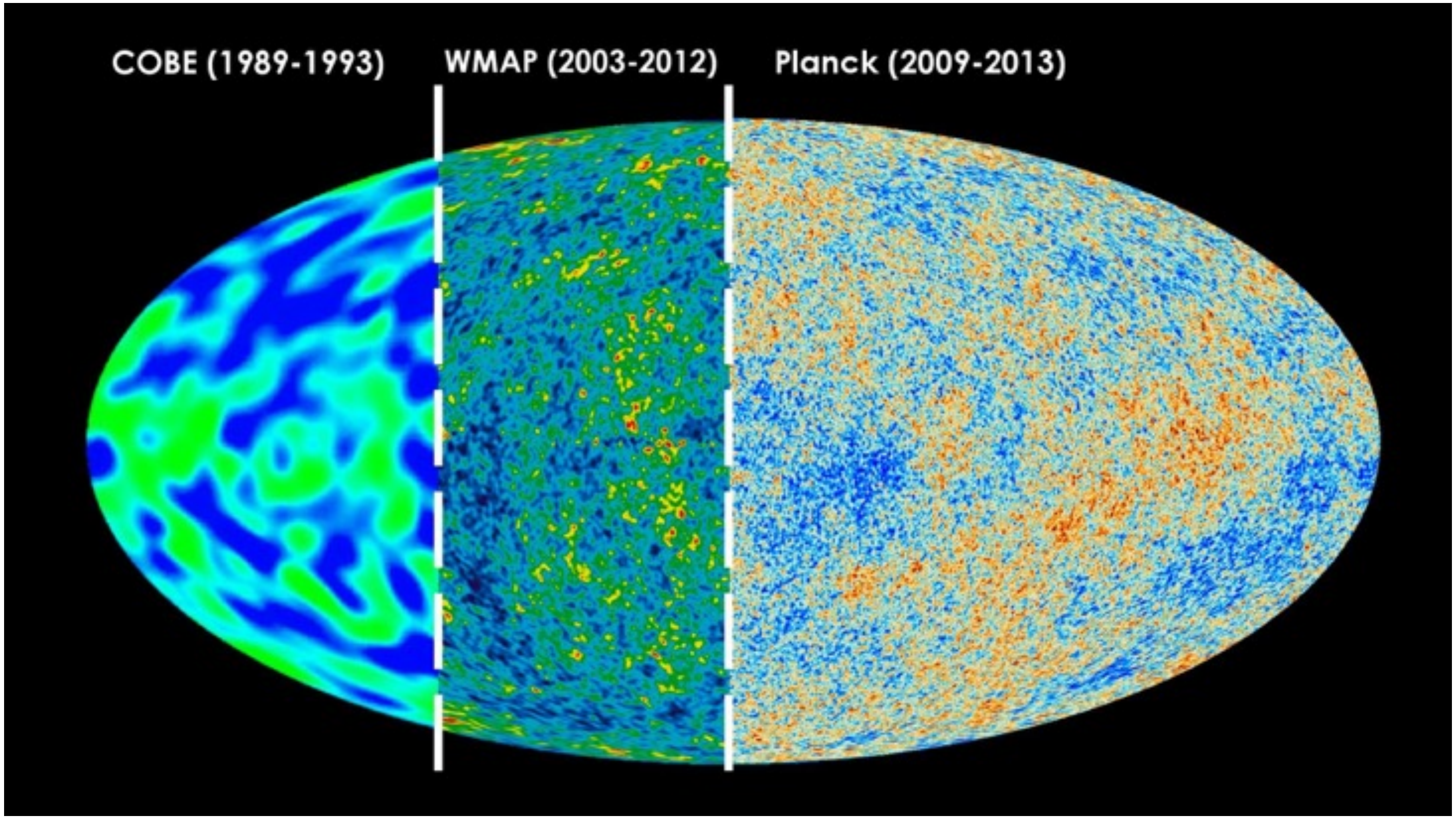
Huge simplification in the treatment of processes from the early to the late Universe, data can be fitted with only a small numbers of parameters.

Good enough?

H_0 (km s^{-1} Mpc^{-1})	
Local Universe [Riess et al. 2016]	73.24 ± 1.74
Planck+WMAP+ACT+SPT+BAO	69.3 ± 0.7

- Experimental systematics
- Control over sources, astrophysical processes, etc.
- Relativistic effects (statistics!)

Concordance Model



Cosmology is emerging as a full-blown experimental science: along with better experiments and better data analysis, we **need better modelling**!

Concordance Model

Cosmological parameters used pervasively in astrophysics and cosmology: errors can propagate and affect other studies.

PRL **116,** 241102 (2016) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2016

Properties of the Binary Black Hole Merger GW150914

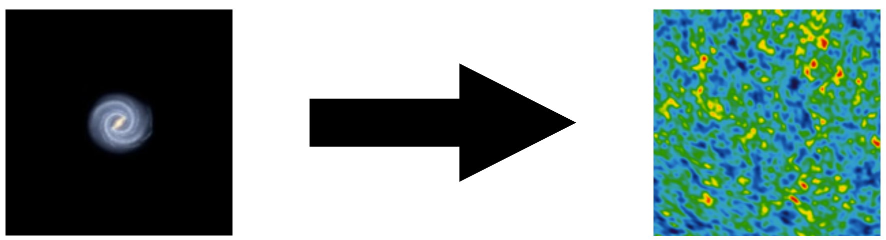
B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 18 February 2016; revised manuscript received 18 April 2016; published 14 June 2016)

To convert the masses measured in the detector frame to physical source-frame masses, we require the redshift of the source. As discussed in the Introduction, GW observations are directly sensitive to the luminosity distance to a source, but not the redshift [98]. We find that GW150914 is at $D_L = 410^{+160}_{-180}$ Mpc. Assuming a flat ΛCDM cosmology with Hubble parameter $H_0 = 67.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and matter density parameter $\Omega_m = 0.306$ [6], the inferred luminosity distance corresponds to a redshift of $z = 0.09^{+0.03}_{-0.04}$.

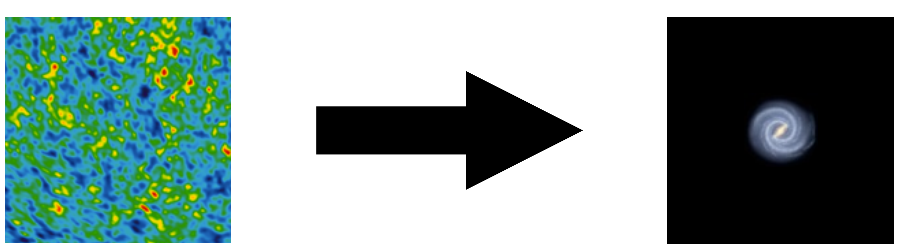
Multimessenger studies may reveal inconsistent information from gravitational and electromagnetic spectrum if models are not accurate enough.

Beyond Concordance

Better modelling?



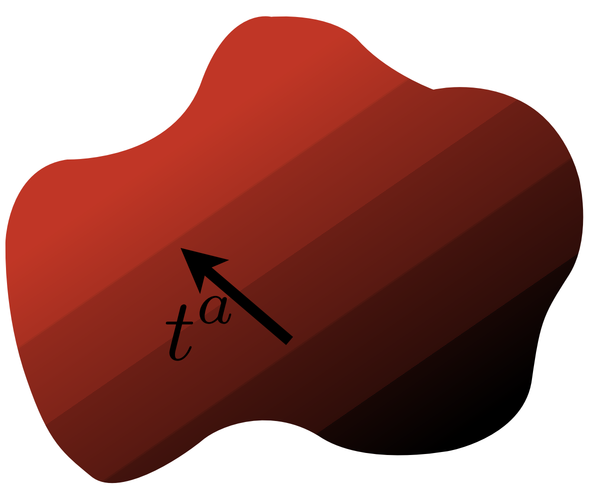
Newtonian two-body interaction plus some relativistic corrections **[Thomas+ 2014, Adamek+ 2014-2016, Rácz 2016]**. (But post-*an approach may hold back relativistic insight.)



Solving Einstein's equations for a cosmological model where nonlinearities are important. **[Clifton+ 2009-2016, Bentivegna+ 2012-2016, Yoo+ 2012-2016, Mertens+ 2015-2016, …]** (But high computational cost, low realism.)

Numerical Relativity for Cosmology

Einstein's equation can be solved exactly by formulating it as an initial-boundary value problem, and integrating numerically. One needs to **choose a time coordinate** and project the equations accordingly.



$$\mathrm{d}s^2 = -\alpha^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i\mathrm{d}t)(\mathrm{d}x^j + \beta^j\mathrm{d}t)$$

$$K_{ij} = -\mathcal{L}_n\gamma_{ij}$$

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$D_jK^j_i - D_iK = 8\pi j_i$$

$$\partial_t\gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\partial_t K_{ij} = -D_iD_j\alpha + \alpha(R_{ij} - 2K_{ik}K^k_j + KK_{ij}) + \beta^kD_kK_{ij} + K_{ik}D_j\beta^k + K_{kj}\Delta_i\beta^k$$

Numerical Relativity for Cosmology

Typical recipe:

- Choose topology and stress-energy content:

$$\rho = n^a n^b T_{ab} \qquad j^a = -\gamma^{ab} n^c T_{bc}$$

- Solve the Einstein constraints to obtain initial data:

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho \qquad D_j K^j_i - D_i K = 8\pi j_i$$

- Choose numerical coordinates

$$\mathrm{d}s^2 = -\alpha^2 \mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i \mathrm{d}t)(\mathrm{d}x^j + \beta^j \mathrm{d}t)$$

- Integrate the evolution equations (with the relevant matter content)

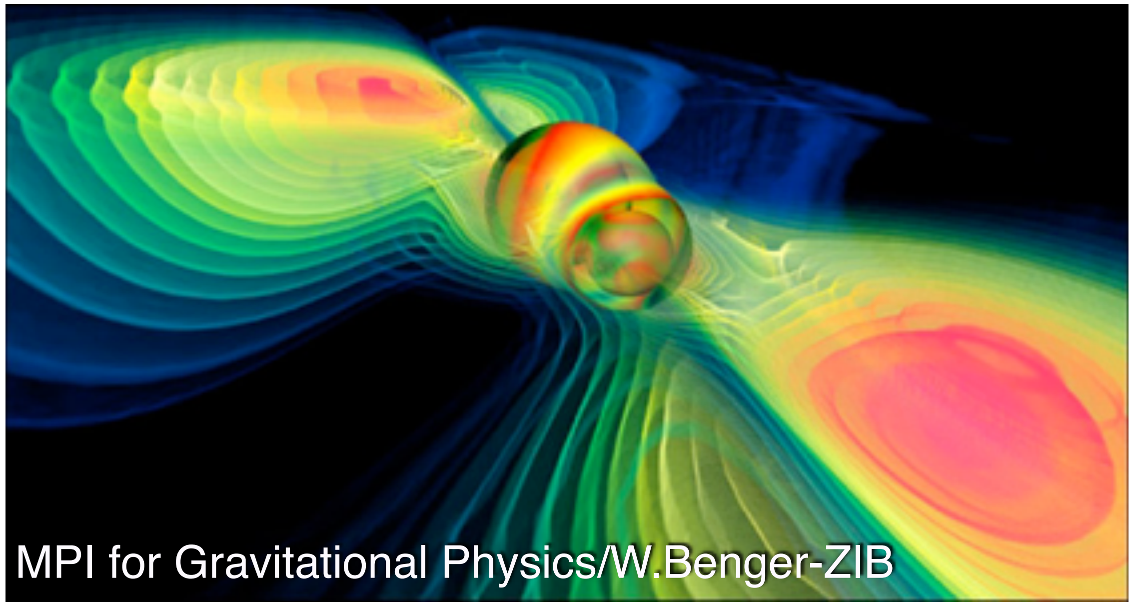
$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\begin{aligned}\partial_t K_{ij} = &-D_i D_j \alpha + \alpha(R_{ij} - 2K_{ik}K^k_j + KK_{ij}) \\ &+ \beta^k D_k K_{ij} + K_{ik} D_j \beta^k + K_{kj} \Delta_i \beta^k\end{aligned}$$

Numerical Relativity for Cosmology

Thanks to numerical relativity, a number of these scenarios have been under scrutiny in the last ten years, with many more being actively pursued now:

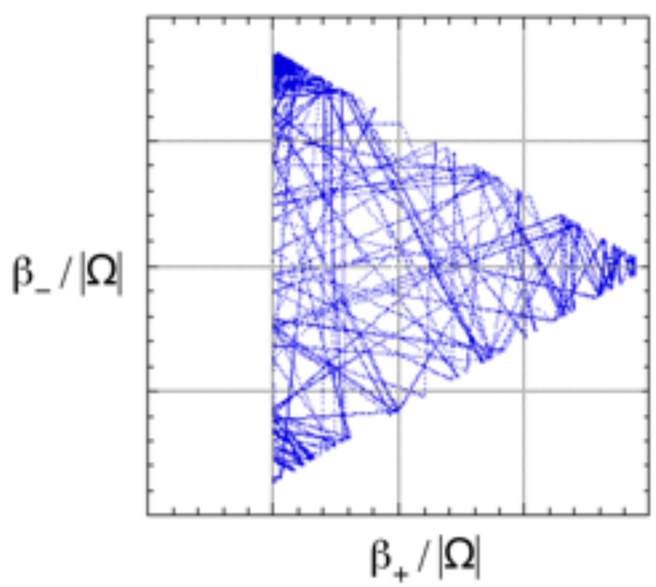
- Black-hole binaries
- Neutron-star binaries
- Mixed binaries
- Gravitational collapse
- and supernovae
- Black holes surrounded by accretion disks



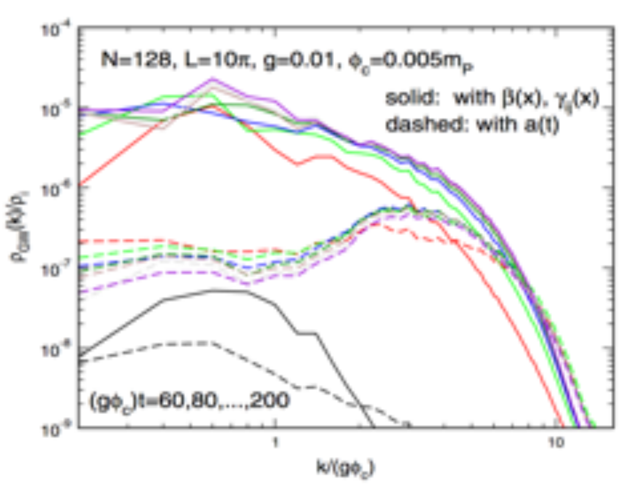
The methods and tools of Numerical Relativity can in principle help construct **general, exact spacetimes**, thereby providing a laboratory to study any system at will (but beware of assumptions…).

Numerical Relativity for Cosmology

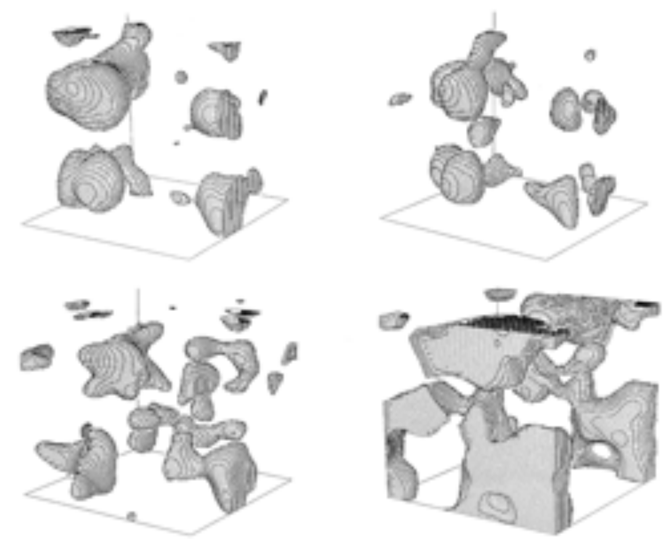
Singularities: **Matzner, Weaver (1970), Berger, Garfinkle (1991, 1997, 2004)**



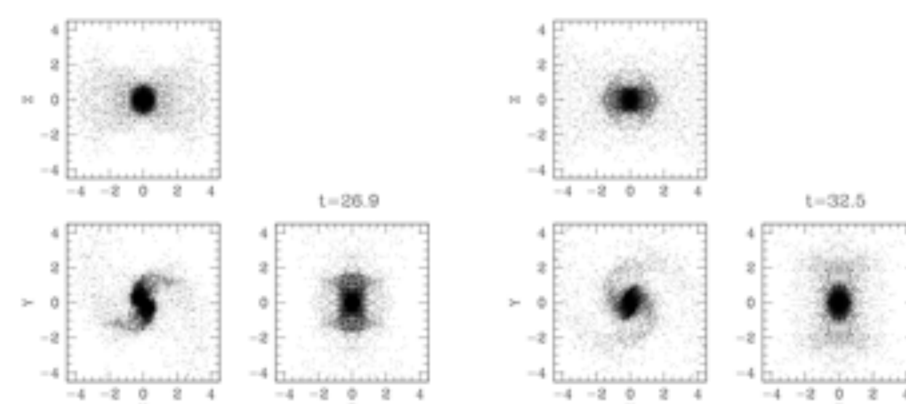
Phase transitions in the early universe and primordial gravitational waves: **Rezzolla, Miller, Pantano (1995), Bastero-Gil, Macias-Perez, Santos (2010), Wainwright, Johnson, Peiris, Aguirre, Lehner, Liebling (2014)**



Inflation: **Centrella, Wilson, Kurki-Suonio, Laguna, Matzner (1983, 1984, 1987, 1993, 1996), Bastero-Gil, Tristram, Macias-Perez, Santos (2007), East, Kleban, Linde, Senatore, Kearney, Shakya, Yoo, Zurek (2015, 2016), Braden, Johnson, Peiris, Aguirre (2016)**



Large-scale structure, black-hole formation: **Anninos, Centrella, McKinney, Wilson (1984, 1985, 1999), Shibata (1999), Bentivegna, Korzyński, Hinder, Bruni (2012-2015), Yoo, Okawa, Nakao (2012-2016), Torres, Alcubierre, Diez-Tejedor, Nunez, de la Macorra (2014-2015), Rekier, Cordero-Carrion, Fuzfa (2015), Mertens, Giblin, Starkman (2015-2016)**

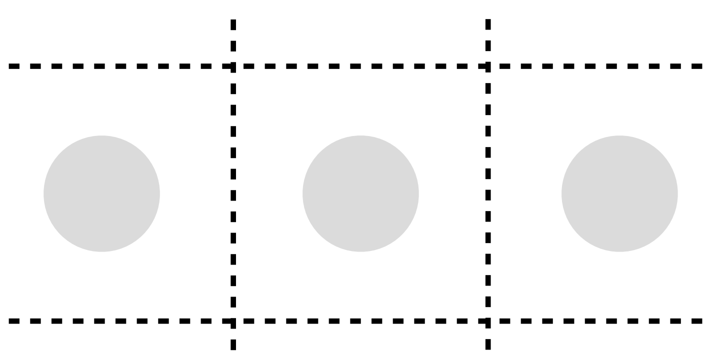


The Late Universe

Construct numerical, fully relativistic spacetimes satisfying the Cosmological Principle above a certain scale but inhomogeneous below it.

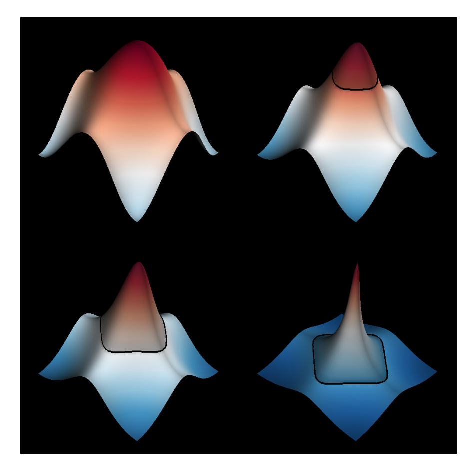
Approach I: discrete

Relativistic "*N*-body", regular lattices of black holes.

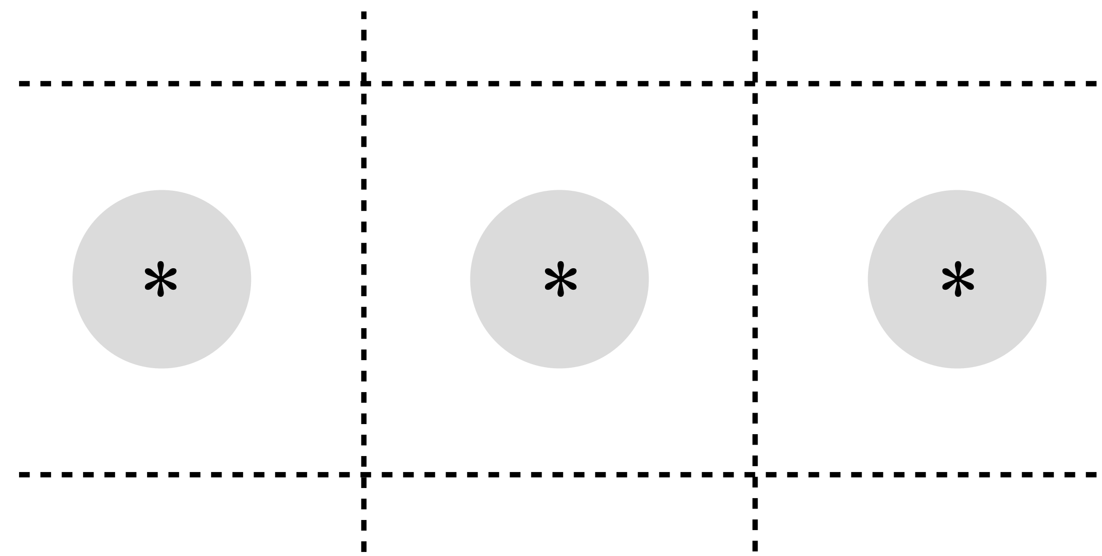


Approach II: continuum

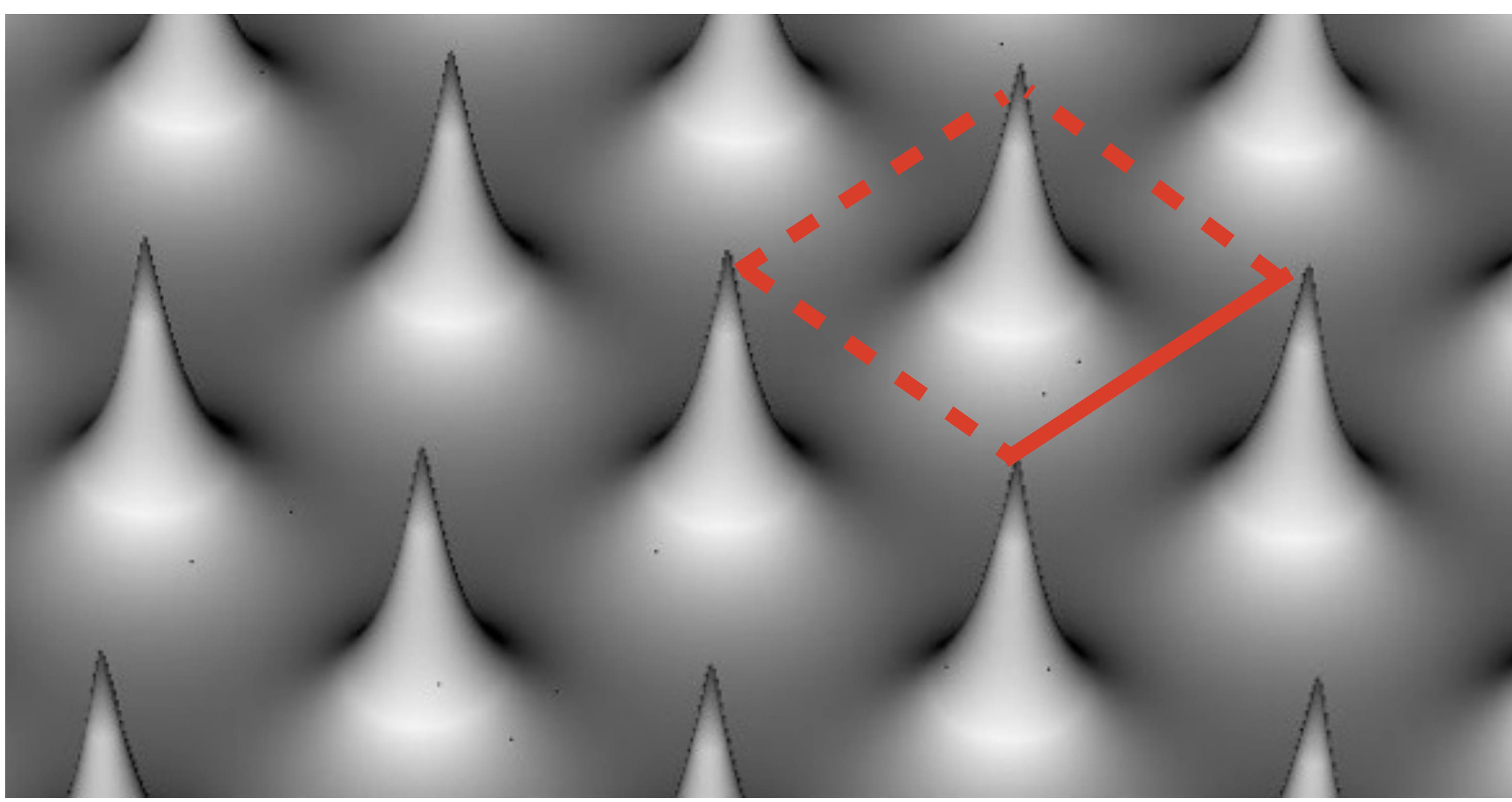
Evolution of perturbed perfect fluids beyond the perturbative regime.



Black-hole lattices



[Lindquist&Wheeler 1957]



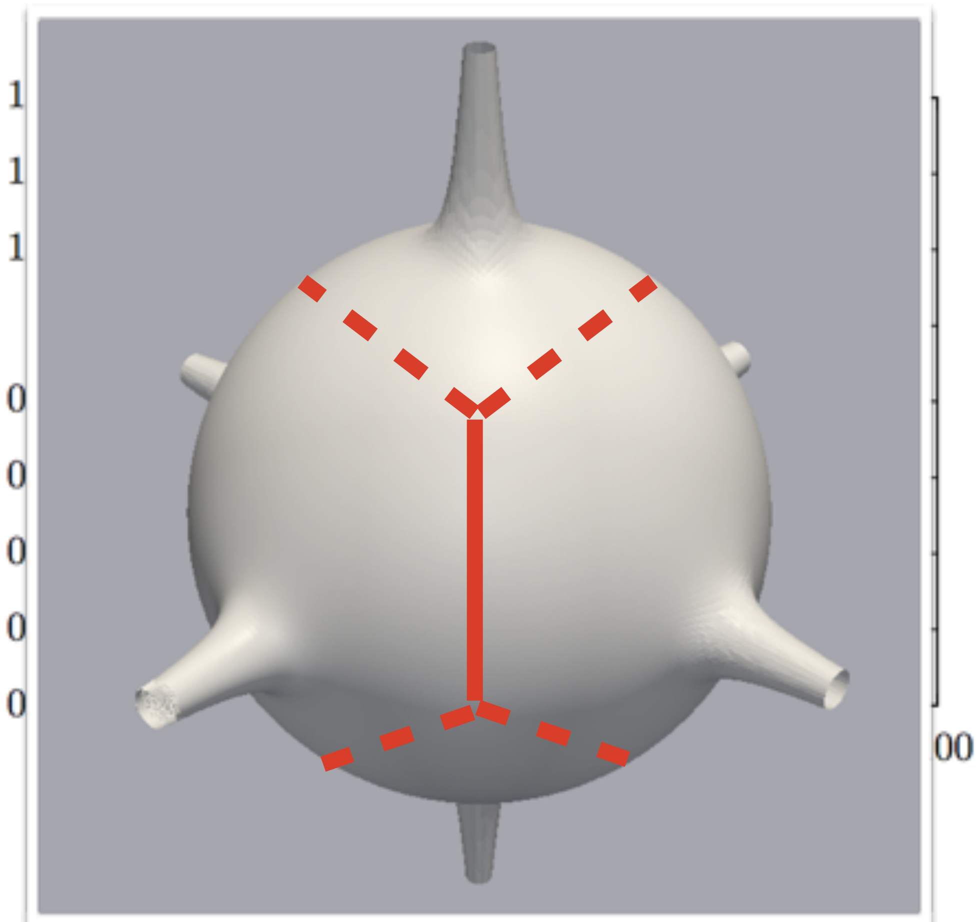
Several roads:

- Junction conditions **[Clifton 2009]**
- Series expansions **[Bruneton&Larena 2012]**
- Solving the GR constraints **[Wheeler 1983, Clifton et al. 2012, Yoo et al. 2012, Bentivegna&Korzyński 2012, Yoo et al. 2013, Bentivegna&Korzyński 2013]**

Black-hole lattices

General principle **[Choquet-Bruhat, Kleban & Senatore 2016]**. Some options:

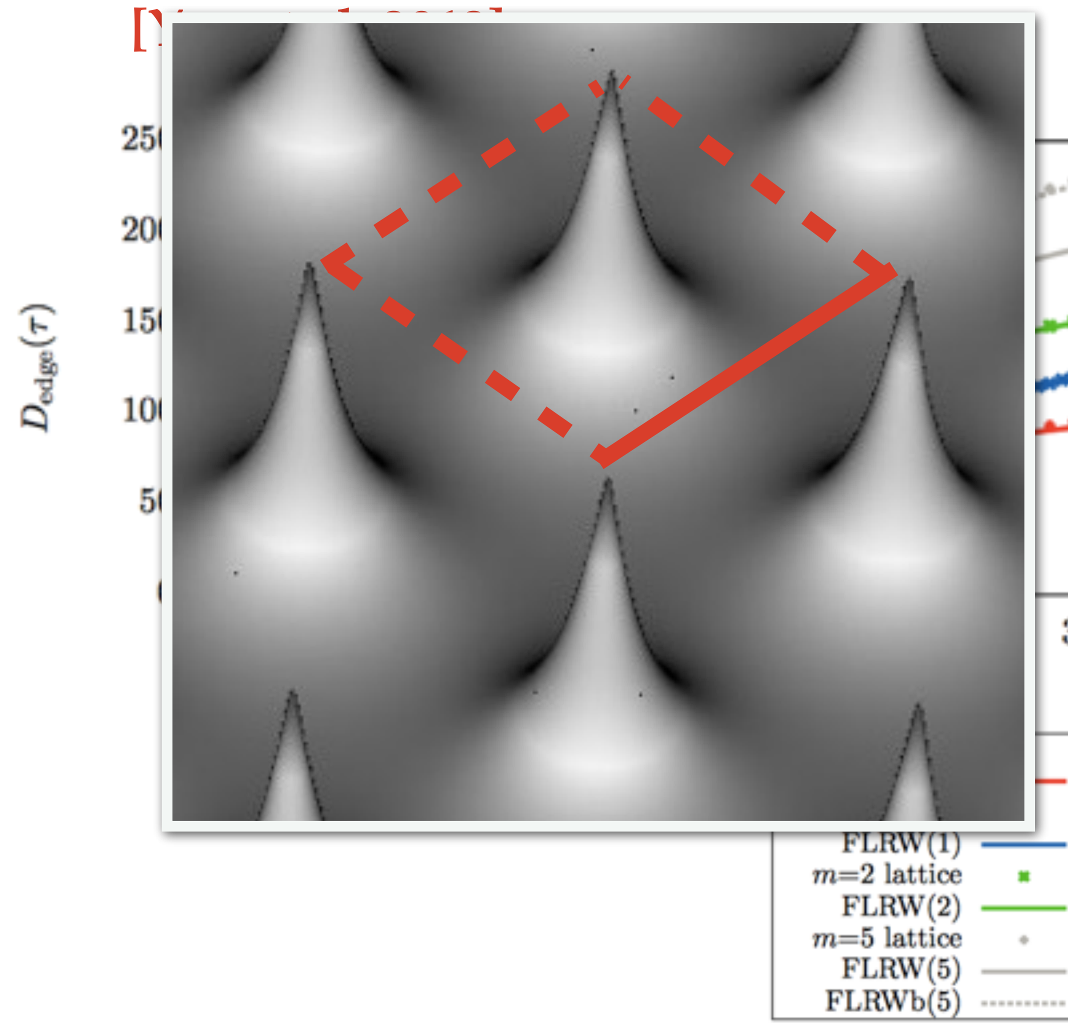
Keep a zero extrinsic curvature, but choose a conformal metric that is



[Bentivegna & Korzyński 2012]

principle to construct multi-black-hole solutions.

Keep a flat conformal metric, but use a non-zero extrinsic curvature

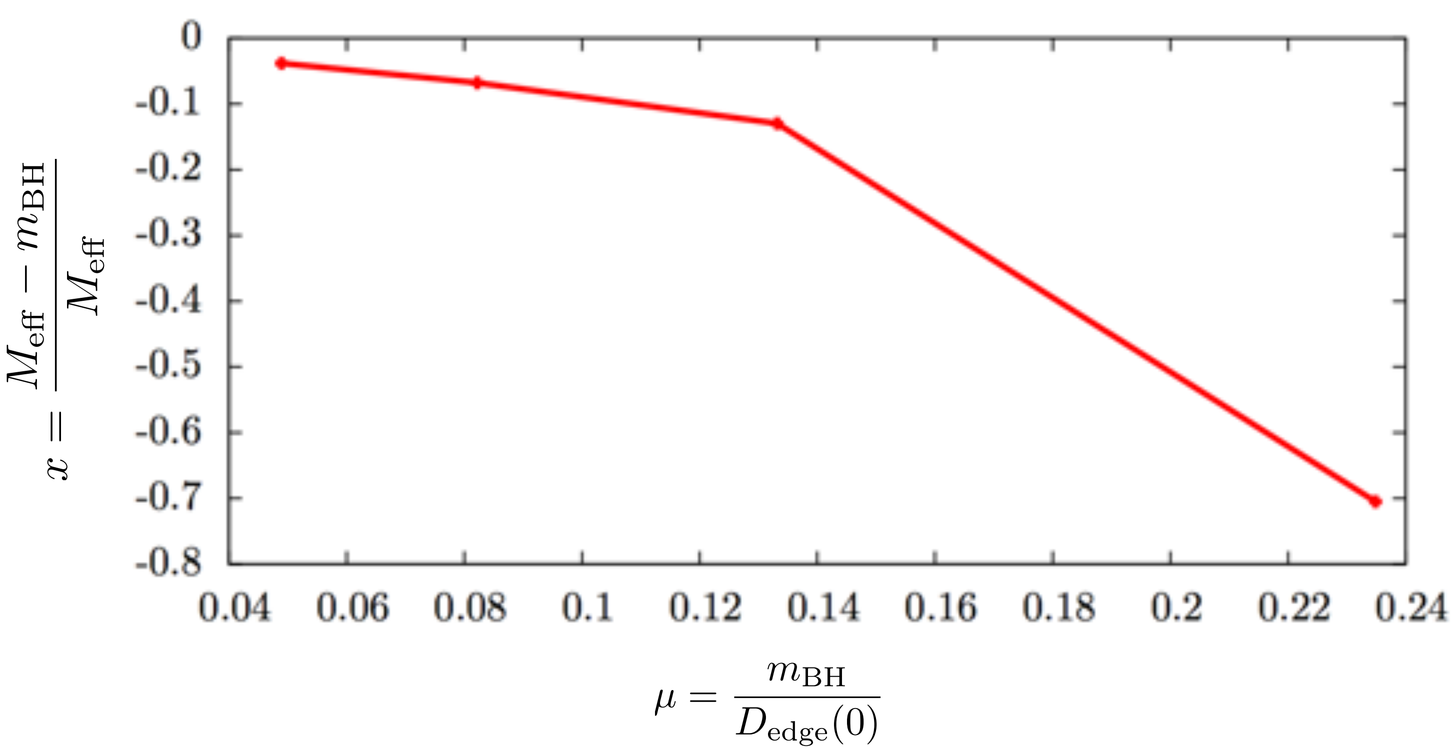


[Bentivegna & Korzyński 2013]

Black-hole lattices

Other properties, however, can be substantially different. In particular, mapping the BH lattices to the FLRW class via their geometric properties leads to counterparts with much larger effective densities **[Bentivegna & Korzyński 2012, 2013]**:

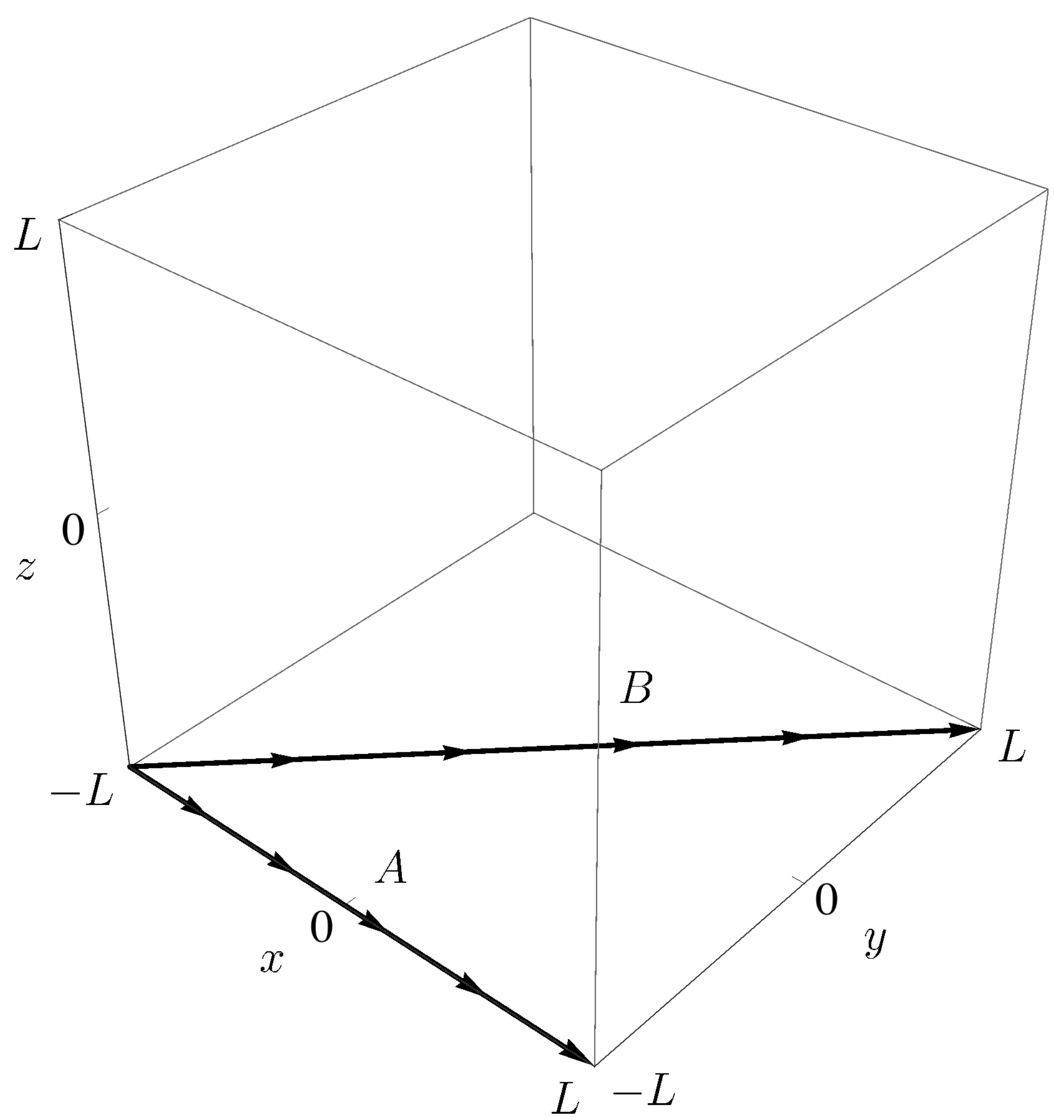
$$M_{\rm eff} = \rho_{\rm eff} 2\pi^2 a_{\rm eff}^3 = 378.78, \quad M_{8\rm BH} = 8 M_{\rm ADM} = 303.53$$



One must choose which mapping to use (fitting problem **non trivial**). Fitting one observable leads to a degradation in the quality of fit to the others.

Black-hole lattices

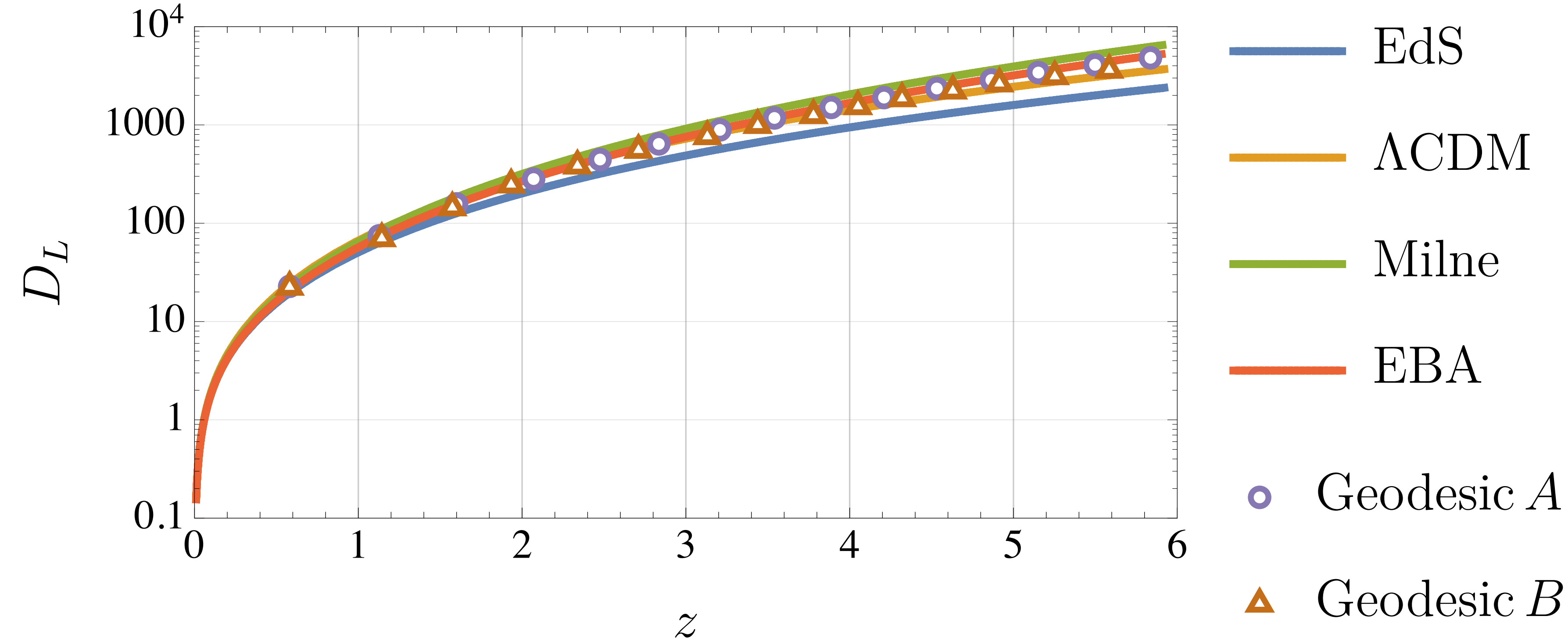
Tracing light helps probe higher-order effects, and is a key element in building cosmological observables.

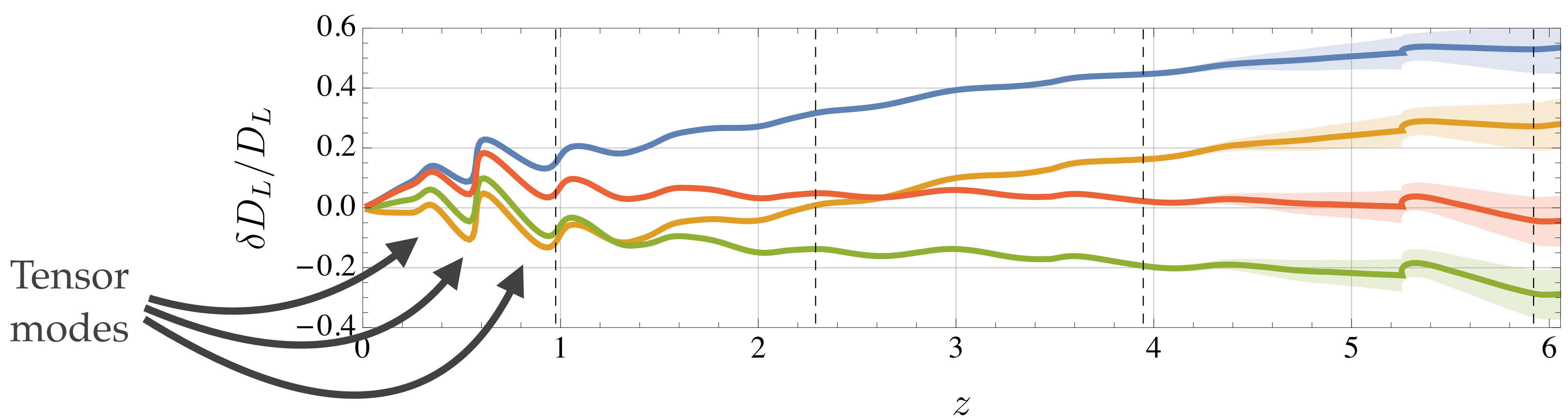


[Bentivegna, Korzyński, Hinder & Gerlicher, arXiv:1611.09275]

- Light propagation through a BHL remains close to the prescription of the empty-beam approximation;
- Decreasing $\mu = M/L$, there is a O(1) difference in the luminosity distance;
- This difference can be mimicked by a negative-pressure fluid.

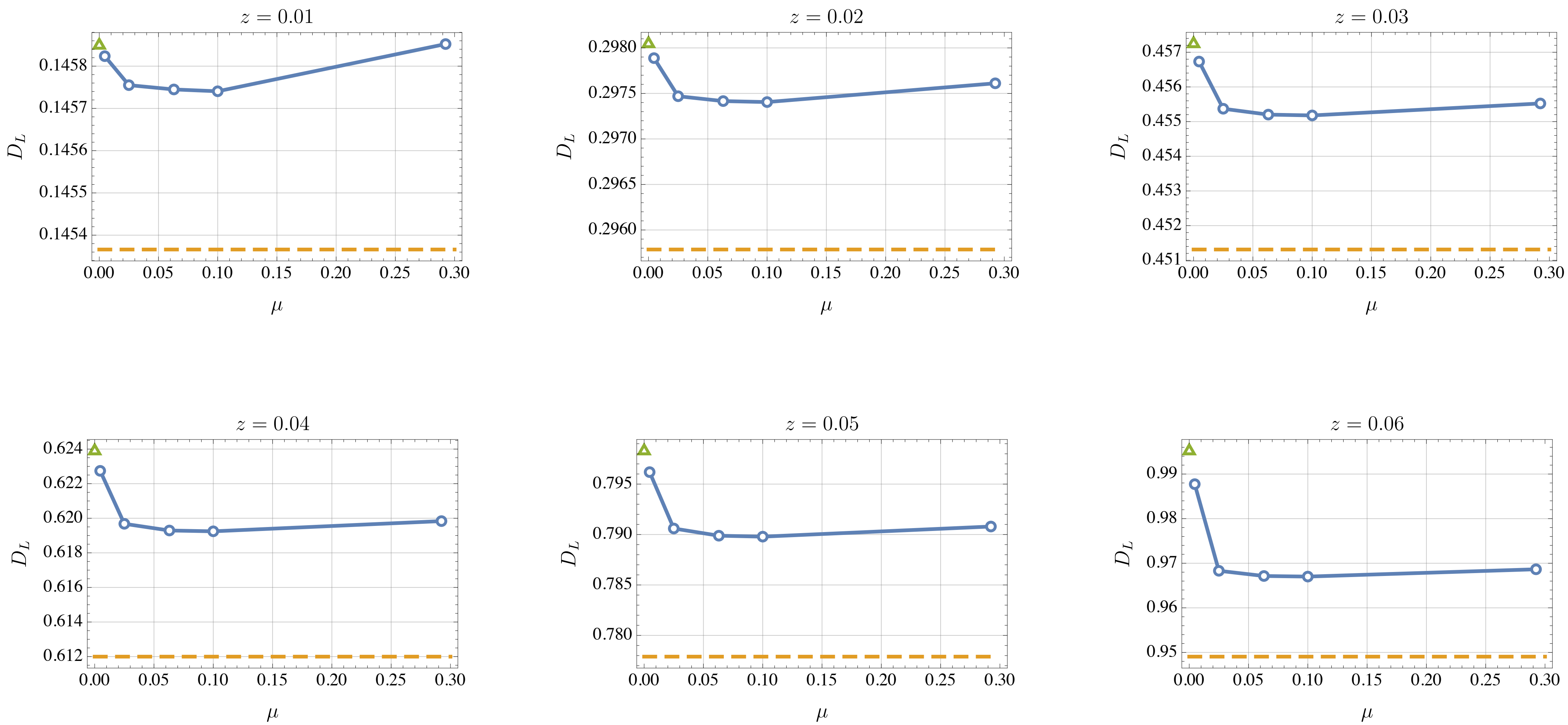
Black-hole lattices



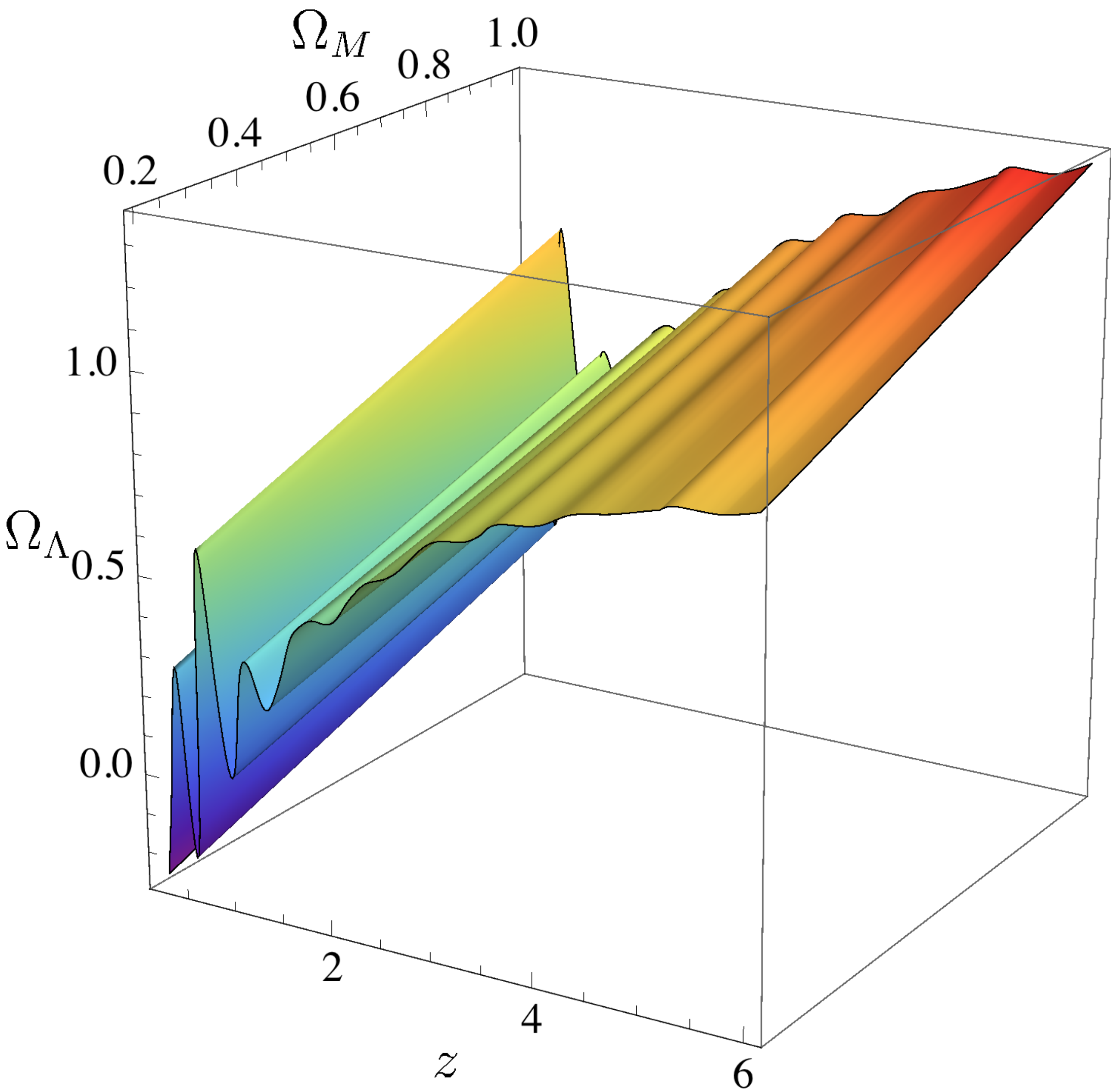


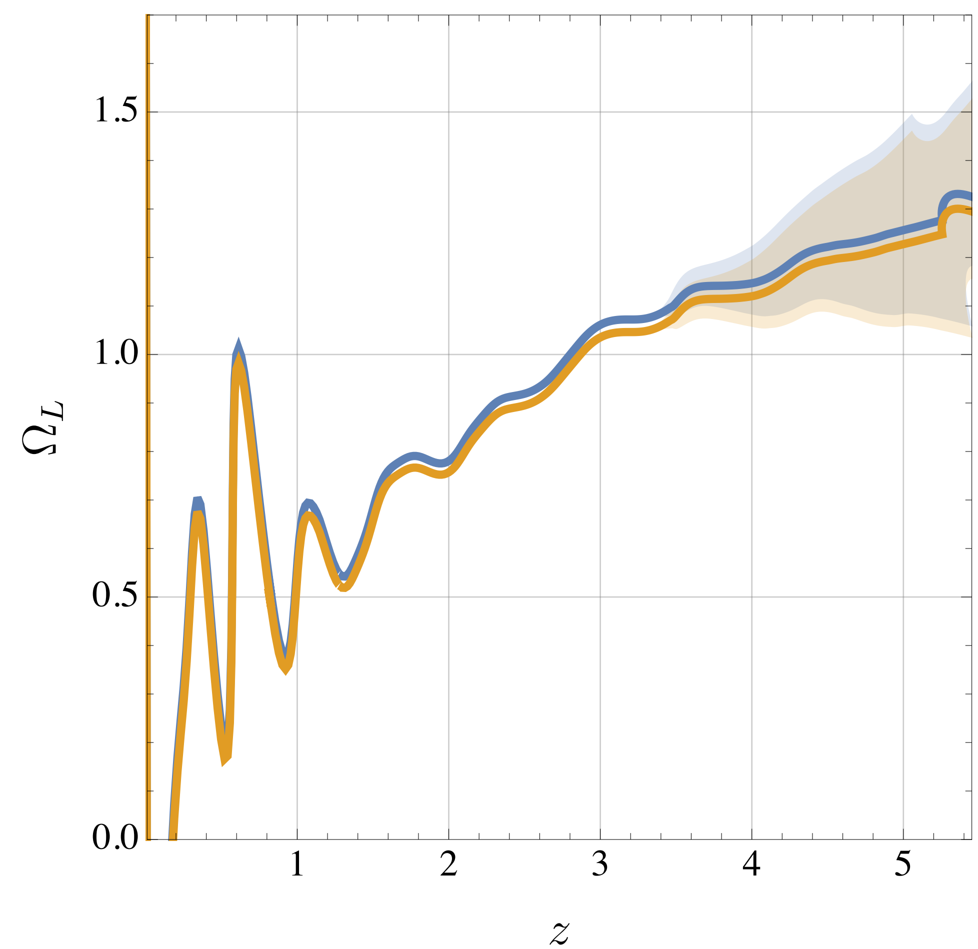
Black-hole lattices

M	L	L_{prop}	μ
0.010	2.15	2.73	0.0046
0.125	5.00	6.28	0.0250
0.500	7.94	9.84	0.0630
1.000	10.00	12.26	0.1000
5.000	17.10	21.77	0.2924



Black-hole lattices

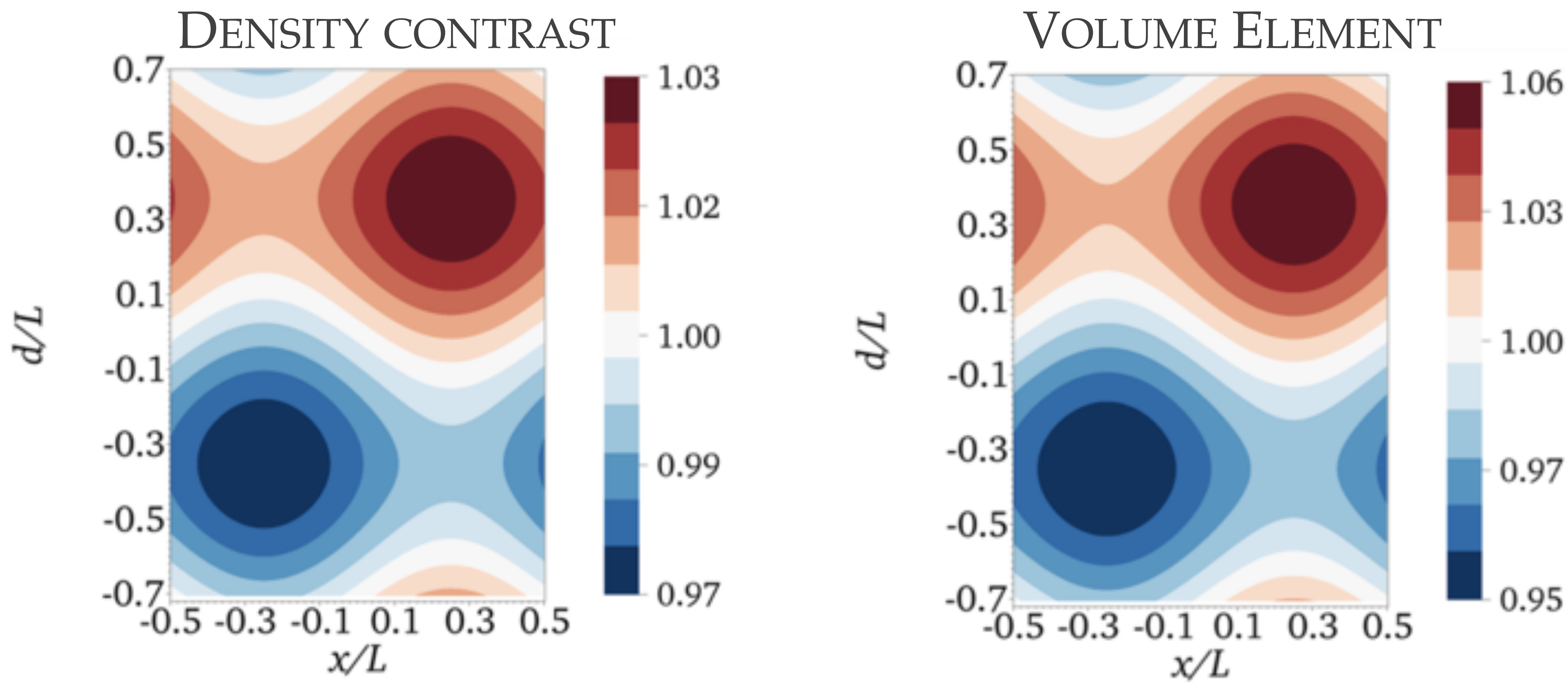




Dust cosmologies

How universal is this result?

Same procedure in non-vacuum spacetimes, same ID restriction **[Anninos 1999, Giblin, Mertens & Starkman 2015, Bentivegna&Bruni, 2015]**:



Approach much more similar to standard cosmological treatments of perturbed fluids. Many analytical approximations available in various regimes.

Dust cosmologies

Perturbation theory around a homogeneous and isotropic background:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

In the background:

$$\gamma_{ij} = a^2\delta_{ij}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\bar{\rho} \qquad a(t) = a_i\left(\frac{t}{t_i}\right)^{2/3}$$

In the perturbed spacetime:

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

$$\delta'' + \frac{3}{2a}\delta' - \frac{3}{2a^2}\delta = 0$$

$$\delta(t) = \delta_+ a(t) + \delta_- a(t)^{-3/2}$$

The **averages** satisfy equations similar to those that hold in FLRW models, but with an extra contribution due to inhomogeneities:

$$V_{\mathcal{D}}(t) = \int_{\mathcal{D}} \sqrt{\gamma}\,\mathrm{d}^3x$$

$$a_{\mathcal{D}} = \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_0}}\right)^{1/3}$$

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi}{3}\frac{M_{\mathcal{D}}}{a_{\mathcal{D}}^3} + \frac{\mathcal{Q}_{\mathcal{D}}}{3}$$

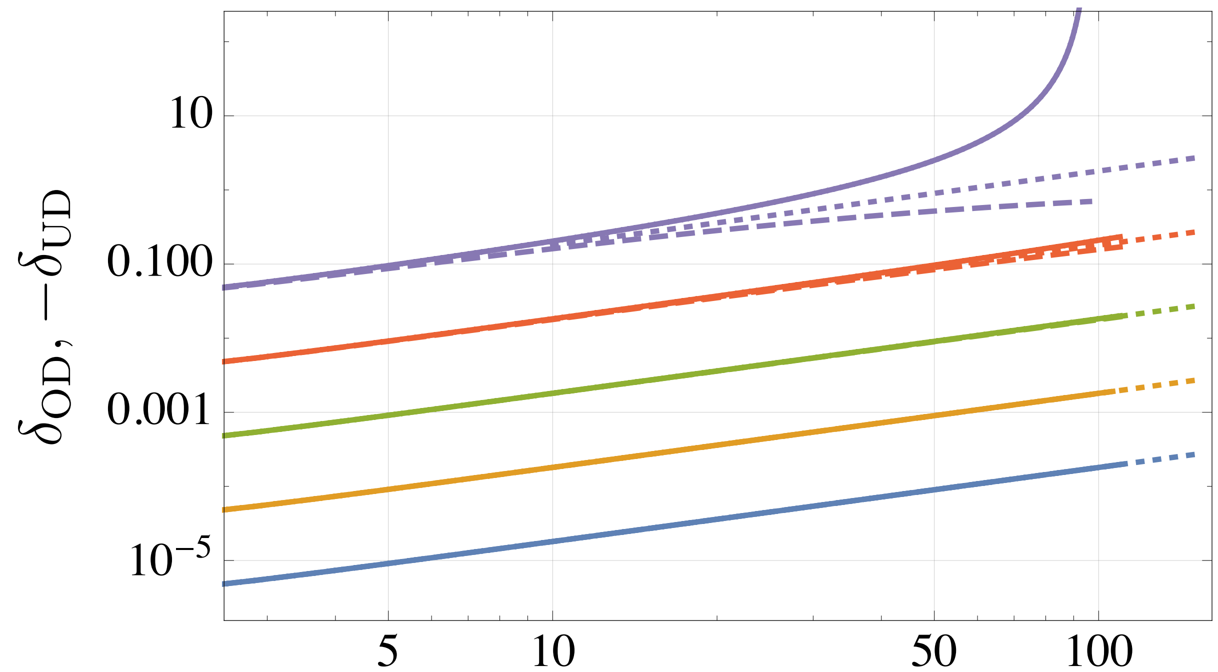
$$\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left(\langle K^2\rangle_{\mathcal{D}} - \langle K\rangle_{\mathcal{D}}^2\right) - 2\langle A^2\rangle_{\mathcal{D}}$$

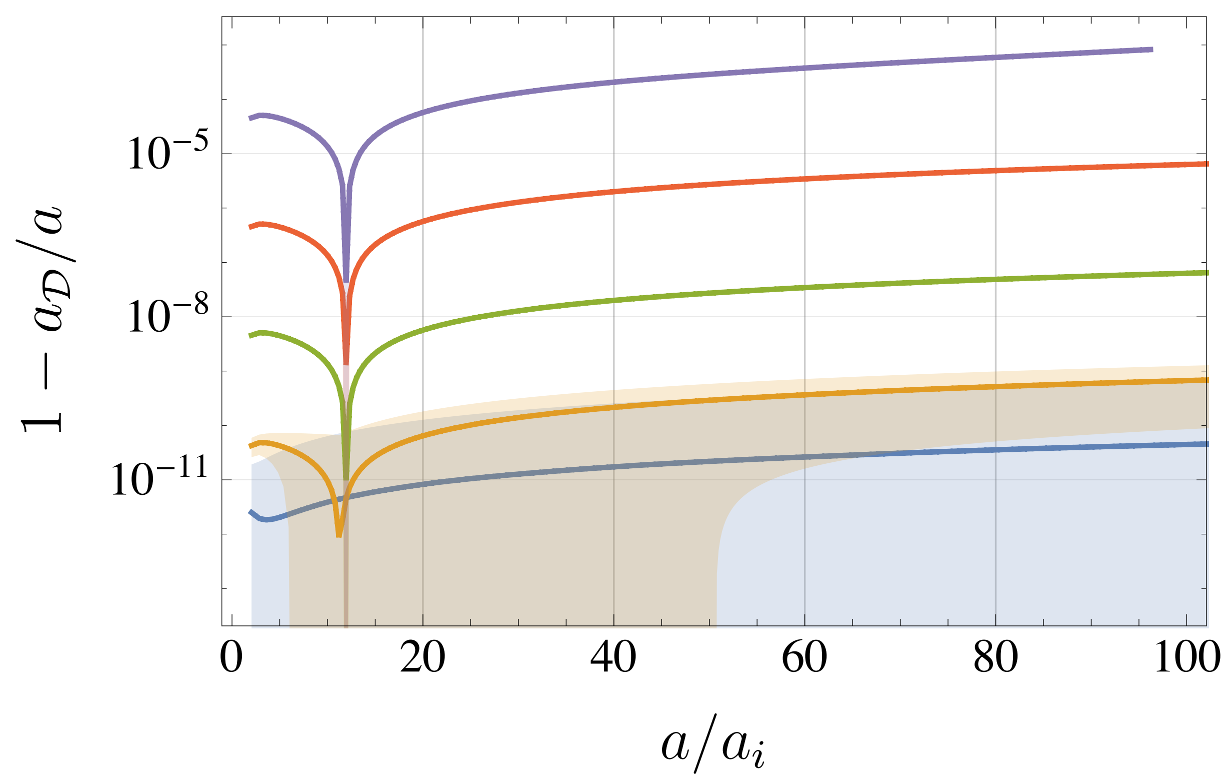
$$\mathcal{Q}_{\mathcal{D}} \sim a_{\mathcal{D}}^{-2}$$

Dust cosmologies

- Start at *z=100*, evolution of initial data with five initial perturbation amplitude: two largest depart from first-order perturbation theory by *z=0*.

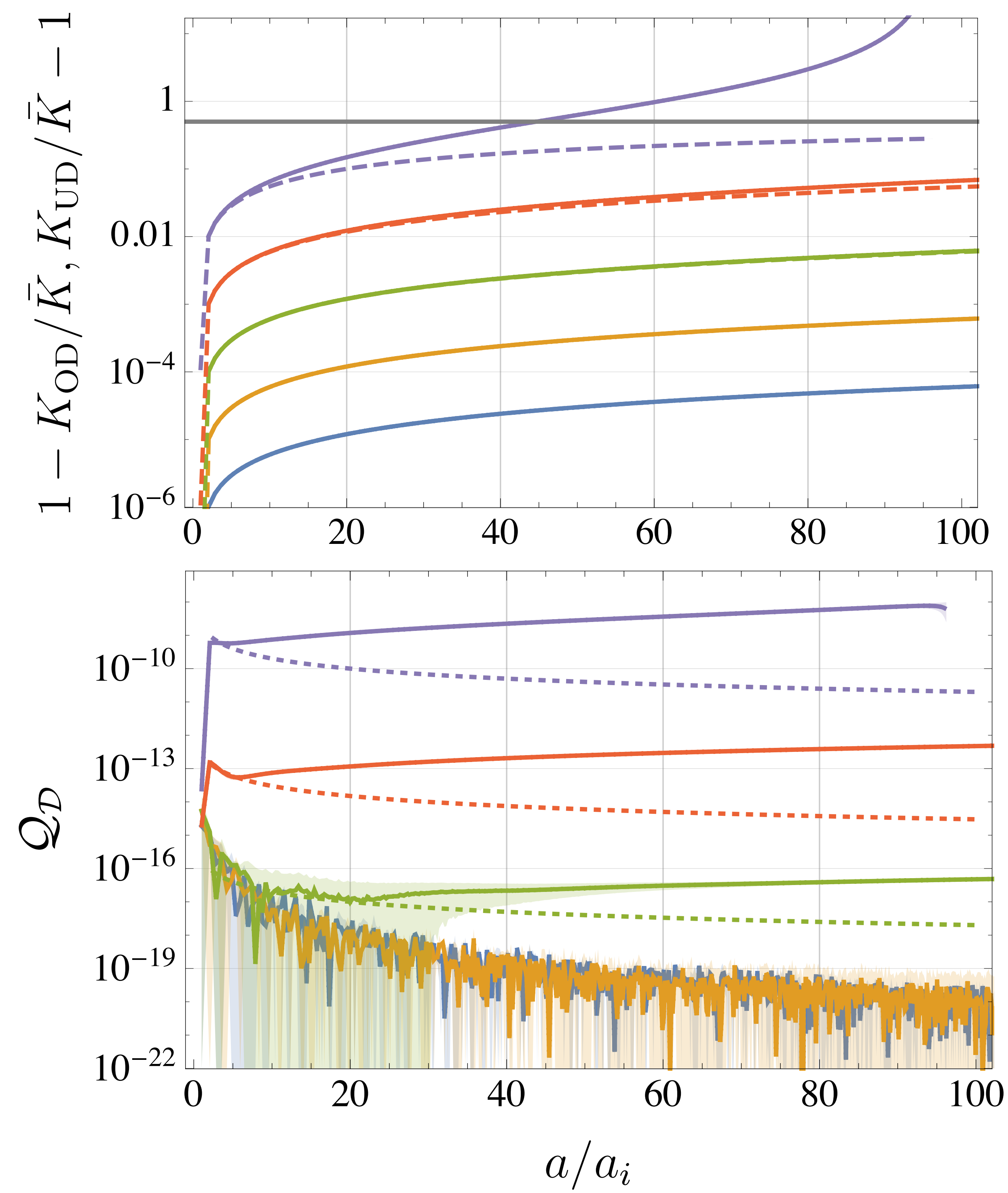
- Average expansion remains close to FLRW solution: departure at most 0.1%.





Dust cosmologies

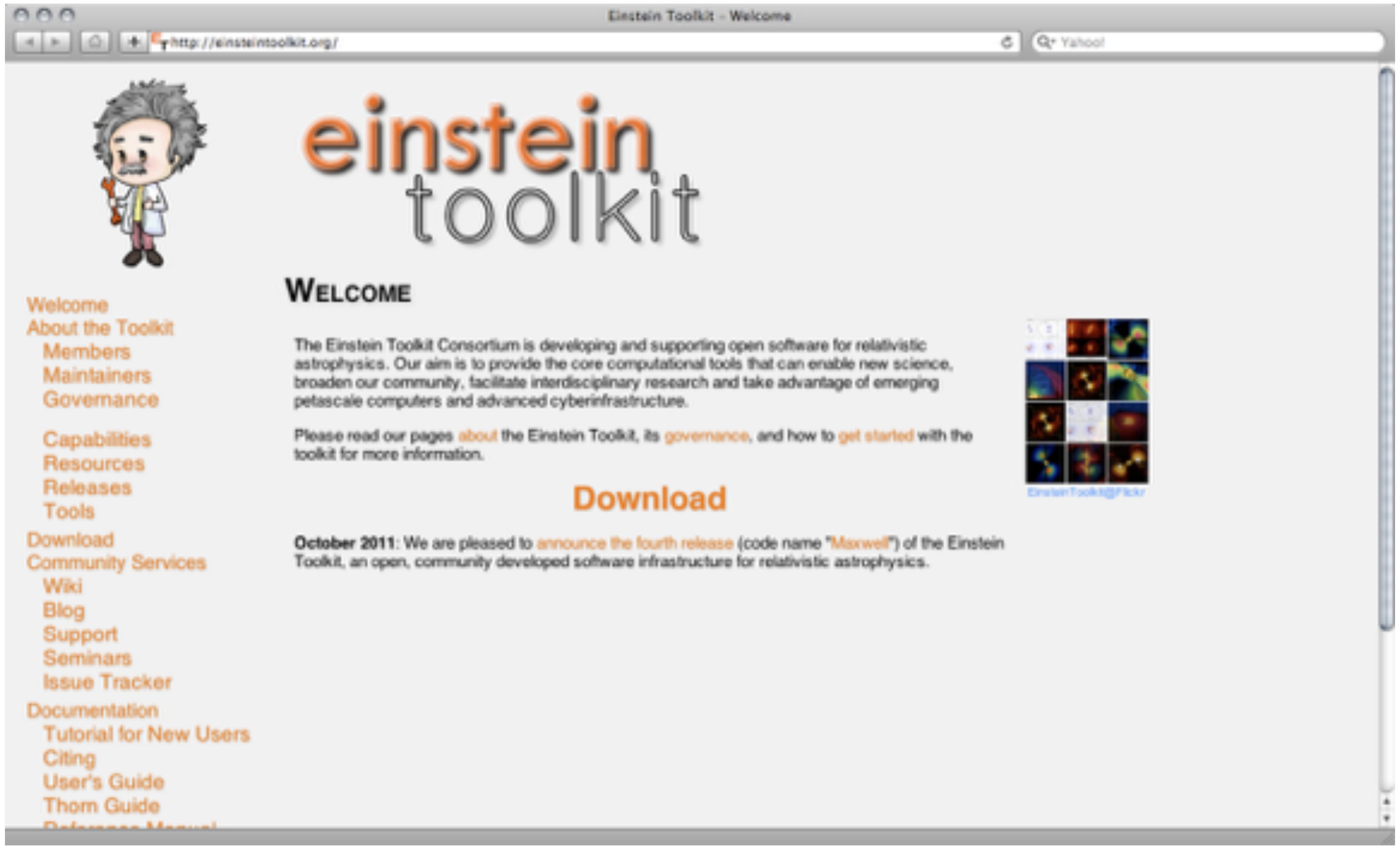
- Local expansion can exhibit departures of order ~30%
- Collapse occurs faster than spherical models (e.g., top-hat)
- Backreaction function *Q* is extremely small, scales like a^{-2} for small density contrasts



Code infrastructure

The Einstein Toolkit:

- Open-source toolkit;
- One code-generating framework;
- Over one hundred components (evolution of the gravitational field and fluids, analysis of spacetimes, I/O);
- AMR capabilities;
- Leveraging HPC systems worldwide;
- Tutorials and demos for new users — **try it out**!



einsteintoolkit.org

Code infrastructure

Einstein Toolkit:

- 3+1 BSSN code (McLachlan);
- ADM analysis module;
- Apparent-horizon finder;
- Utilities (I/O, visualization, simulation management).

Not in the Toolkit:

- Cosmological ID solver **[arXiv:1305.5576]**;
- Dust evolution **[arXiv:1610.05198]**;
- Cosmological analysis modules (backreaction);

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- Ray tracing module **[arXiv:1611.09275]**

Open Issues

- Initial-data prescriptions
  - Represent physically "reasonable" class of observers
  - Provide easy control over physical content of prescription
- Dynamical coordinates
  - Must represent physical prescription of observers at all times
  - Must lead to well-behaved simulations
- Efficient use of numerical resolution
  - Better AMR algorithms

Summary

- 1 -

A consistently relativistic model of the large-scale Universe is necessary, and feasible.

- 2 -

Early results are intriguing:

Universal initial-data no-go

Length scaling follows FLRW class

Light propagation probes higher-order effects