Weak field and full GR cosmological simulations

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Inhomogeneous Cosmologies, Torun 3/07/17

Outline

- standard ΛCDM cosmology and a basic question
- non-linear Post-Friedmann ΛCDM: a weak-field/post-Newtonian type approximation scheme for cosmology
- cosmological frame dragging from Newtonian N-body simulations
- full Numerical Relativity cosmological simulations

Credits: first part

- Irene Milillo, Daniele Bertacca, MB and Andrea Maselli, *The missing link: a nonlinear post-Friedmann framework for small and large scales* [arXiv: 1502.02985], Physical Review D, **92**, 023519 (2015)
- MB, Dan B. Thomas and David Wands, *Computing General Relativistic effects from Newtonian N-body simulations: Frame dragging in the post-Friedmann approach,* Physical Review D, **89**, (2014) 044010 [arXiv:1306.1562]
- Dan B. Thomas, MB, Kazuya Koyama, Baojiu Li and Gong-bo Zhao *f(R) gravity on non-linear scales: The post-Friedmann expansion and the vector potential,* JCAP, 1507 (2015) 07, 051 [arXiv:1503.07204]
- C. Rumpf, E. Villa, D. Bertacca and M. Bruni, *Lagrangian theory for cosmic structure for- mation with vorticity: Newtonian and post-Friedmann approximations*, Phys. Rev. D **94** (2016) 083515 [arXiv:1607.05226]
- A. Maselli, B. Bruni & D. Thomas, Interacting vacuum-energy in a Post-Friedmann expanding Universe (to be submitted)

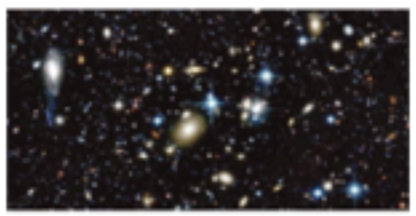
Credits: second part

Featured in Physics | Editors' Suggestion

Departures from the Friedmann-Lemaitre-Robertston-Walker Cosmological Model in an Inhomogeneous Universe: A Numerical Examination

John T. Giblin, Jr., James B. Mertens, and Glenn D. Starkman

Phys. Rev. Lett. **116**, 251301 (2016) – Published 24 June 2016



Cosmologists have begun using fully relativistic models to understand the effects of inhomogeneous matter distribution on the evolution of the Universe.

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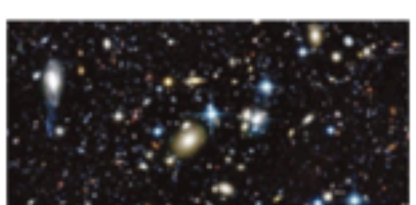
Effects of Nonlinear Inhomogeneity on the Cosmic Expansion with Numerical Relativity

Eloisa Bentivegna and Marco Bruni

Phys. Rev. Lett. **116**, 251302 (2016) – Published 24 June 2016



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Show Abstract +

Standard ΛCDM Cosmology

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- Recipe for modeling based on 3 main ingredients:

1. Homogeneous isotropic background, FLRW models
2. Relativistic Perturbations (e.g. CMB), good for large scales I-order, II order, gradient expansion
3. Newtonian study of non-linear structure formation (N-body simulations or approx. techniques, e.g. 2LPT) at small scales

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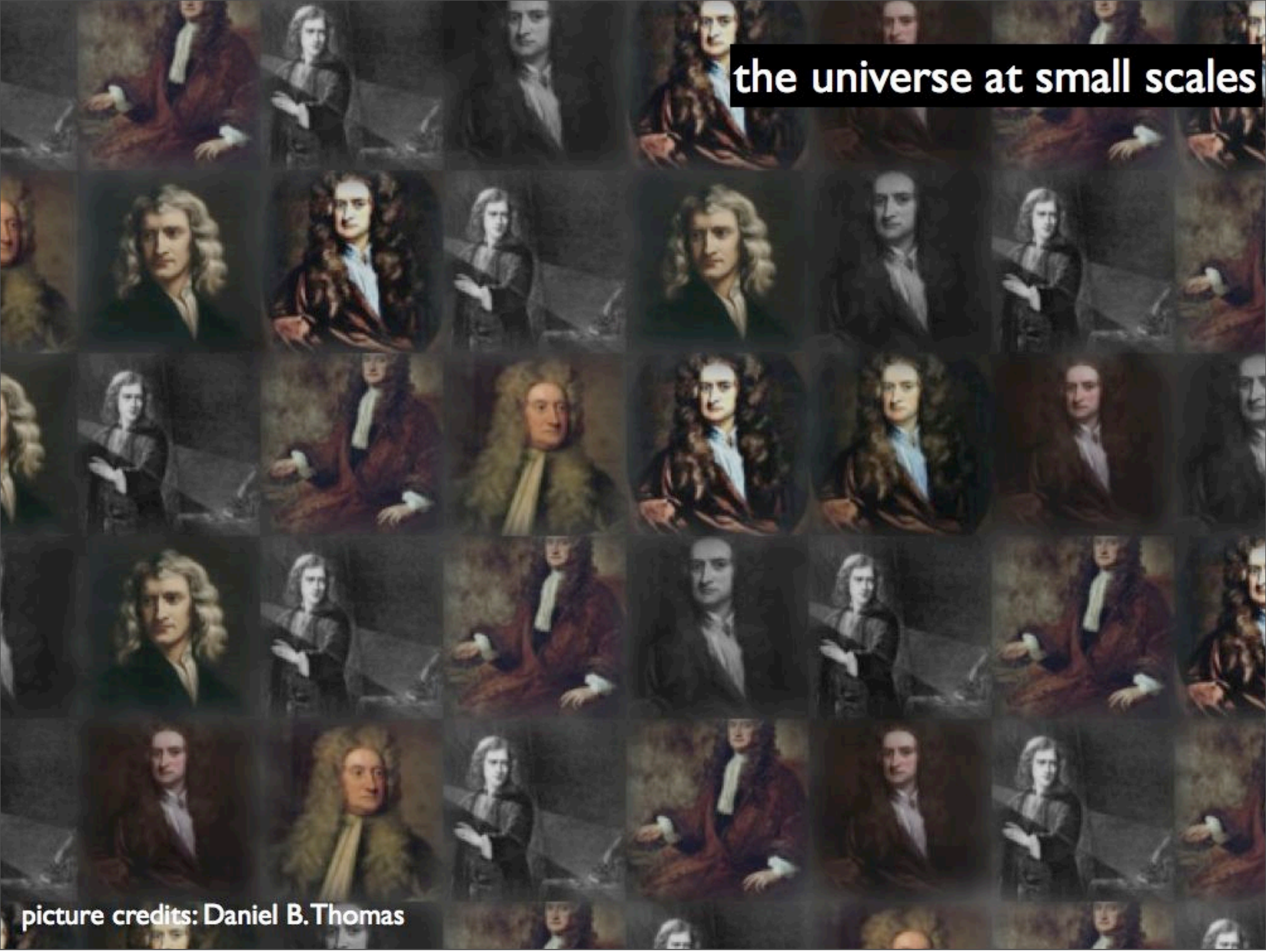
- on this basis, well supported by observations, the flat ΛCDM model has emerged as the Standard "Concordance" Model of cosmology.


the universe at very large scales: GR
picture credits: Daniel B. Thomas


the universe at small scales
picture credits: Daniel B. Thomas

Questions on ΛCDM

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FRW models
  2. Relativistic Perturbations (e.g. CMB; linear, nonlinear)
  3. Newtonian study of non-linear structure formation (numerical simulations or approx. techniques)
- Is 3 enough? (more data, precision cosmology, observations and simulations covering large fraction of H^{-1}, etc...)
  - We need to bridge the gap between 2 and 3

Accuracy versus Precision

Accuracy describes how close a measurement is to the true value. **Precision** describes how reproducible a measurement is. These are distinct and relate to different types of errors.

Accurate but not precise.

Large random errors, but on average correct.

Precise but not accurate.

Large systematic error (biased.)

Errors can be random or systematic (reproducible but wrong).

Systematic errors could arise from incorrect calibration of measurement apparatus, or incorrect assumptions.

Ideally measurements should be both accurate and precise!

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- N-body simulations for Euclid aim at 1% precision
- we should be equally accurate: target is GR

nonlinear post-Friedmann framework

nonlinear post-Friedmann framework

- GR, flat ΛCDM background
- fully non-linear density field
- post-F: weak-field + small peculiar velocities
  - start with a post-Minkowski (weak field) approach on a FLRW background, Hubble flow is not slow but peculiar velocities are small
- post-F: non-linear framework including both Newtonian regime and first-order GR perturbations

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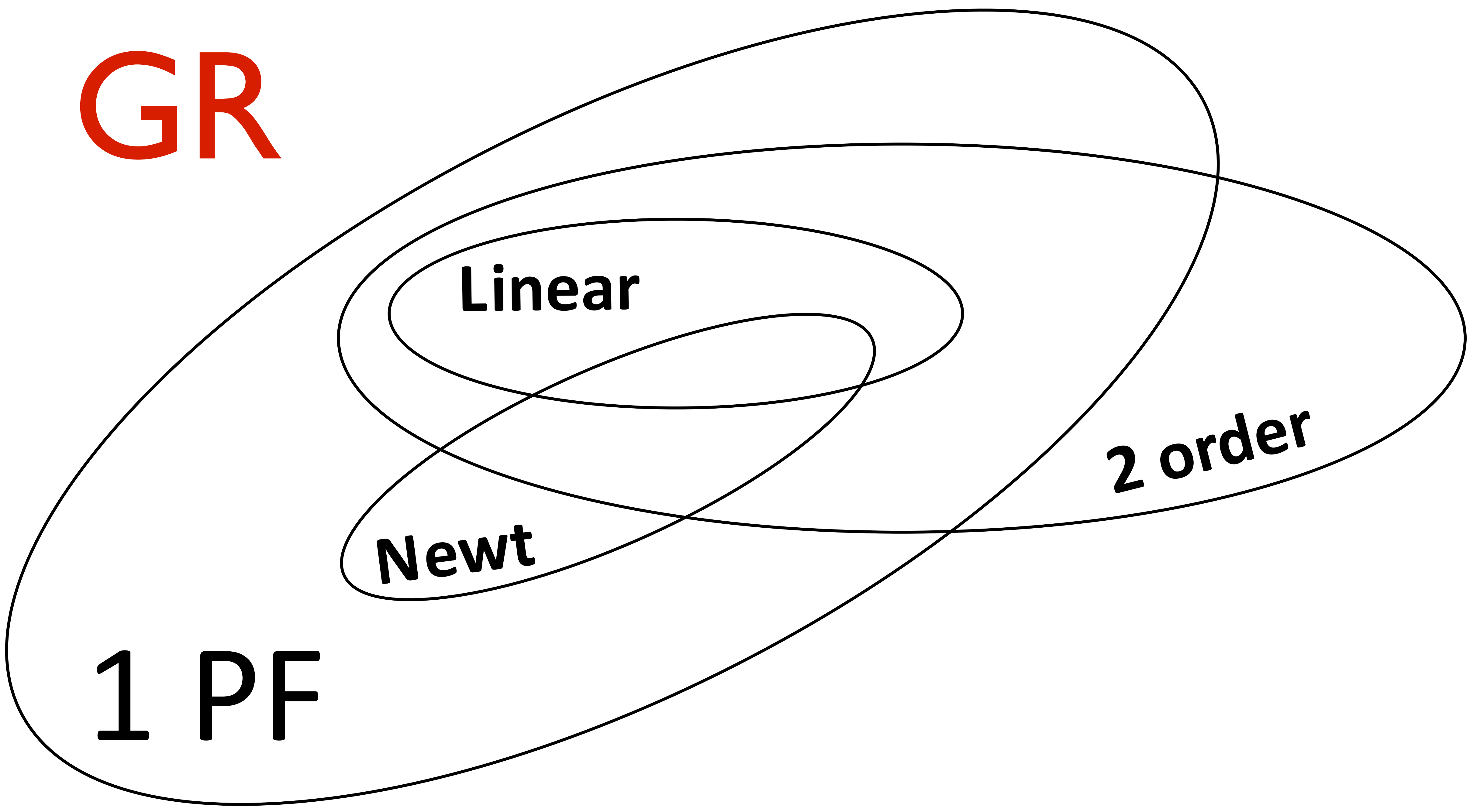
post-Friedmann framework

- spaces of equations (not solutions!)



metric and matter I

starting point: the 1-PN cosmological metric
(cf. Chandrasekhar 1965)

$$g_{00} = -\left[1 - \frac{2U_N}{c^2} + \frac{1}{c^4}(2U_N^2 - 4U_P)\right] + O\left(\frac{1}{c^6}\right),$$

$$g_{0i} = -\frac{a}{c^3}B_i^N - \frac{a}{c^5}B_i^P + O\left(\frac{1}{c^7}\right),$$

$$g_{ij} = a^2\left[\left(1 + \frac{2V_N}{c^2} + \frac{1}{c^4}(2V_N^2 + 4V_P)\right)\delta_{ij} + \frac{1}{c^4}h_{ij}\right] + O\left(\frac{1}{c^6}\right),$$

we assume a Newtonian-Poisson gauge: B_i is solenoidal and h_{ij} is TT, at each order 2 scalar DoF in g_{00} and g_{ij}, 2 vector DoF in frame dragging potential B_i and 2 TT DoF in h_{ij} (not GW!)

Newtonian ΛCDM, with a bonus

- insert leading order terms in E.M. conservation and Einstein equations
- subtract the background, getting usual Friedmann equations
- introduce usual density contrast by $\rho=\rho_b(1+\delta)$

from E.M. conservation:
Continuity & Euler equations

$$\dot{\delta} + \frac{v^i \delta_{,i}}{a} + \frac{v^i{}_{,i}}{a}(\delta + 1) = 0\,,$$
$$\dot{v}_i + \frac{v^j v_{i,j}}{a} + \frac{\dot{a}}{a} v_i = \frac{1}{a} U_{N,i}\,.$$

Poisson

$$G^0{}_0 + \Lambda = \frac{8\pi G}{c^4} T^0{}_0 \;\rightarrow\; \frac{1}{c^2}\frac{1}{a^2}\nabla^2 V_N = -\frac{4\pi G}{c^2}\bar{\rho}\delta$$

Newtonian ΛCDM, with a bonus

what do we get from the ij and 0i Einstein equations?

$$\text{trace of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4} T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{2}{a^2}\nabla^2(V_N - U_N) = 0\,,$$

zero "Slip"

$$\text{traceless part of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4} T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{1}{a^2}\left[(V_N - U_N)_{,i}{}^{,j} - \frac{1}{3}\nabla^2(V_N - U_N)\delta^j_i\right] = 0$$

bonus

$$G^0{}_i = \frac{8\pi G}{c^4} T^0{}_i \;\rightarrow\; \frac{1}{c^3}\left[-\frac{1}{2a^2}\nabla^2 B^N_i + 2\frac{\dot a}{a^2} U_{N,i} + \frac{2}{a}\dot V_{N,i}\right] = \frac{8\pi G}{c^3}\bar\rho(1+\delta)v_i$$

- Newtonian dynamics at leading order, with a bonus: the frame dragging potential B_i is not dynamical at this order, but cannot be set to zero: doing so would forces a constraint on Newtonian dynamics
- result entirely consistent with vector relativistic perturbation theory
- in a relativistic framework, gravitomagnetic effects cannot be set to zero even in the Newtonian regime, cf. Kofman & Pogosyan (1995), ApJ 442:

magnetic Weyl tensor at leading order

$$H_{ij} = \frac{1}{2c^3}\left[B^N_{\mu,\nu(i}\varepsilon_{j)}{}^{\mu\nu} + 2v_\mu (U_N + V_N)_{,\nu(i}\varepsilon_{j)}{}^{\mu\nu}\right]$$

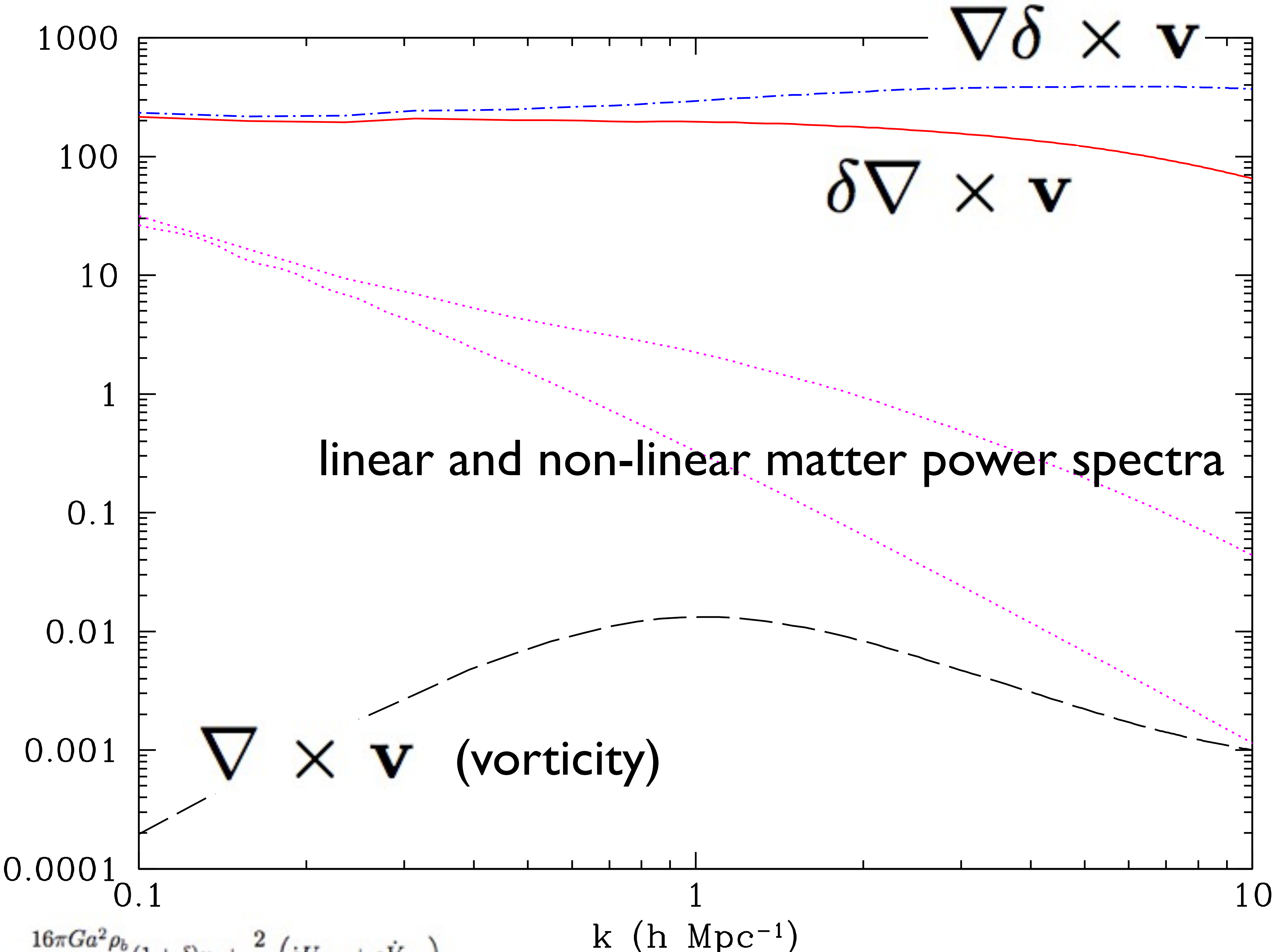
nonlinear post-Friedmann framework: applications

frame-dragging potential from N-body simulations

- Simulations at leading order in the post-Friedmann expansion
- dynamics is Newtonian, but a frame-dragging vector potential is sourced by the vector part of the Newtonian energy current

$$\nabla \times \nabla^2 \mathbf{B}^N = -(16\pi G\bar{\rho} a^2)\nabla \times [(1+\delta)\mathbf{v}].$$

power spectra: sources

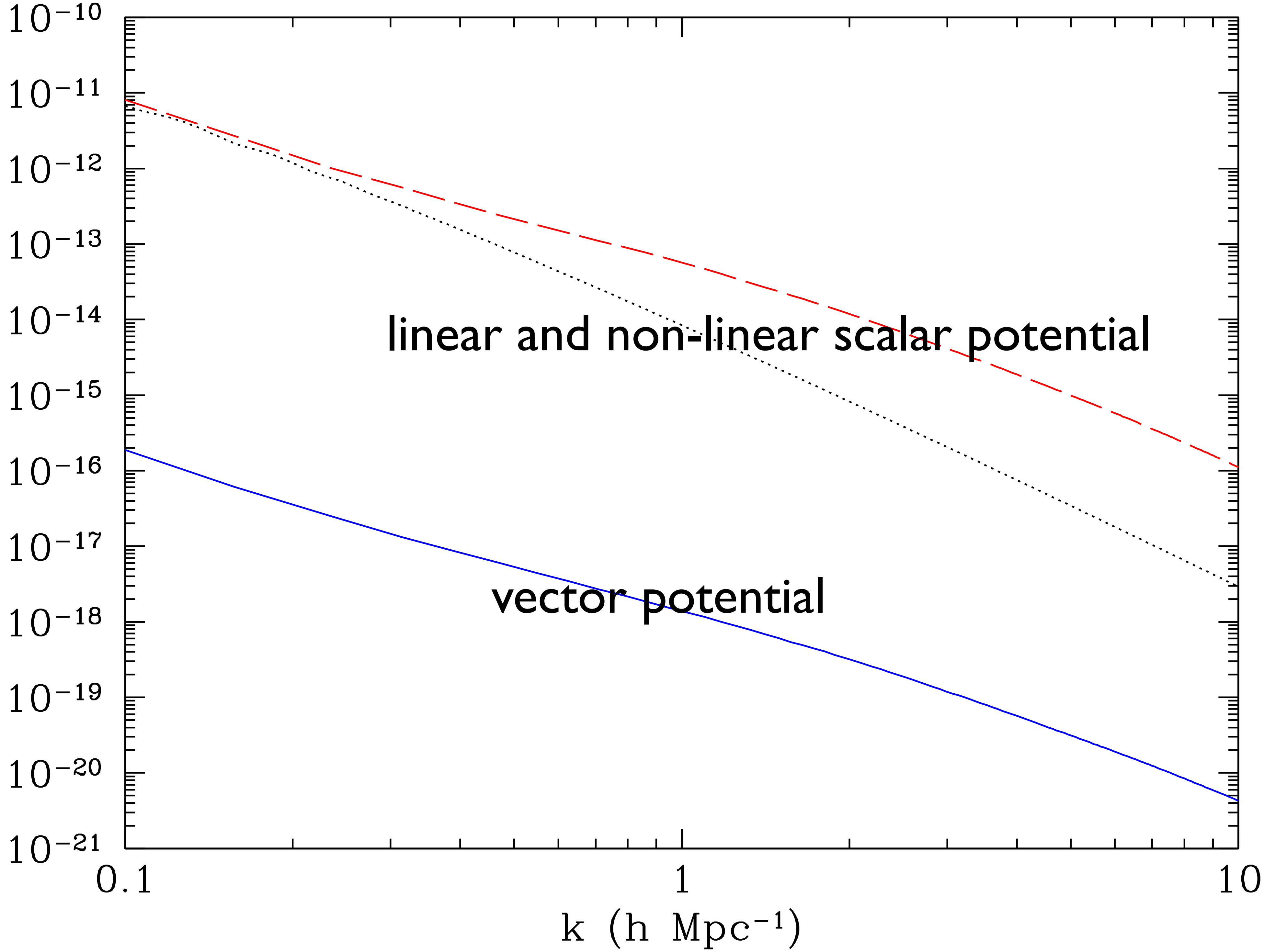


$$\frac{1}{c^3}\nabla^2 P_i^N = -\frac{16\pi G a^2 \rho_b}{c^3}(1+\delta)v_i + \frac{2}{c^3}\left(\dot{a}U_{N,i} + a\dot{V}_{N,i}\right)$$

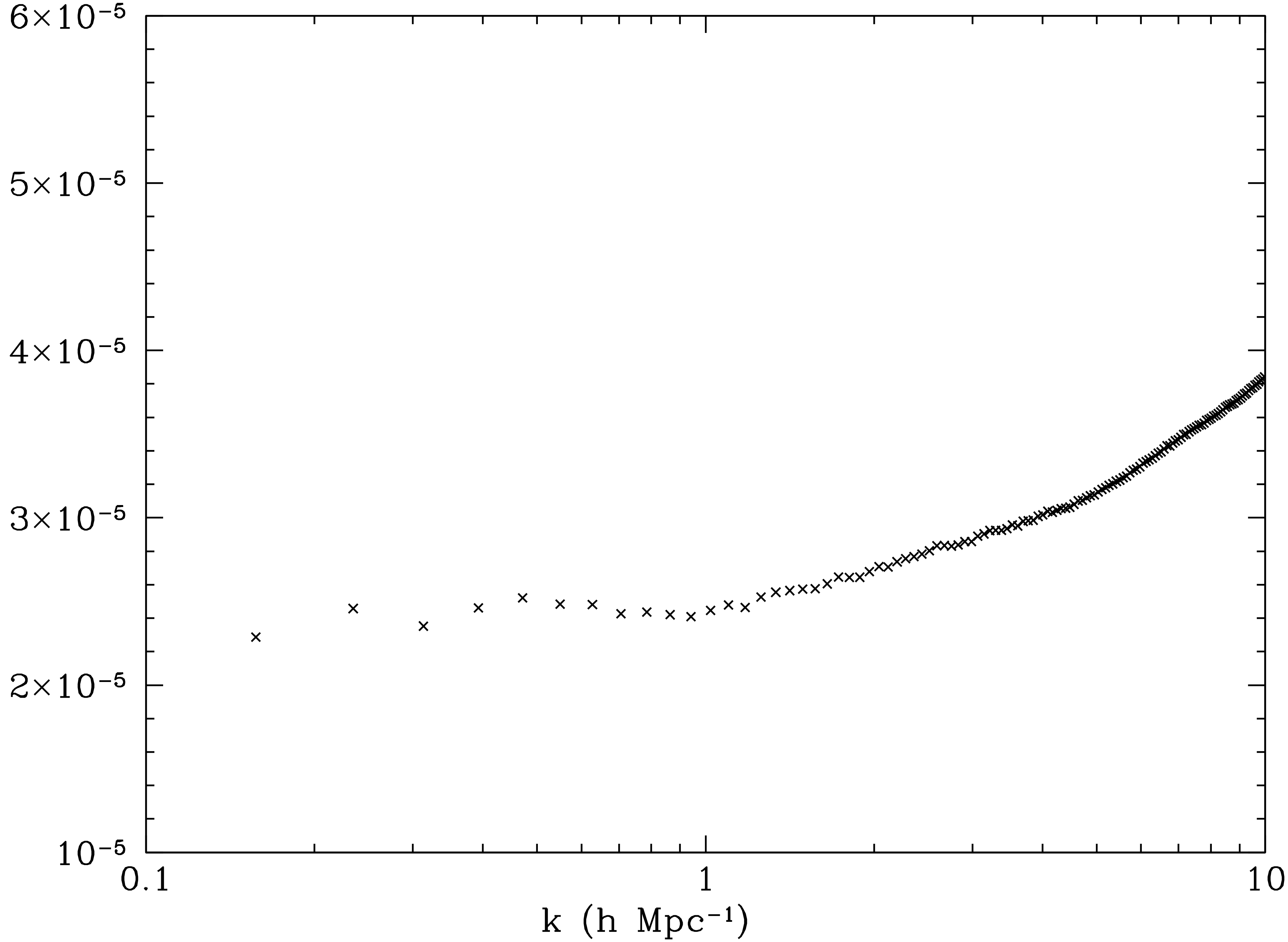
$$\nabla\times\nabla^2\vec{P}^N = -\left(16\pi G\rho_b a^2\right)\nabla\times[(1+\delta)\vec{v}]$$

$$\nabla\times[(1+\delta)\vec{v}] = (\nabla\delta)\times\vec{v} + (1+\delta)\nabla\times\vec{v}$$

scalar and vector potentials



ratio of the potentials



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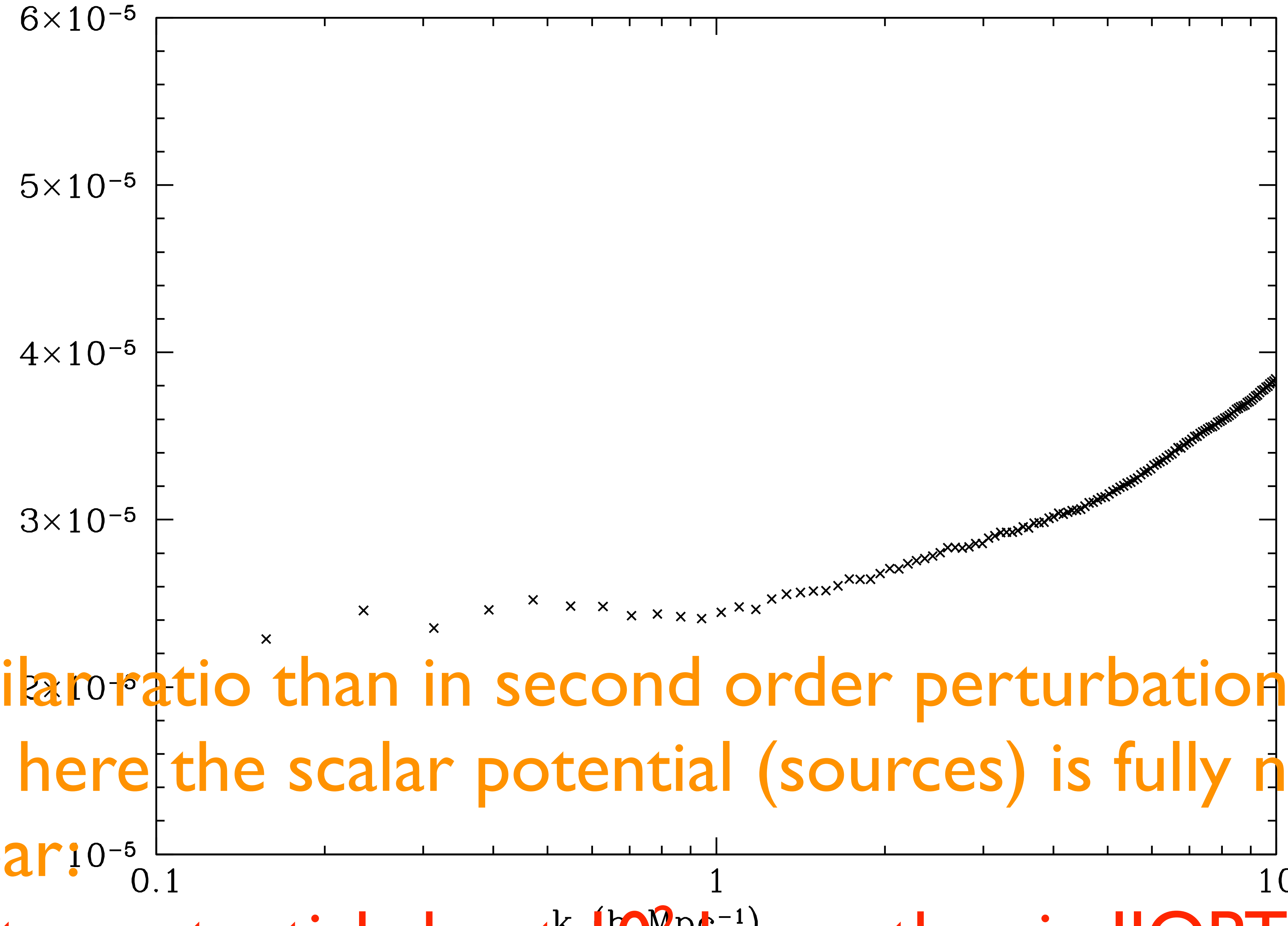

similar ratio than in second order perturbation theory but here the scalar potential (sources) is fully non-linear:

vector potential about 10^2 larger than in IIOPT

cf. Lu, Ananda, Clarkson & Maartens (2009)

Full GR
Numerical Relativity
Simulations

Eloisa Bentivegna & MB, PRL 116, 251302 (2016)

cf. J.T. Giblin Jr., J.B. Mertens & G.D. Starkman, PRL 2016, 251301 (2016) back to basic...

Newtonian Cosmology

1. Newtonian self-gravitating fluid: described by the continuity, Euler and Poisson equations

2. rescale physical coordinates to comoving coordinates $\dot{\vec{r}} = H\vec{r} + a\vec{v}$

dust: p=0

$$\frac{d\delta}{dt} + \frac{\vec{\nabla}\cdot\vec{v}}{a}(1+\delta)$$

$$\frac{d\vec{v}}{dt} + \frac{\dot{a}}{a}\vec{v} = -\vec{\nabla}\phi$$

$$\nabla^2\phi = 4\pi G\rho_b\delta$$

note: convective time derivative

Linear perturbations

- for dust, linearise, combine continuity and Euler, substitute from Poisson, to get

$$\delta'' + \frac{3}{2a}\delta' - \frac{3}{2a^2}\delta = 0\,,$$

- In GR, for a w=constant fluid, use energy and momentum conservation equations, and the Energy constraint, to get (Δ gauge-invariant)

$$\Delta'' + \frac{3}{2S}(1-3w)\Delta' + \frac{3}{2S^2}(3w^2-2w-1)\Delta - \frac{wD^2\Delta}{H_0^2\Omega_0}S^{1+3w} = 0$$

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Solution in EdS and top-hat

$$a(t) = a_i \left(\frac{t}{t_i}\right)^{2/3} ,$$

$$\delta(t) = \delta_+ a(t) + \delta_- a(t)^{-3/2}$$

- top-hat turnaround and collapse time: characterized by the value of δ at these events:

$$\delta_T = 1.06 \quad \delta_c = 1.696$$

Buchert's averaging

- From V, we can then define the average scale factor

$$V_{\mathcal{D}} := \int_{\mathcal{D}} d^3x\sqrt{h}$$

$$a_{\mathcal{D}} \equiv (V_{\mathcal{D}}/V_{\mathcal{D}ini})^{1/3}$$

- then, the key to getting BR through averaging is the non-commutativity of the time derivative and the spatial averaging

$$\partial_t\langle\Psi\rangle_{\mathcal{D}} - \langle\partial_t\Psi\rangle_{\mathcal{D}} = \langle\Theta\Psi\rangle_{\mathcal{D}} - \langle\Theta\rangle_{\mathcal{D}}\langle\Psi\rangle_{\mathcal{D}}$$

- then, averaging the continuity equation, Hamiltonian constraints and the Raychaudhuri equation gives effective Friedmann equations

$$\dot{\langle\rho\rangle}_{\mathcal{D}} = -3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\langle\rho\rangle_{\mathcal{D}}$$

$$\begin{aligned}\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 &= \frac{8\pi G}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \langle\mathcal{R}\rangle_{\mathcal{D}}) \\ \left(\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right) &= -\frac{4\pi G}{3}\langle\rho\rangle_{\mathcal{D}} + \frac{1}{3}\mathcal{Q}_{\mathcal{D}},\end{aligned}$$

Bucherts averaging

- in the effective Friedmann equations

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \langle\mathcal{R}\rangle_{\mathcal{D}})$$

$$\left(\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right) = -\frac{4\pi G}{3}\langle\rho\rangle_{\mathcal{D}} + \frac{1}{3}\mathcal{Q}_{\mathcal{D}},$$

- the term $\langle\mathcal{R}\rangle_{\mathcal{D}}$ represents the average of the spatial Ricci scalar, while

$$\mathcal{Q}_{\mathcal{D}} \equiv \frac{2}{3}\left(\langle\Theta^2\rangle_{\mathcal{D}} - \langle\Theta\rangle_{\mathcal{D}}^2\right) - 2\langle\sigma^2\rangle_{\mathcal{D}}.$$

- is the back-reaction term, which can be positive. If this term satisfies $\mathcal{Q}_{\mathcal{D}} > 4\pi G\langle\rho\rangle_{\mathcal{D}}$ then clearly it can act as Dark Energy

Full GR
Numerical Relativity
Simulations

Eloisa Bentivegna & MB, PRL 116, 251302 (2016)

cf. J.T. Giblin Jr., J.B. Mertens & G.D. Starkman, PRL 2016, 251301 (2016)

Assumptions and procedure

- Initial conditions: a small δ 10^{-2}-10^{-6} on EdS background

$$\rho_i = \bar{\rho}_i \left(1 + \delta_i \sum_{j=1}^{3} \sin \frac{2\pi x^j}{L}\right)$$

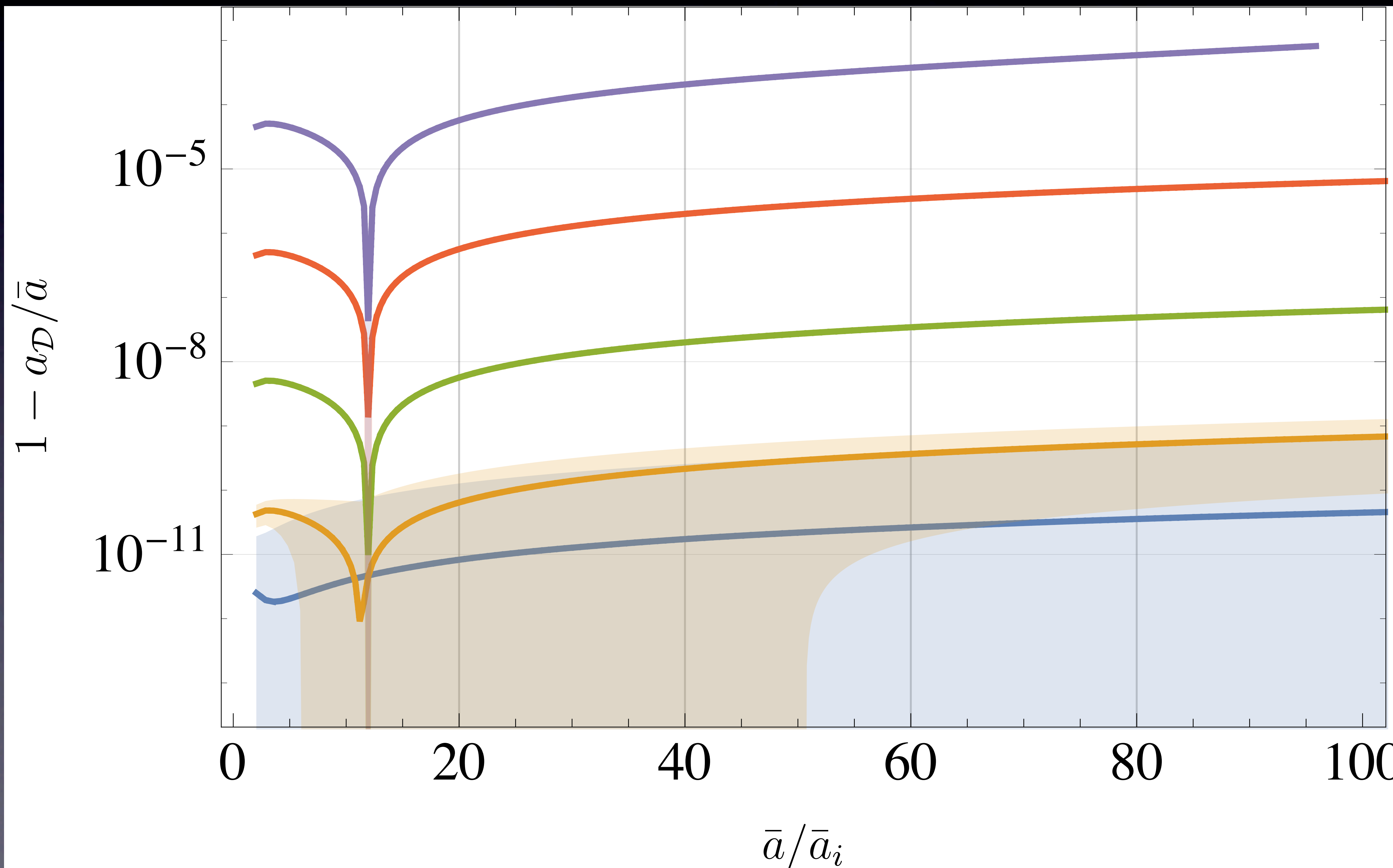
- synchronous-comoving gauge, irrotational fluid (Lagrangian approach)
- Integrate EFE using the Einstein Toolkit, freey available open source infrastructure for Numerical Relativity
- use a variant of BSSN formulation of EFE

Assumptions and procedure

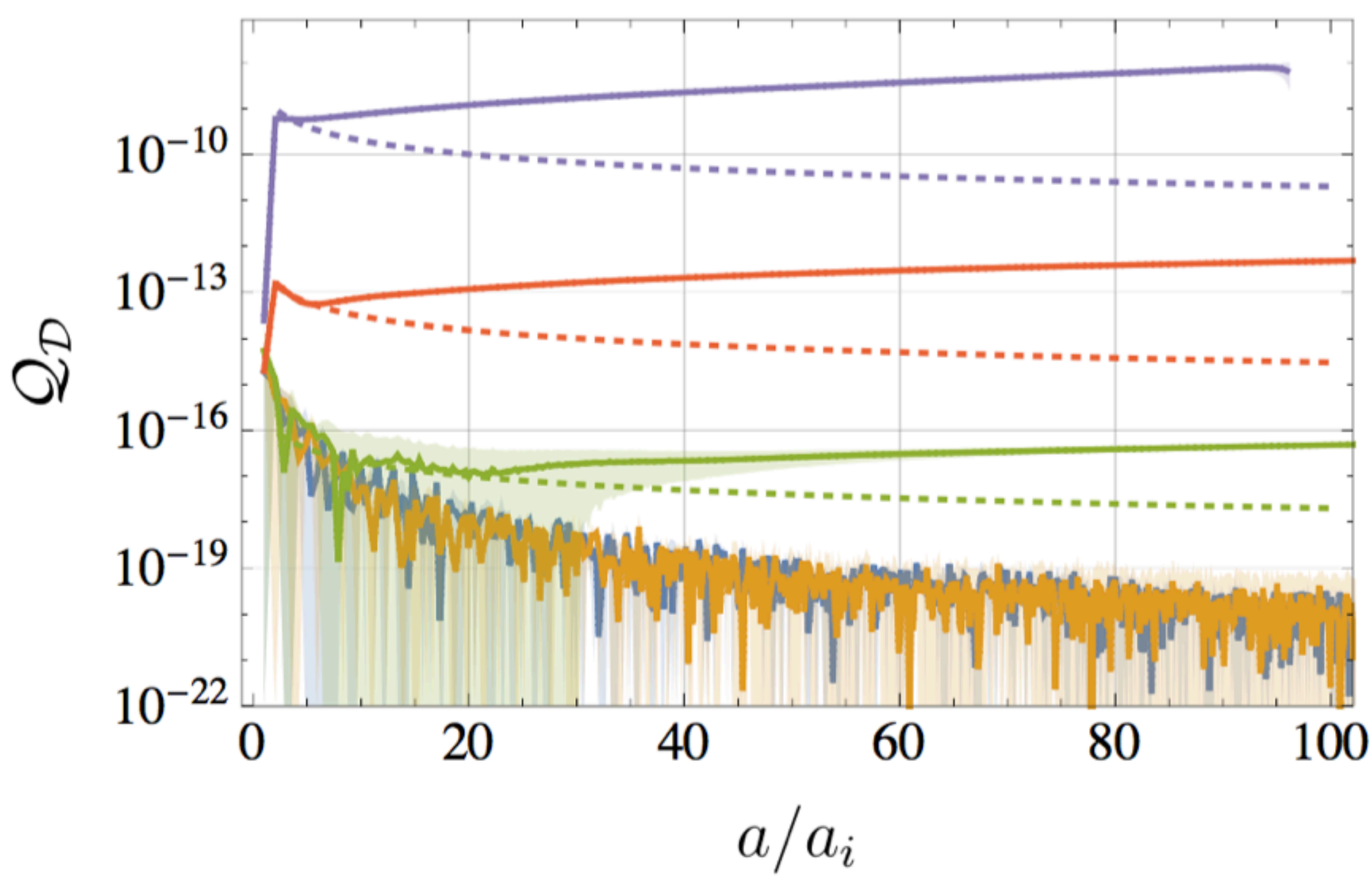
- solve initial constraint
- evolve EFE with periodic boundary conditions on comoving box of size L
- initial conditions: perturbations of EdS with $H_i^{-1}=L/4$
- domain discretised with 160^3 points
- compare average quantities and EdS evolution
- measure local quantities (expansion and density)

backreaction

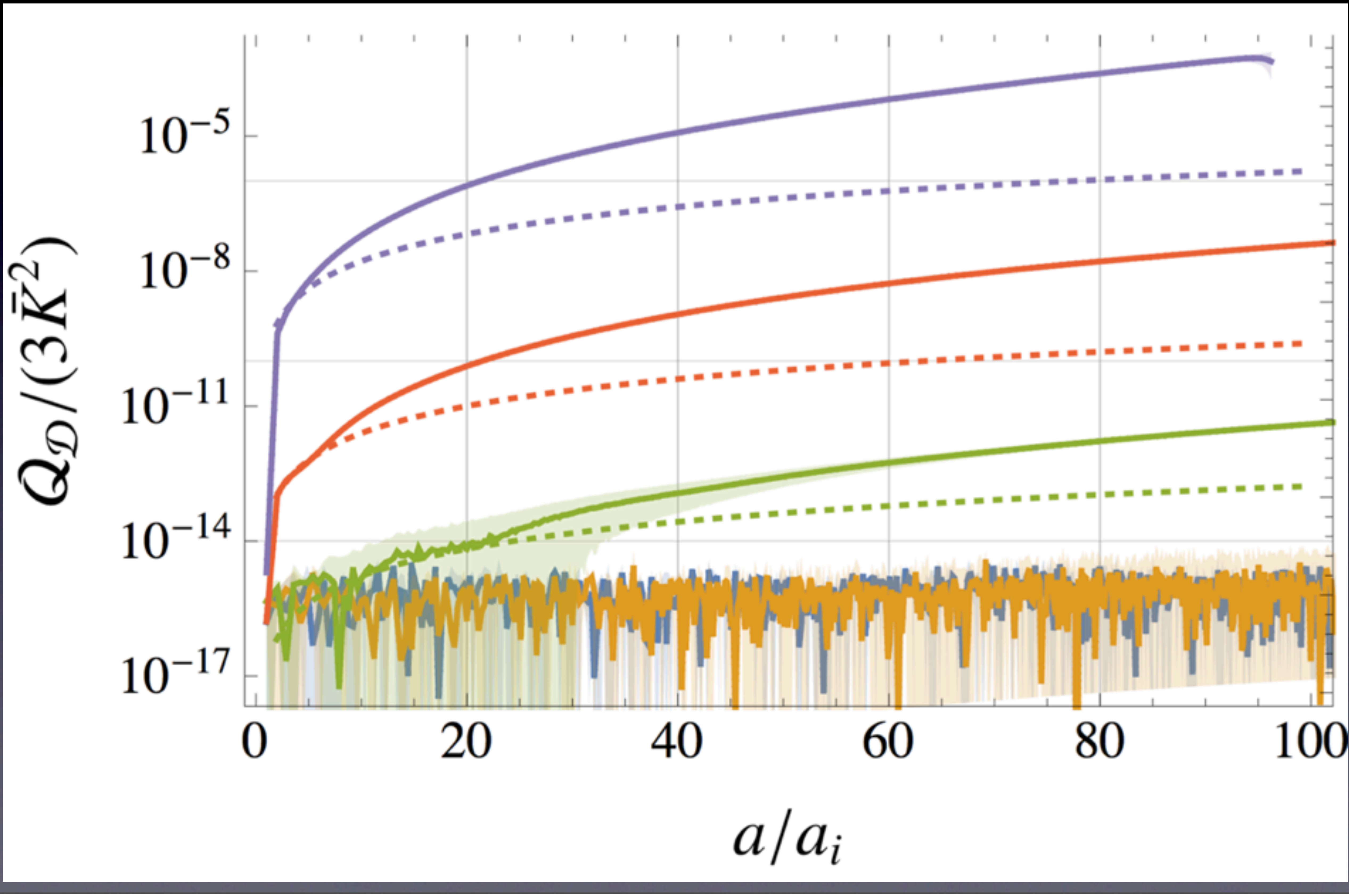
average expansion



backreaction

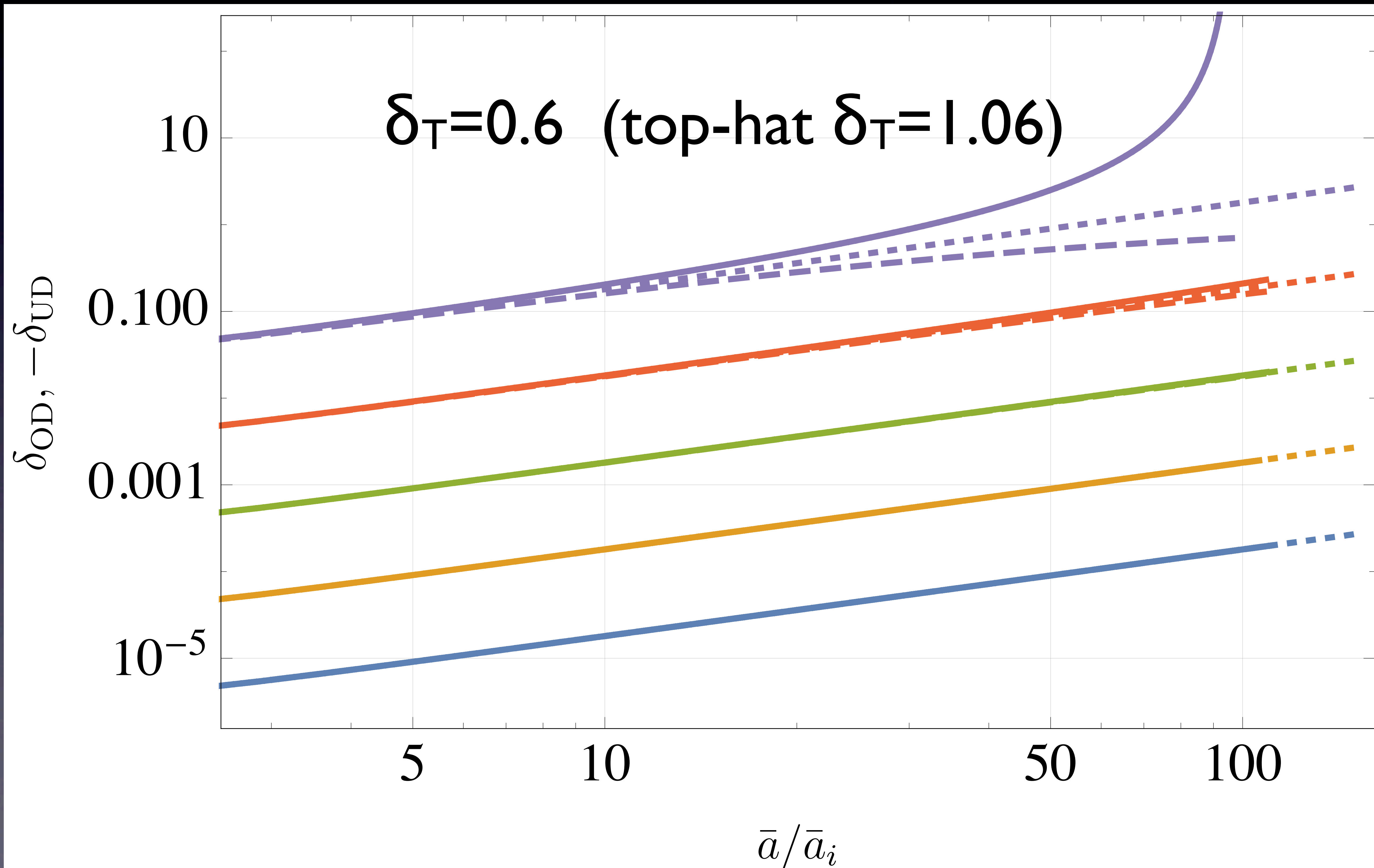


backreaction: Ω_Q

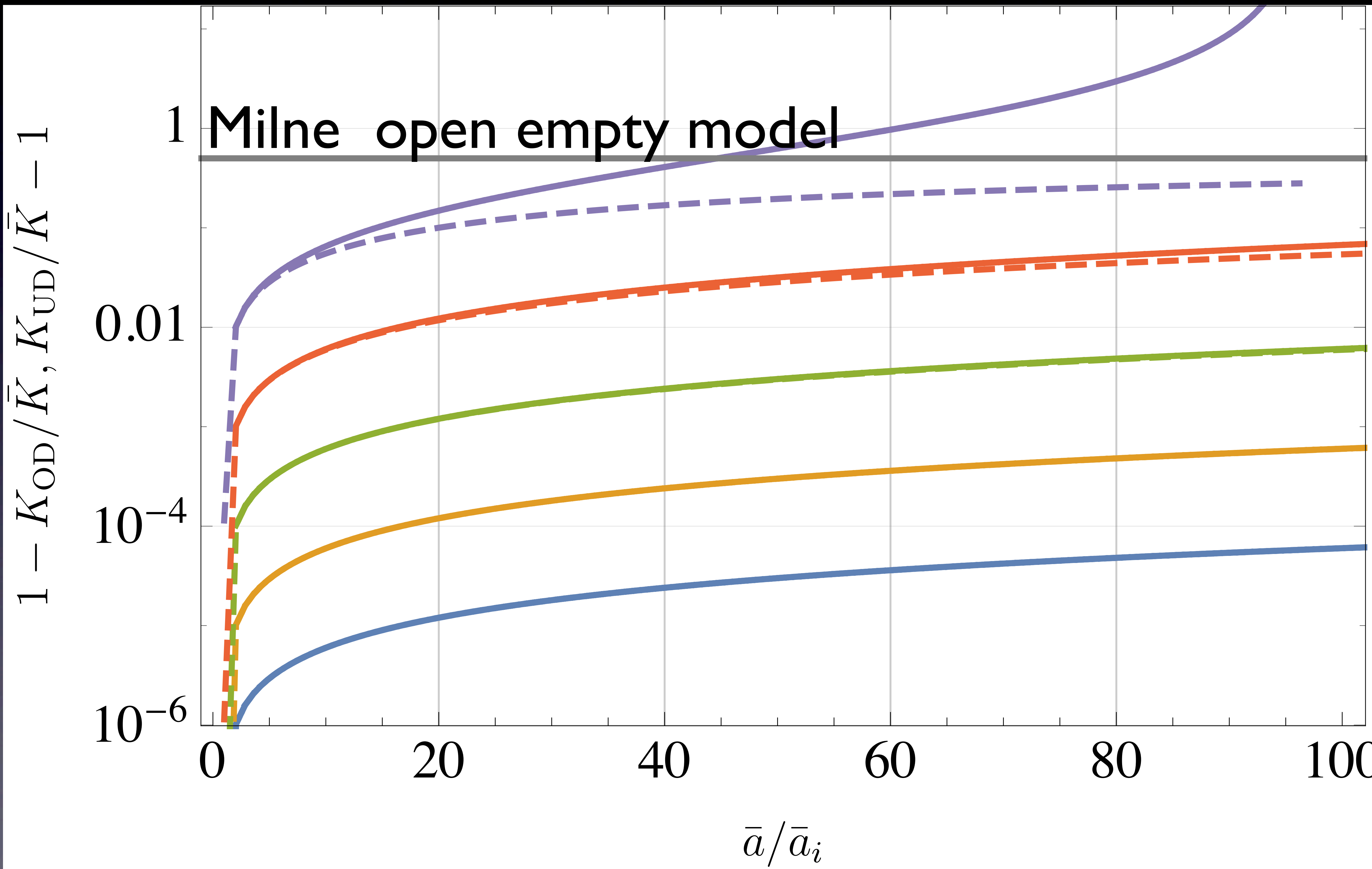


peaks, collapse and voids

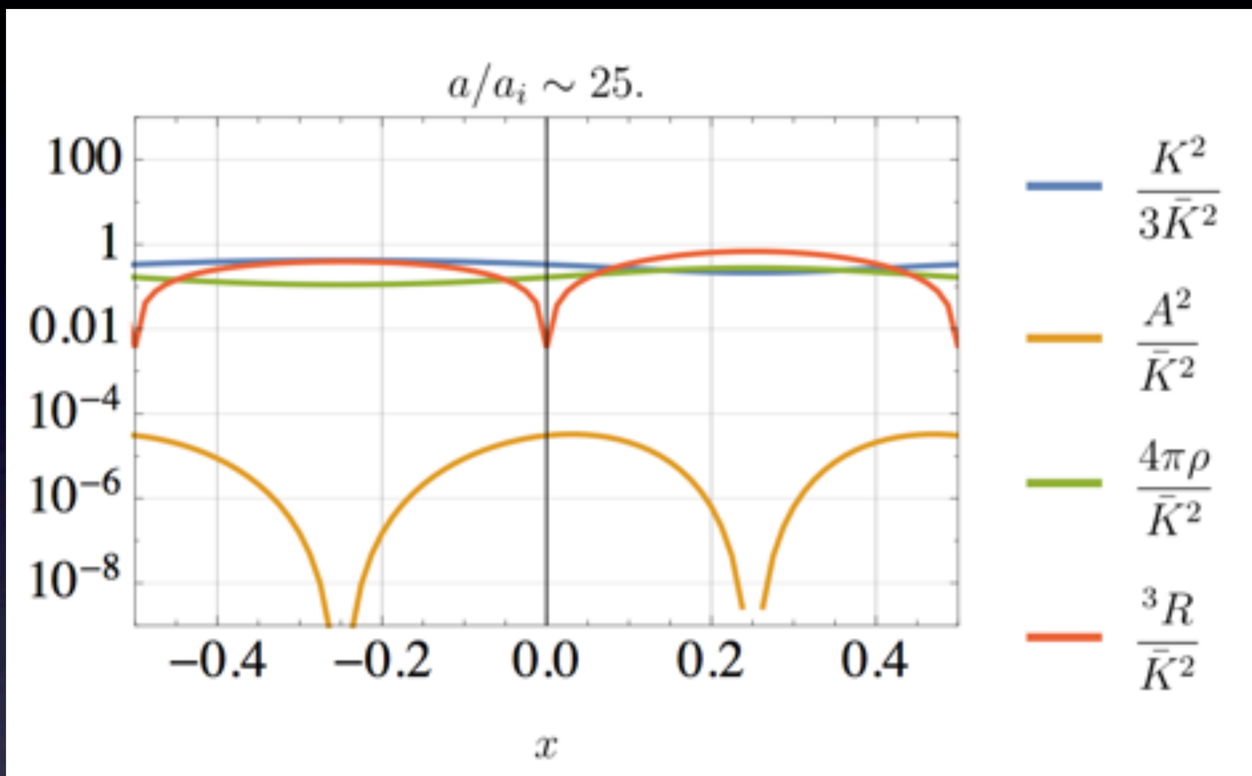
over and under densities

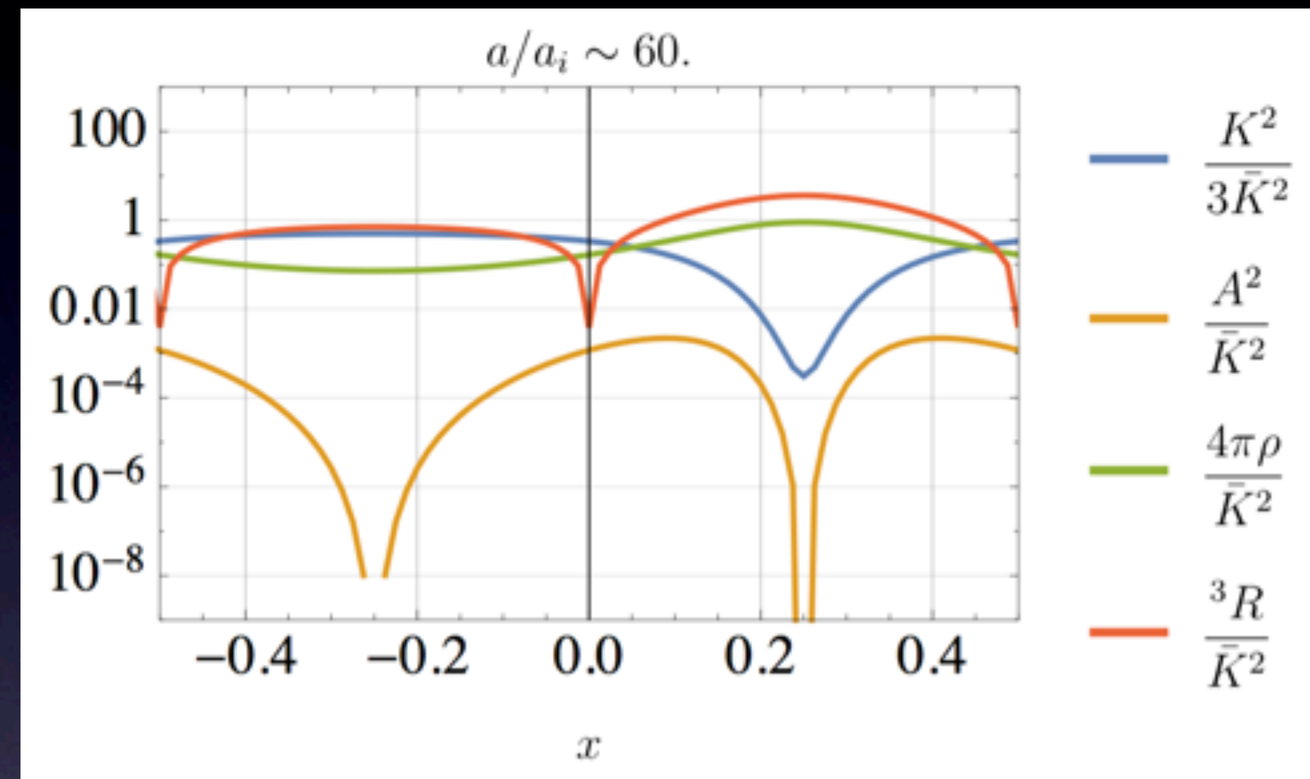


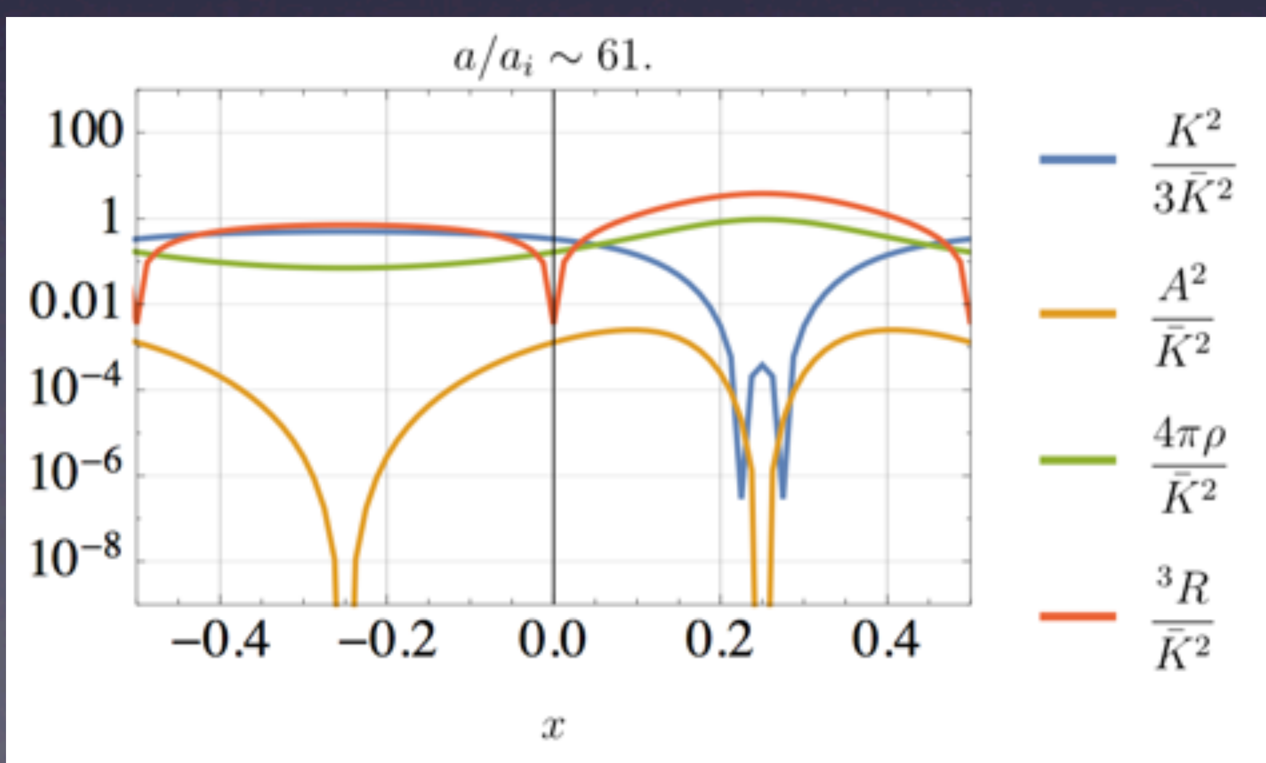
local expansion of peaks and voids

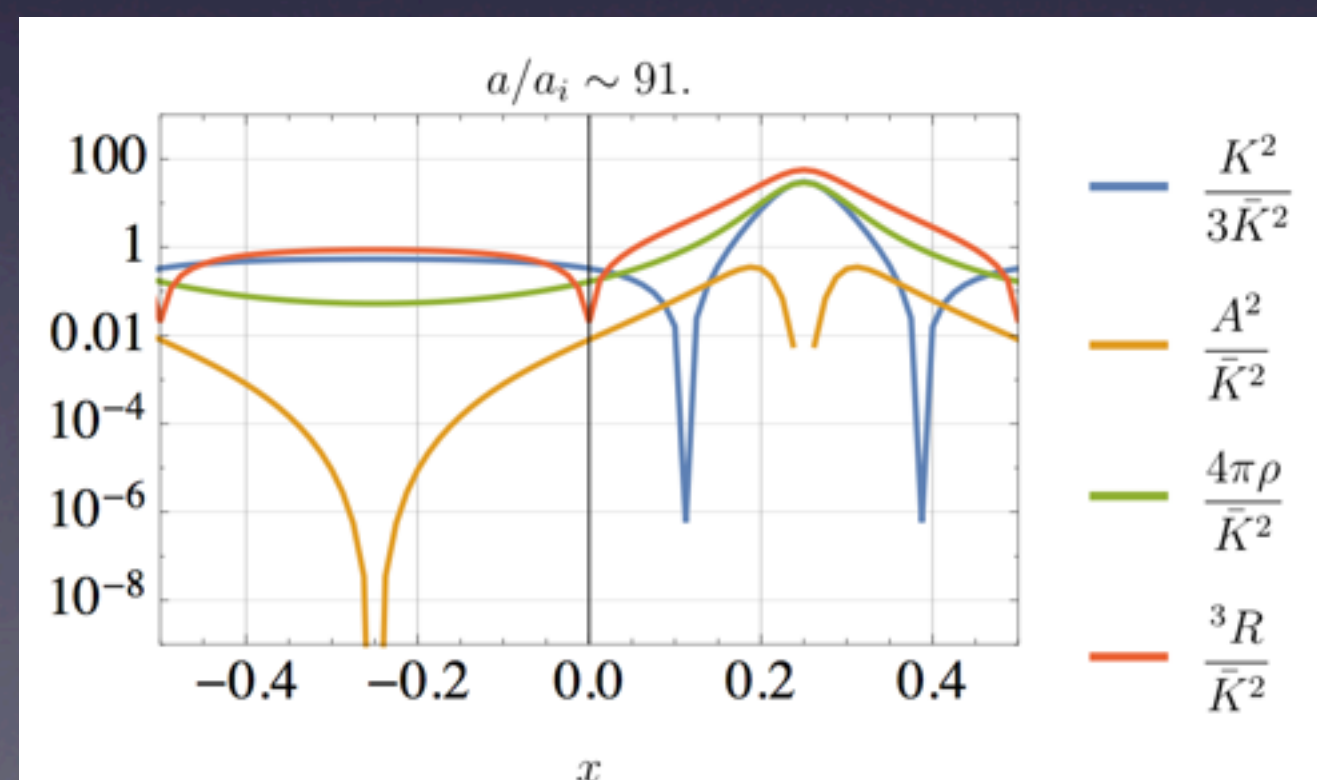


local contribution to Raychaudhuri equation









Conclusions

- post-F: framework including Newtonian and 1 GR order
  - Frame dragging small, but further work needed, e.g. lensing
  - Adamek et al.: consistent results, plus $\Phi=\Psi$ at leading order
- Full GR Numerical Relativity simulations:
  - within the fluid assumption (stop before shall crossing), backreaction is small and the box expands like EdS
  - peaks collapse much faster than standard Top-Hat
  - voids expand up to 28% faster than average (background)
  - Gibling, Mertens & Starkman work fully consistent with ours

recent progress

- Bentivegna, *An automatically generated code for relativistic inhomogeneous cosmologies*, [arXiv:1610.05198]; Bentivegna et al, *Light propagation through black-hole lattices*, [arXiv:1611.09275]
- Giblin, Mertens, & Starkman, *Observable Deviations from Homogeneity in an Inhomogeneous Universe* [arXiv:1608.04403]; *A cosmologically motivated reference formulation of numerical relativity* [arXiv:1704:04307]
- Macpherson et al. *Inhomogeneous Cosmology with Numerical Relativity* [arXiv:1611.05447];
- Daverio et al. *A numerical relativity scheme for cosmological simulations* [arXiv:1611.03437]

recent progress

- N-body with weak field GR:
  - Adamek et al., *g-evolution: a cosmological N-body code based on General Relativity* [arXiv:1604.06065]
- Initial conditions for Newtonian N-body evolution:
  - Chisari & Zaldarriaga (2011), Green & Wald (2012)
  - Fidler et al. *Relativistic initial conditions for N-body simulations* [arXiv:1702.03221], [arXiv:1606.05588], [arXiv:1505.04756]
  - Adamek et al, *The effect of early radiation in N-body simulations of cosmic structure formation* [arXiv:1703.08585]

Outlook

- MB+Bentivegna: work in progress to try different initial conditions and different gauges, testing and understanding the collapse and voids results
- various: work in progress to compare results from different codes
- is backreaction totally negligible or it may contribute, e.g. at the level or radiation?
- goals: "realistic" initial conditions, extract observable quantities, discover and quantify GR effects
- Much further work needed to obtain "realistic" simulations and compare with Newtonian N-body simulations

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but at least we have started!

Why?

- "Because it is there." — George Mallory, answer to the question 'Why do you want to climb Mt. Everest?