Fully nonlinear and exact cosmological perturbation theory

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6 July 2017
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Perturbation method:

- Perturbation expansion
- All perturbation variables are ~~small~~
- ~~Weakly~~ nonlinear
- Strong gravity; fully relativistic
- Valid in all scales
- Fully nonlinear and Exact perturbations

Post-Newtonian method:

- Abandon geometric spirit of GR: recover the good old absolute space and absolute time
- Newtonian equations of motion with GR corrections
- Expansion in strength of gravity $\frac{\delta\Phi}{c^2} \sim \frac{GM}{Rc^2} \sim \frac{v^2}{c^2} \ll 1$
- Fully nonlinear
- No strong gravity; weakly relativistic
- Valid far inside horizon
- Case of the Fully nonlinear and Exact perturbations

Fully NL & Exact Pert. Theory

JH, Noh, MN (2013) **433**, 3472
JH, Noh, Park, MN (2016) **461**, 3239
Gong, JH, Noh, Yoo, **arXiv: 0706.07753**

Metric convention **without** fixing temporal gauge (slicing) condition:

TT

$$\widetilde{g}_{00} = -a^2\left(1+2\alpha\right), \quad \widetilde{g}_{0i} = -a\chi_i, \quad \widetilde{g}_{ij} = a^2\left[\left(1+2\varphi\right)\delta_{ij} + 2h_{ij}\right].$$

raised and lowered using δ_{ij}

T

$$+\,2\gamma_{,ij} + 2C^{(v)}_{(i,j)} \equiv 0$$

Spatial Gauge taken

=spatial Harmonic to 1PN

Exact inverse metric:

$$\widetilde{g}^{00} = -\frac{1}{a^2\mathcal{N}^2}, \quad \widetilde{g}^{0i} = -\frac{\delta^{ij}+H^{ij}}{a^3\mathcal{N}^2(1+2\varphi+I)}\chi_j,$$

$$\widetilde{g}^{ij} = \frac{1}{a^2(1+2\varphi+I)}\left(\delta^{ij}+H^{ij} - \frac{(\delta^{ik}+H^{ik})(\delta^{j\ell}+H^{j\ell})}{a^2\mathcal{N}^2(1+2\varphi+I)}\chi_k\chi_\ell\right).$$

$$H^{ij} \equiv -2\frac{(1+2\varphi)h^{ij} - 2h^{ik}h^j_k}{(1+2\varphi)^2 - 2h^{k\ell}h_{k\ell}}, \quad I \equiv \frac{8}{3}\frac{h_{k\ell}h^k_m h^{\ell m}}{(1+2\varphi)^2 - 2h^{k\ell}h_{k\ell}}$$

$$N = a\sqrt{1+2\alpha+\frac{\delta^{ij}+H^{ij}}{a^2(1+2\varphi+I)}\chi_i\chi_j} \equiv a\mathcal{N}$$

Temporal gauge (slicing, hypersurface):

$$
\begin{aligned}
&\text{comoving gauge}: && v \equiv 0, \\
&\text{zero-shear gauge}: && \chi \equiv 0, \\
&\text{uniform-curvature gauge}: && \varphi \equiv 0, \\
&\text{uniform-expansion gauge}: && \kappa \equiv 0, \\
&\text{uniform-density gauge}: && \delta \equiv 0, \\
&\text{synchronous gauge}: && \alpha \equiv 0.
\end{aligned}
$$

Longitudinal, Newtonian, Poisson, …

Perturbed trace of extrinsic curvature, K

K = - 3H + κ

~Maximal Slicing (K ≡ 0)

→ Remnant gauge mode

Applicable to fully NL orders!

Except for synchronous gauge, complete gauge fixing.
Remaining variables are gauge-invariant to fully NL order!

Post-Newtonian Approximation

Chandrasekhar, ApJ (1965) **142**, 1488: **1PN, Minkowski**

JH, Noh, Puetzfeld, JCAP (2008) **03**, 010: **cosmological**

Noh, JH, JCAP (2013) **08**, 040: **as a limit of FNL PT**

1PN Hydrodynamics (Minkowski):

$$\dot{\overline{\varrho}} + \nabla\cdot(\overline{\varrho}\overline{\mathbf{v}}) = -\frac{1}{c^2}\overline{\varrho}\frac{d}{dt}\left(\frac{1}{2}\overline{v}^2 + 3U\right),$$

$$\dot{\overline{\varrho}} + \nabla\cdot(\overline{\varrho}\overline{\mathbf{v}}) = -\frac{1}{c^2}\left[\overline{\varrho}\frac{d}{dt}\left(\frac{1}{2}\overline{v}^2 + 3U + \Pi\right) + p\nabla\cdot\overline{\mathbf{v}}\right],$$

$$\dot{\overline{\mathbf{v}}} + \overline{\mathbf{v}}\cdot\nabla\overline{\mathbf{v}} - \nabla U + \frac{1}{\overline{\varrho}}\nabla p = \frac{1}{c^2}\Big[-2\nabla\left(U^2 - \widetilde{\Phi}\right) + \dot{P}_i + \overline{v}^j\left(P_{i,j} - P_{j,i}\right)$$
$$-\overline{\mathbf{v}}\frac{d}{dt}\left(\frac{1}{2}\overline{v}^2 + 3U\right) + \overline{v}^2\nabla U + \left(\overline{v}^2 + 4U + \Pi + \frac{p}{\overline{\varrho}}\right)\frac{1}{\overline{\varrho}}\nabla p - \overline{\mathbf{v}}\frac{1}{\overline{\varrho}}\frac{d}{dt}p\Big],$$

$$\Delta U + 4\pi G\overline{\varrho} = -\frac{1}{c^2}\left[3\ddot{U} - 2U\Delta U + 2\Delta\widetilde{\Phi} + \dot{P}^i{}_{,i} + 8\pi G\left(\overline{\varrho}\overline{v}^2 + \frac{1}{2}\overline{\varrho}\Pi + \frac{3}{2}p\right)\right],$$

$$0 = \frac{1}{4}\left(P^j{}_{,ji} - \Delta P_i\right) + \nabla\dot{U} - 4\pi G\overline{\varrho}\overline{\mathbf{v}},$$

$$0 = U - V.$$

General gauge: $P^i{}_{,i} + n\dot{U} = 0.$

Harmonic gauge: n ≡ 4
Maximal Slicing: n ≡ 3
Zero-shear Slicing: n ≡ 0

$$g_{00} = -\left[1 - \frac{1}{c^2}2U + \frac{1}{c^4}\left(2U^2 - 4\widetilde{\Phi}\right)\right], \quad g_{0i} = -\frac{1}{c^3}P_i, \quad g_{ij} = \left(1 + \frac{1}{c^2}2V\right)\delta_{ij}.$$

$$u^i \equiv u^0\frac{\overline{v}^i}{c} \qquad v_i = \overline{v}_i + \frac{1}{c^2}\left[(U + 2V)\overline{v}_i - P_i\right]$$

Special Rel. Hydrodynamics with Gravity

Special Relativistic Hydrodynamics + ~0PN
Weak gravity and Action-at-a-distance
With relativistic pressure, velocity, stress
JH, Noh, ApJ (2016) ***833***, 180

Minkowski background:

Metric:

$$\chi_i \equiv c\chi_{,i} + \chi_i^{(v)} \text{ with } \chi^{(v)i}{}_{,i} \equiv 0$$

$$ds^2 = -\left(1 - \frac{2\Phi}{c^2}\right) c^2 dt^2 - 2\chi_i c dt dx^i + \left(1 + \frac{2\Psi}{c^2}\right) \delta_{ij} dx^i dx^j$$

Assumptions:

Weak Gravity

Action-at-a-distance

$$\frac{\Phi}{c^2} \ll 1, \quad \frac{\Psi}{c^2} \ll 1, \quad \gamma^2 \frac{t_\ell^2}{t_g^2} \ll 1$$

$$t_g \sim 1/\sqrt{G\varrho} \qquad t_\ell \sim \ell/c \sim 2\pi/(kc)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \qquad u_i \equiv \gamma \frac{v_i}{c}$$

Maximal slicing: K ≡ 0

Uniform-expansion slicing (κ ≡ 0) in cosmology

SR Hydrodynamics with Gravity

Maximal Slicing: K ≡ 0

Continuity:

$$\frac{d\overline{\varrho}}{dt} + \overline{\varrho}\nabla \cdot \mathbf{v} = \frac{\overline{\varrho}}{c^2}\frac{1}{\varrho + p/c^2}\left(\frac{dp}{dt} - \frac{1}{\gamma^2}\dot{p}\right),$$

E conservation:

$$\frac{d\varrho}{dt} + \left(\varrho + \frac{p}{c^2}\right)\nabla \cdot \mathbf{v} = \frac{1}{c^2}\left(\frac{dp}{dt} - \frac{1}{\gamma^2}\dot{p}\right),$$

M conservation:

$$\frac{d\mathbf{v}}{dt} = \boxed{\nabla\Phi} - \frac{1}{\gamma^2}\frac{1}{\varrho + p/c^2}\left(\nabla p + \frac{1}{c^2}\mathbf{v}\dot{p}\right),$$

Poisson eq:

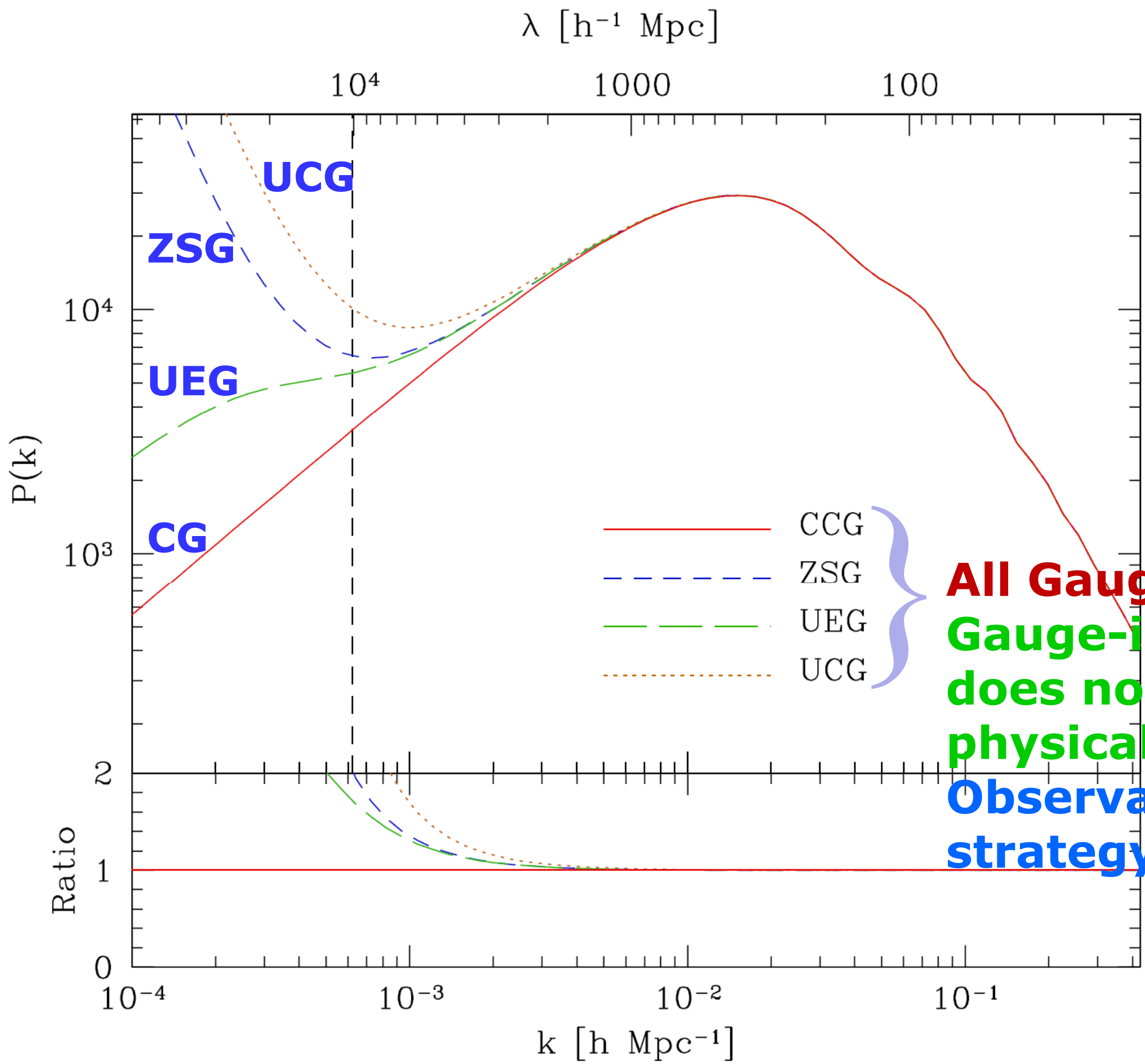
$$\Delta\Phi + 4\pi G\left(\varrho \boxed{\underline{+ 3\frac{p}{c^2}}\Big) = -8\pi G\left(\varrho + \frac{p}{c^2}\right)\gamma^2\frac{v^2}{c^2}}$$

$$\varrho \equiv \overline{\varrho}(1 + \Pi/c^2), \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$$

Missing in ZSG:
In trouble with TOV!

Nonlinear perturbations and Gauge dependence

Baryonic matter power spectrum in the CDM model: linear order

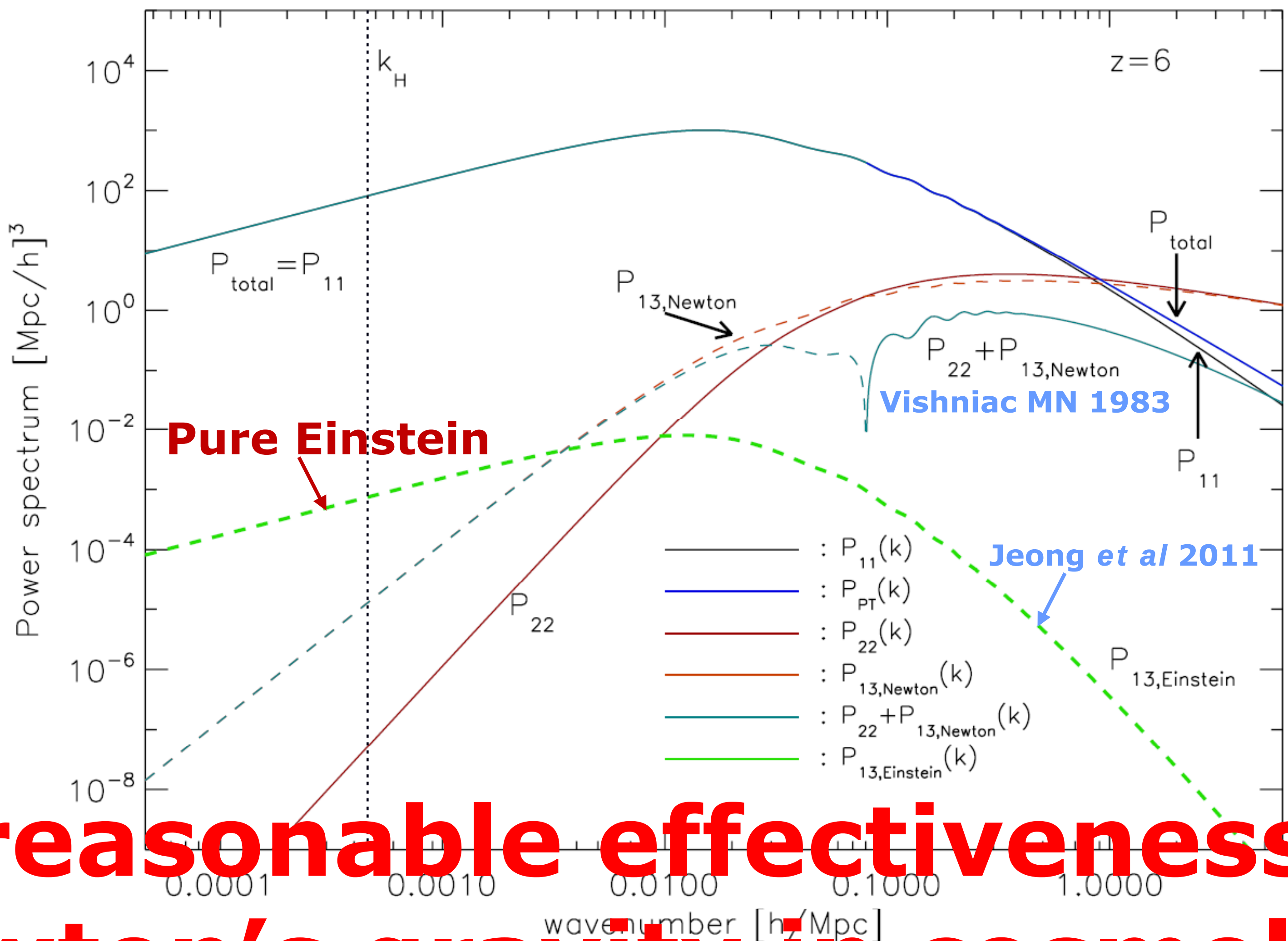


All Gauge-invariant!

Gauge-invariance does not guarantee physical observable!

Observational strategy may tell.

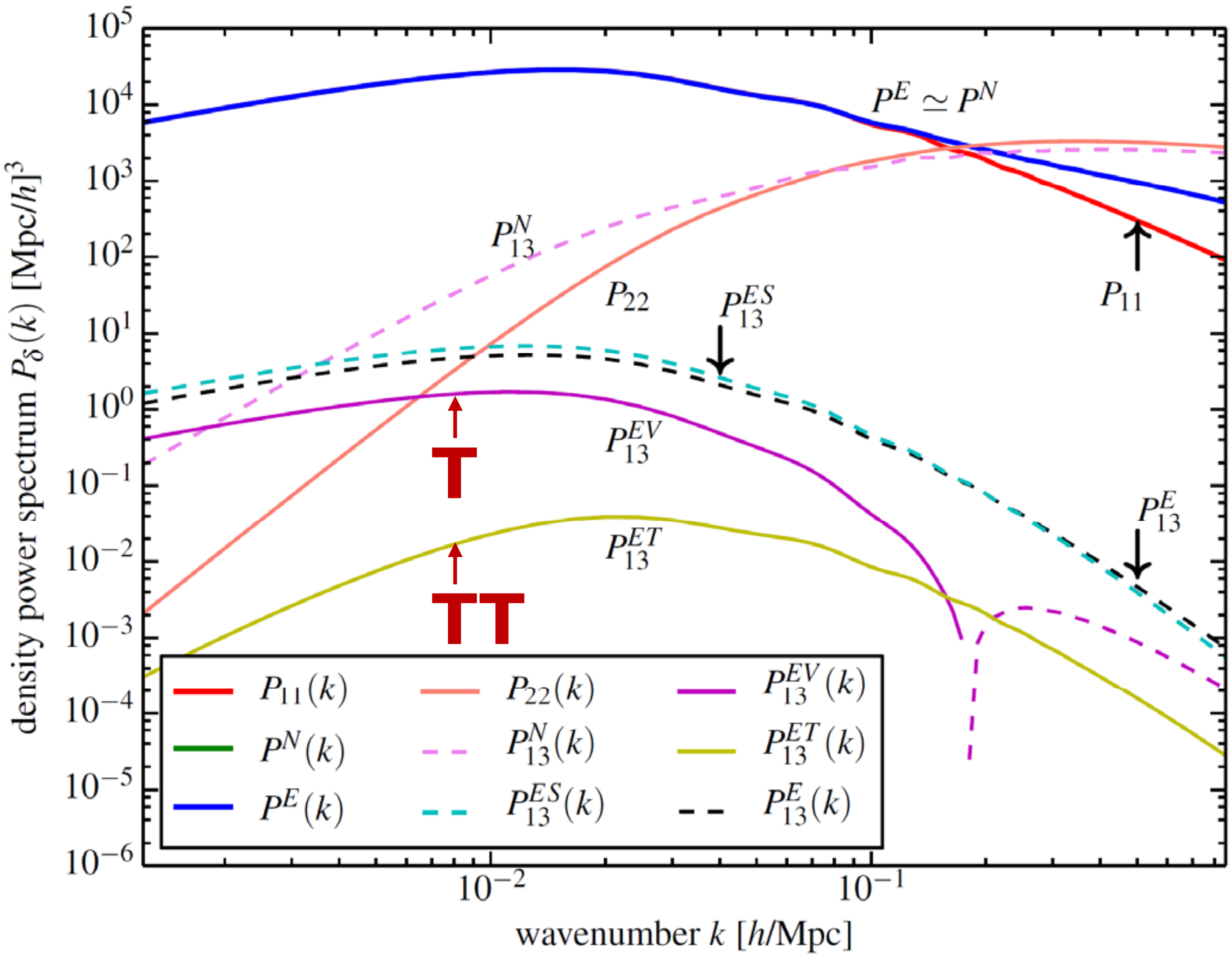
Leading Nonlinear Density Power-spectrum in the Comoving gauge



Unreasonable effectiveness of Newton's gravity in cosmology!

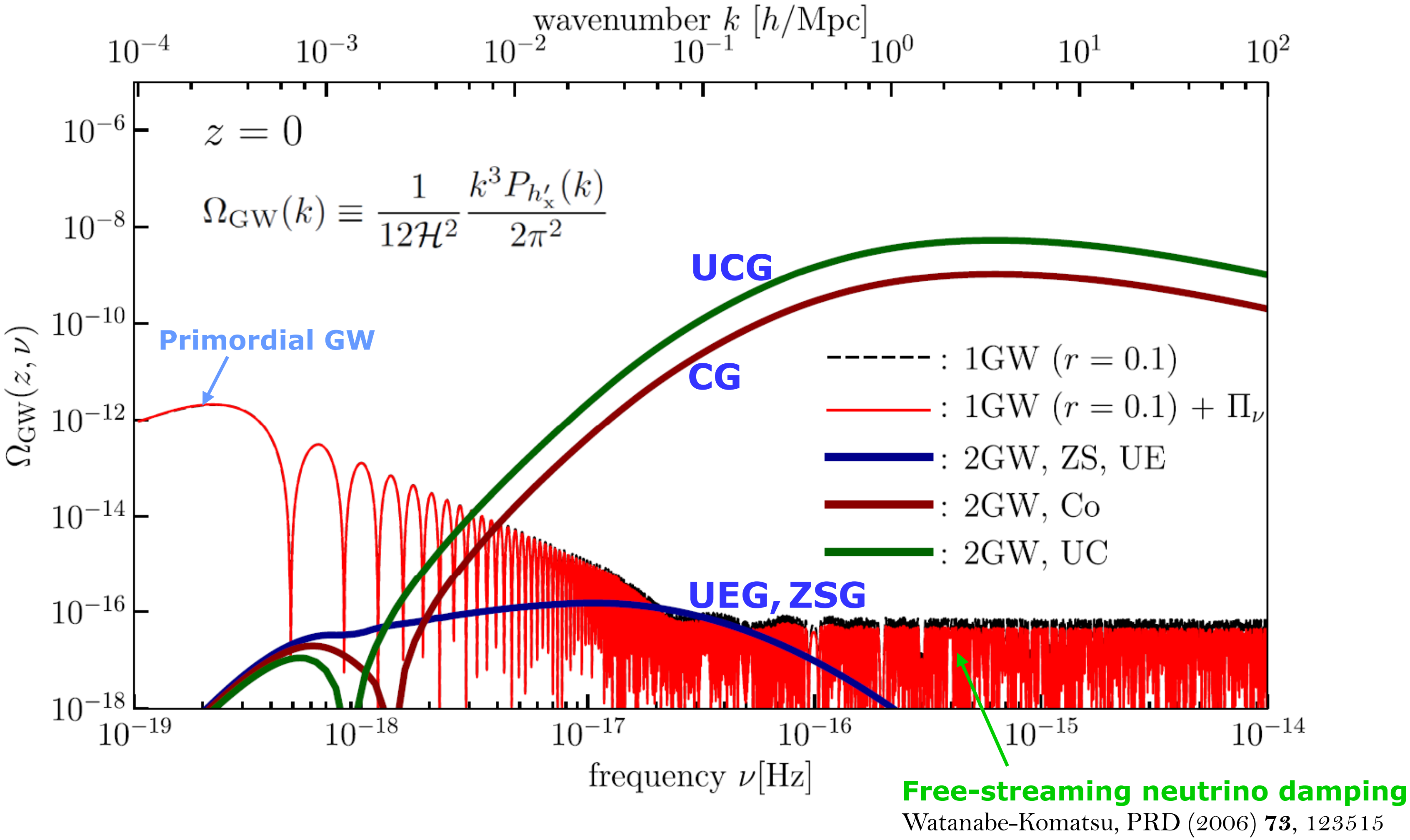
Jeong, Gong, Noh, JH, ApJ (2011) ***722***, 1

NL Density Power-spectrum in the CG with vector and tensor contributions:



JH, Jeong, Noh, MN (2016) **459**, 1124

TT perturbation generated from Galaxy Clustering



Pulsar Timing Array: 10^{-11}~10^{-7}Hz, LISA: 10^{-5}~1Hz, LIGO: 10~10^4Hz

JH, Jeong, Noh, ApJ (2017) **842**, 46

Fully NL and exact cosmological pert.

1. Multi-component fluids
2. Minimally coupled scalar fields
3. 1PN hydrodynamics
4. Special Relativistic Hydrodynamics with gravity
5. Now including TT, most general!

Future extentions

1. Special Relativistic Magneto-hydrodynamics with gravity
2. Special Relativistic Hydrodynamics with 1PN gravity
3. Light propagation (geodesic, Boltzmann)
4. Higher order PN equations
5. Gauge-invariant combinations

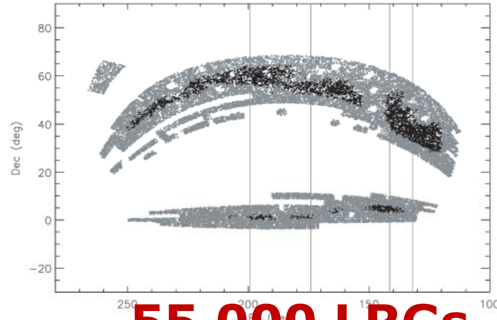
Applications

1. Fitting and Averaging
2. Backreaction
3. Relativistic (cosmological) numerical simulation

“Spatial homogeneity is one of the foundations of standard cosmology, so any chance to check those foundations observationally should be welcomed with open arms.”

George F. R. Ellis (2008)

THE ASTROPHYSICAL JOURNAL, 624:54–58, 2005 May 1



55,000 LRGs

COSMIC HOMOGENEITY DEMONSTRATED WITH LUMINOUS RED GALAXIES

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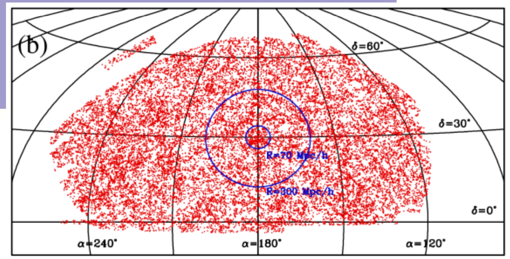
Claimed Homogeneity Scale: R~70h^{-1}Mpc

Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **469,** 1924–1931 (2017)
Advance Access publication 2017 April 25



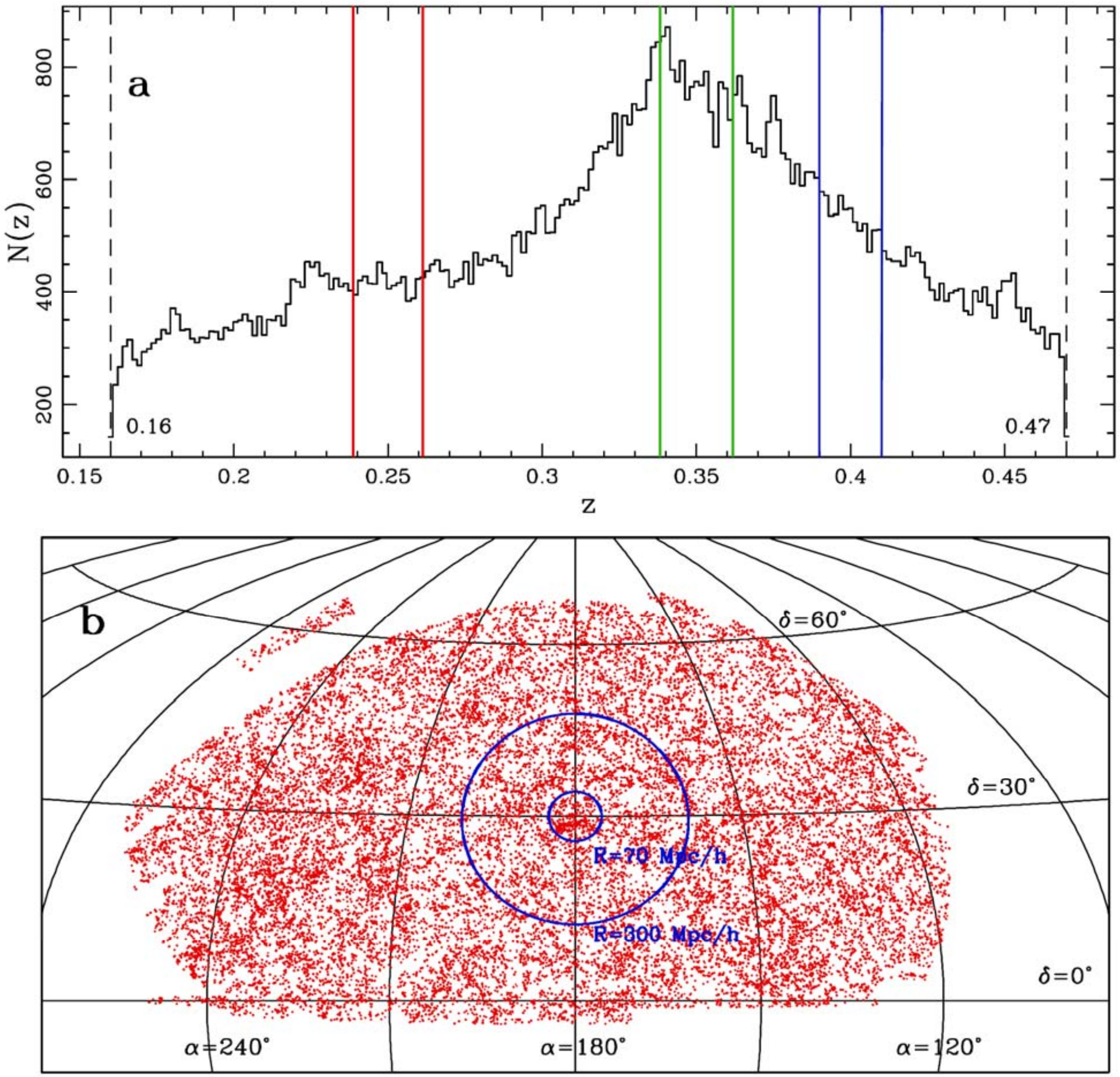
105,831 LRGs

The cosmological principle is not in the sky

Park, Hyun, Noh, JH, MN (2017) **469**, 1924

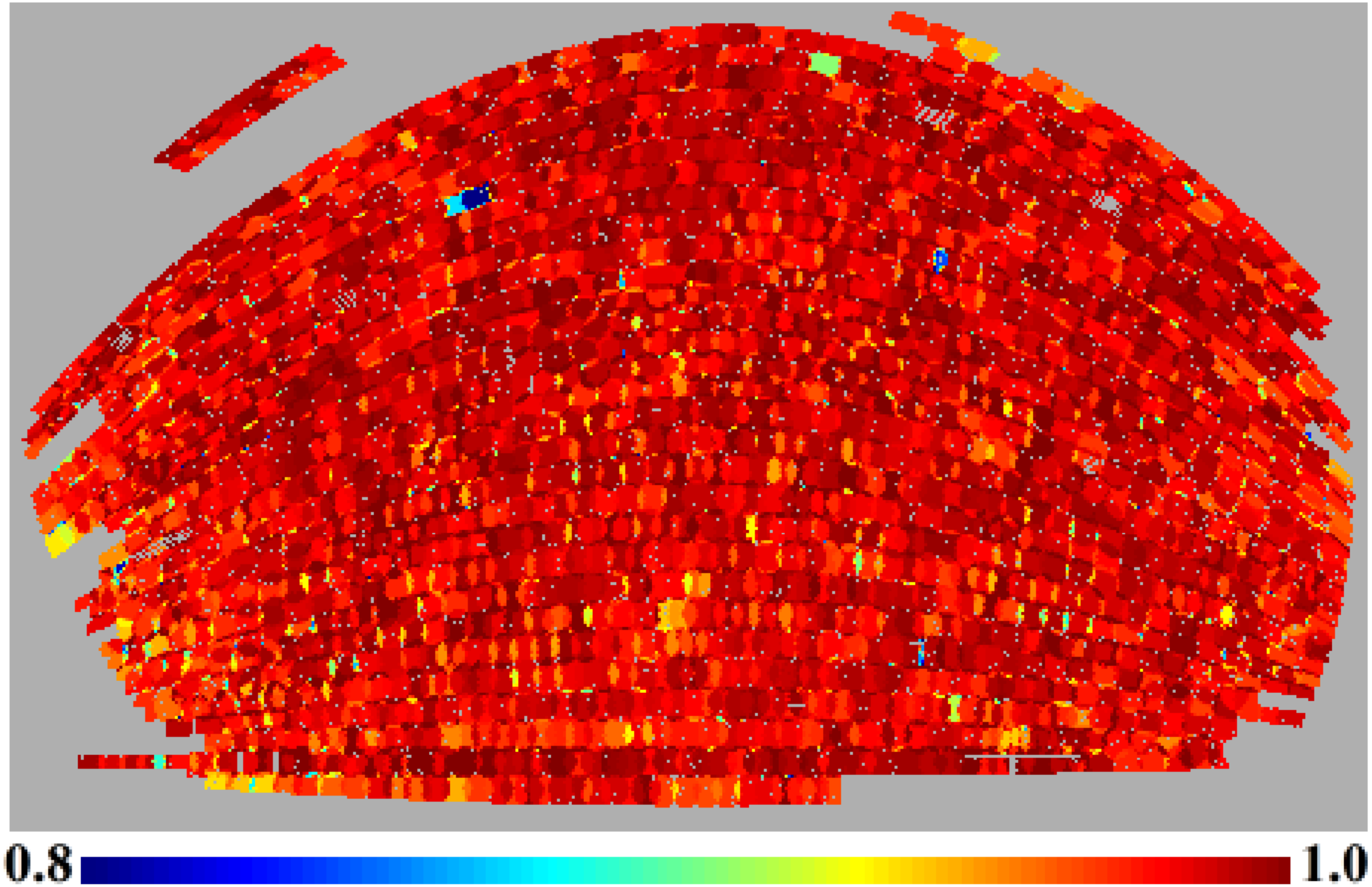
Chan-Gyung Park

Redshift and Angular distributions of SDSS DR7 105,831 LRGs

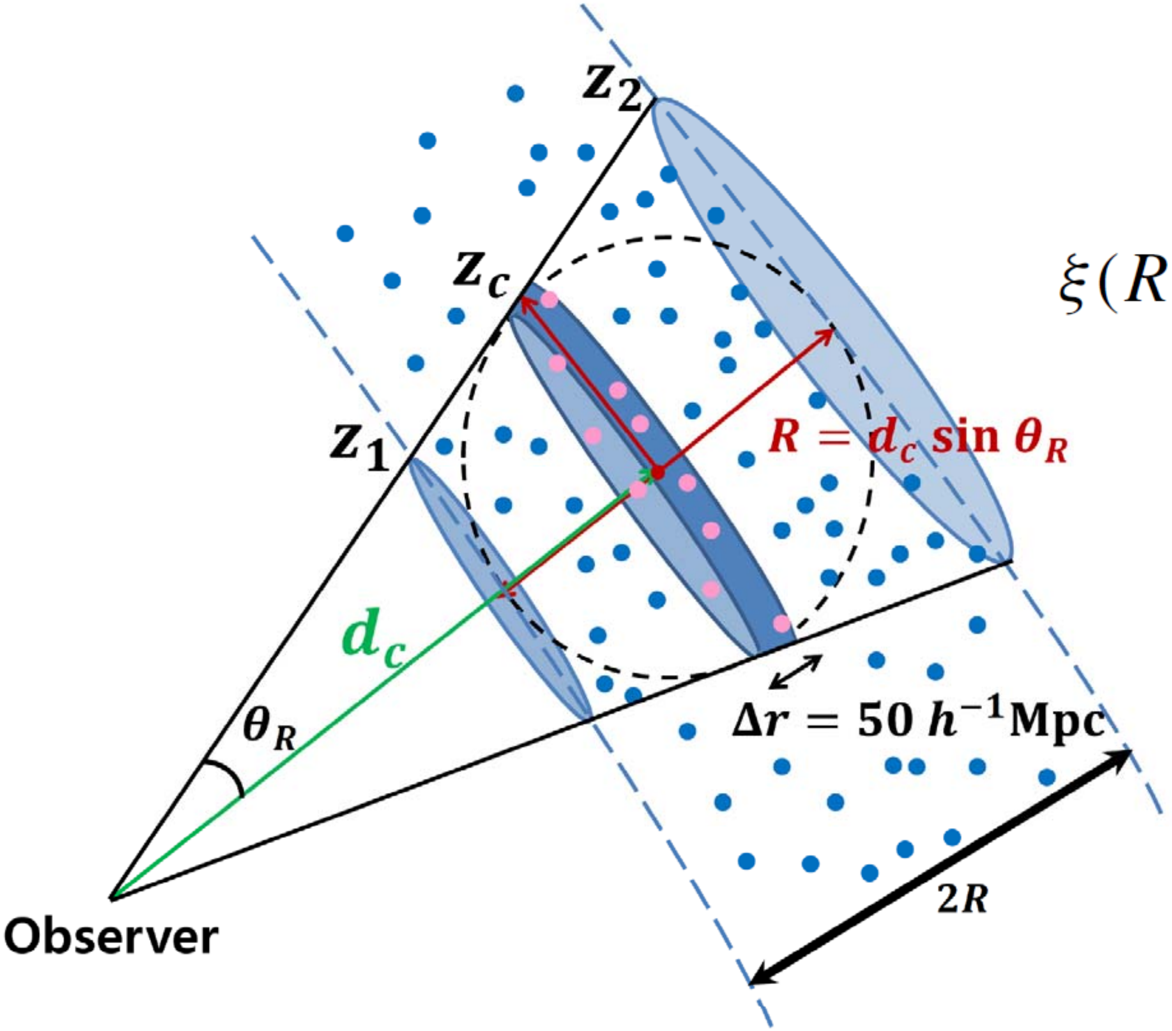


Galaxies within a slice with thickness of 140 h^{-1} Mpc centred at $z_c = 0.35$ in the Hammer–Aitoff equal-area projection with equatorial coordinates

Angular selection function



Geometry of a truncated cone



$$\xi(R) = \frac{N_{\mathrm{trc}}/V_{\mathrm{trc}}}{N_{\mathrm{slice}}/V_{\mathrm{slice}}}$$

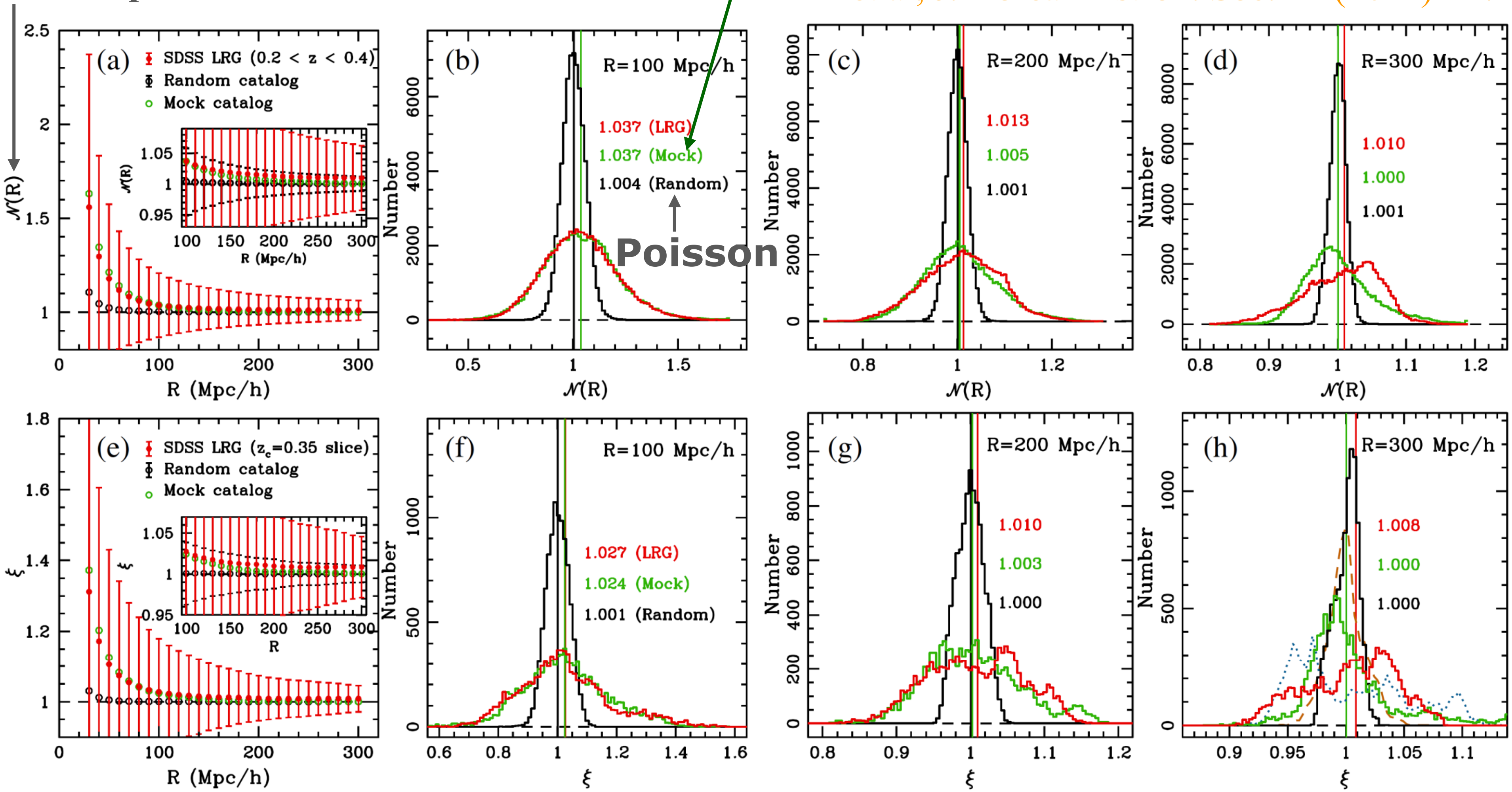
Counting galaxies within a sphere with varying radius (a)-(d)

Mock: *Horizon Run 3* numerical N-body simulation

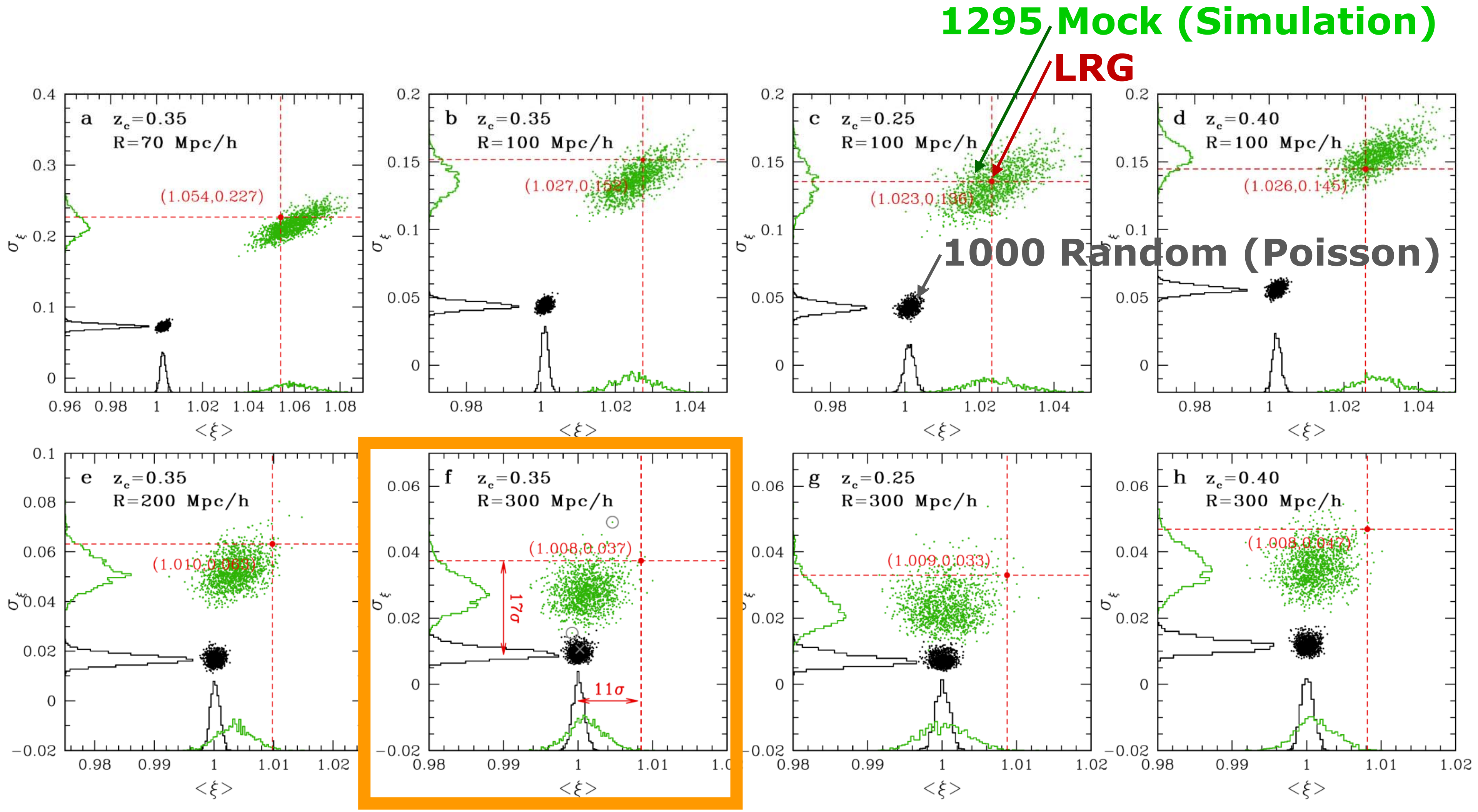
7210^3 particles, $(10.815h^{-1}\mathrm{Gpc})^3$ volume

Kim et al, J. Korean Astron. Soc. **44** (2011) 217

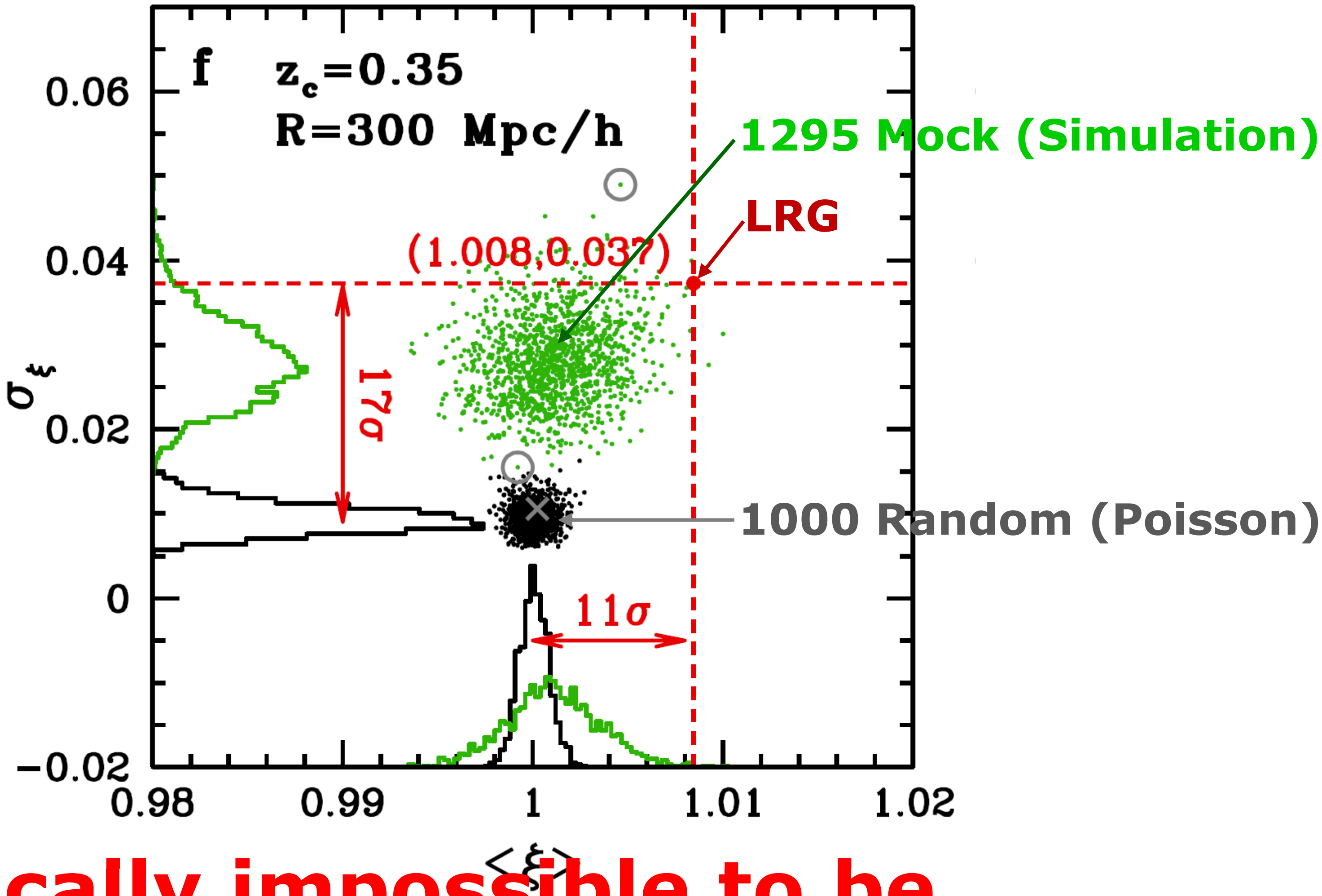
Normalized number of LRGs within sphere with radius R



Counting galaxies within redshift ranges with varying radius

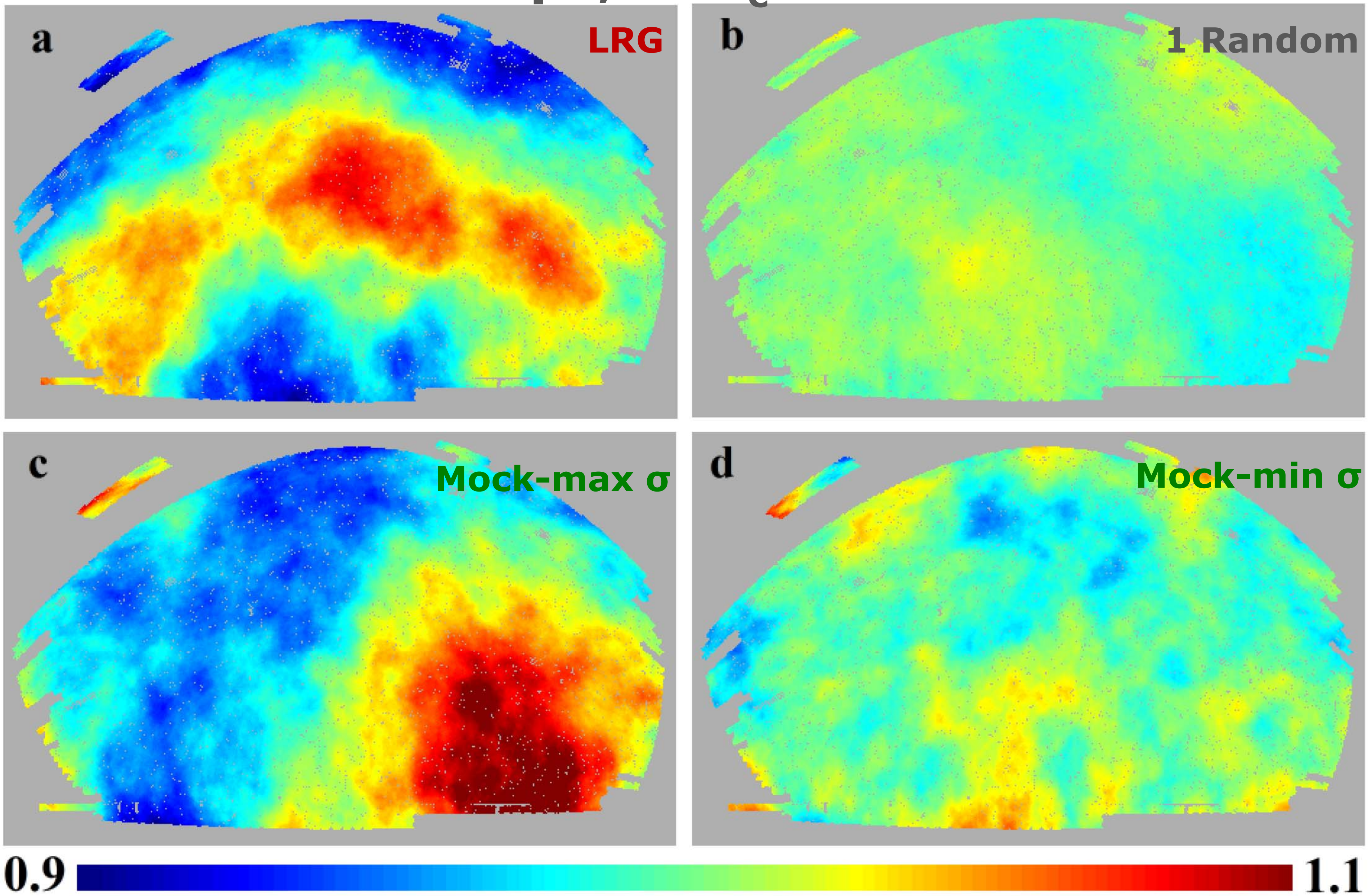


Counting galaxies within redshift ranges with R = 300h^{-1}Mpc



Statistically impossible to be homogeneous even at R = 300h^{-1}Mpc!

Angular distribution of normalized numbers with R = 300h^{-1}Mpc, at z_c = 0.35



Angular distributions of galaxies at 0.235 < z < 0.470