Drift effects in cosmology

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in collaboration with

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Jarosław Kopiński (FUW Warsaw)

CosmoToruń17 Inhomogeneous Cosmology Workshop

Centre for Astronomy at Nicolaus Copernicus University

UNIWERSYTET MIKOŁAJA KOPERNIKA W TORUNIU

Toruń, 2nd-7th July 2017

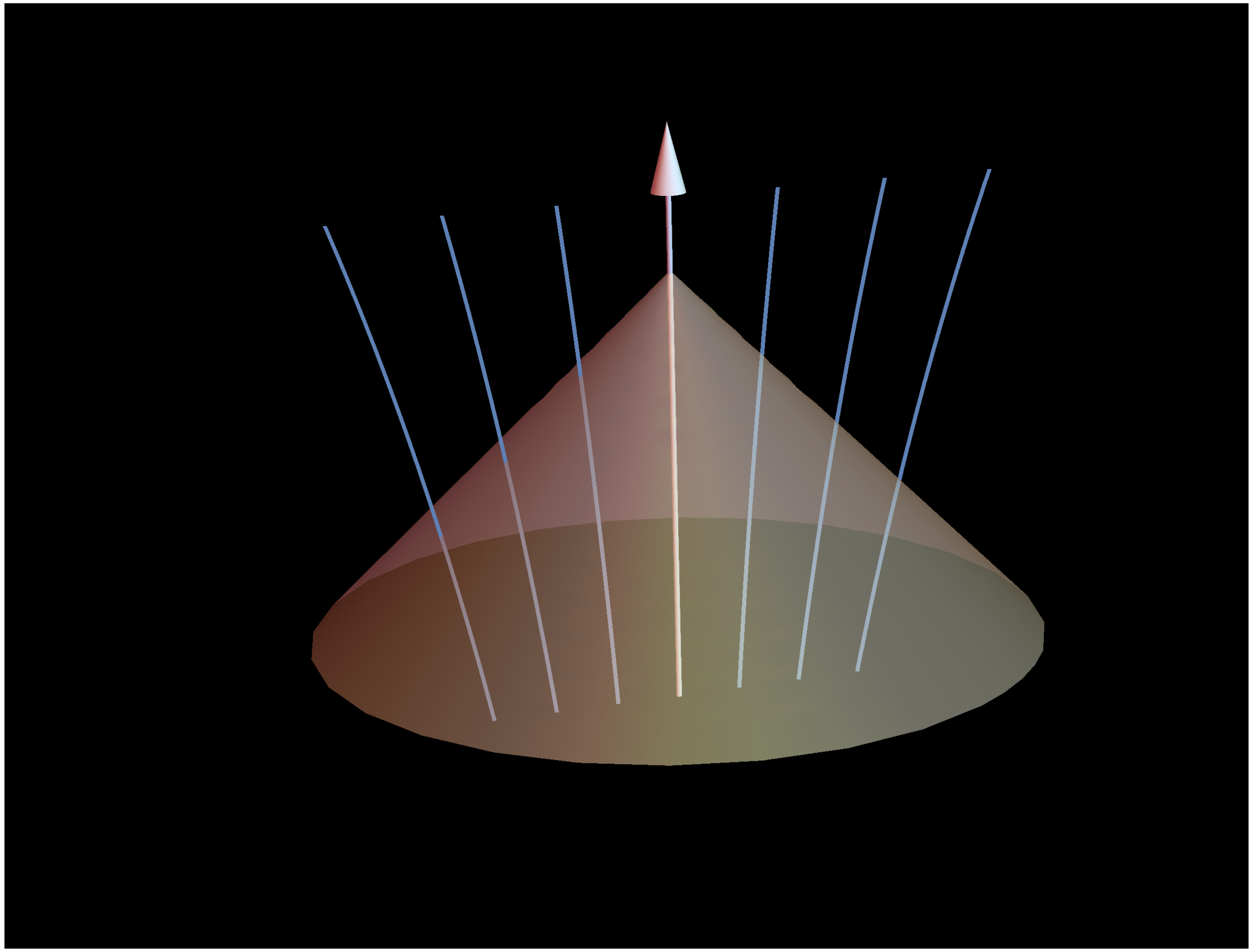
Outline

1. Motivation
2. Geometry
3. Drift effects
4. Applications

NCN project SONATA BIS No 2016/22/E/ST9/00578
“Local relativistic perturbative framework in hydrodynamics and general relativity and its application to cosmology”

Motivation

Observations in cosmology effectively on a single lightcone



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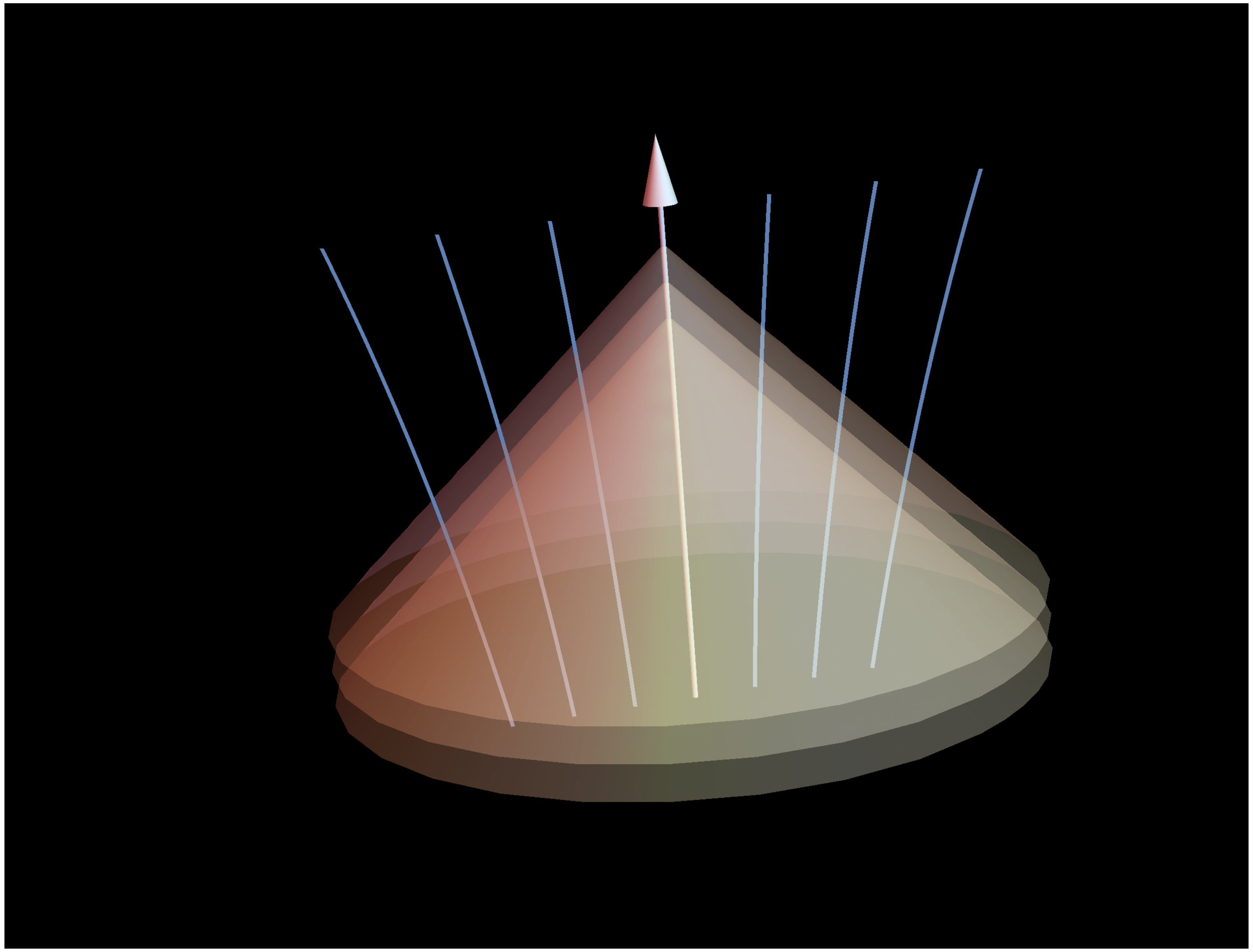

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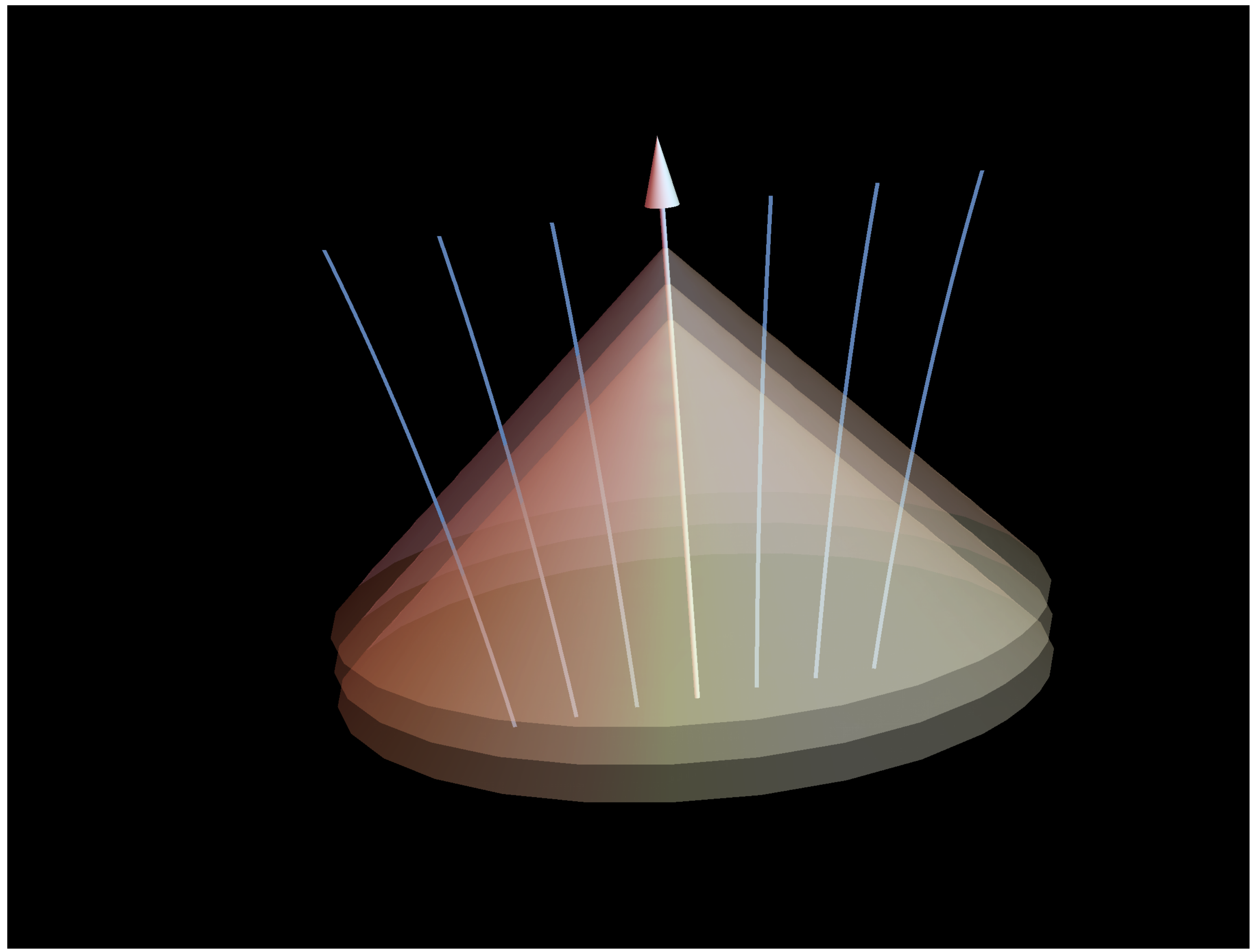
$$\frac{t_{obs}}{t_H} = \frac{10\,\mathrm{ys}}{1.4 \cdot 10^{10}\,\mathrm{ys}}$$



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Possible to measure directly *H(z)*

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In practice: peculiar motions, inhomogeneities (lensing, time-dependent light bending), time-dependent potential wells …

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Feasibility of observations

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- E-ELT, CODEX spectrograph (Ly-α forest z drift, after 1-2 decades of observations)
- VLT ESPRESSO spectrograph
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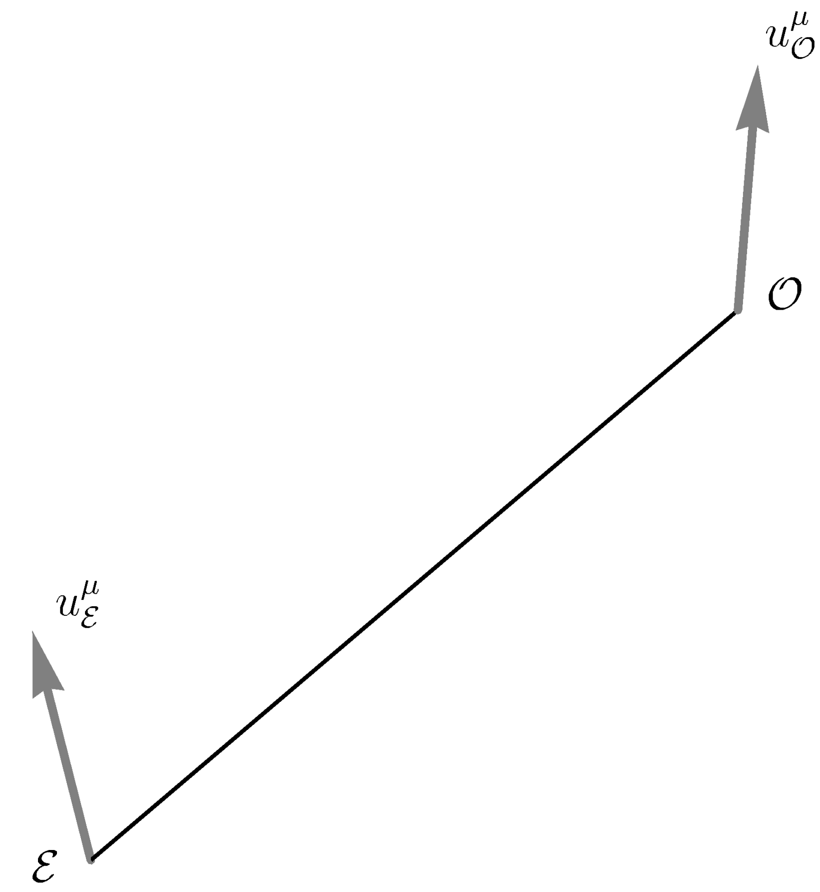
Theory

very simplified: Loeb 1998, Balbi, Quercellini 2007, Uzan et al 2008…

exact models: Krasiński 2011, Krasiński, Bolejko 2012, Quercellini et al 2009…

Motivation

Lack of a general theory



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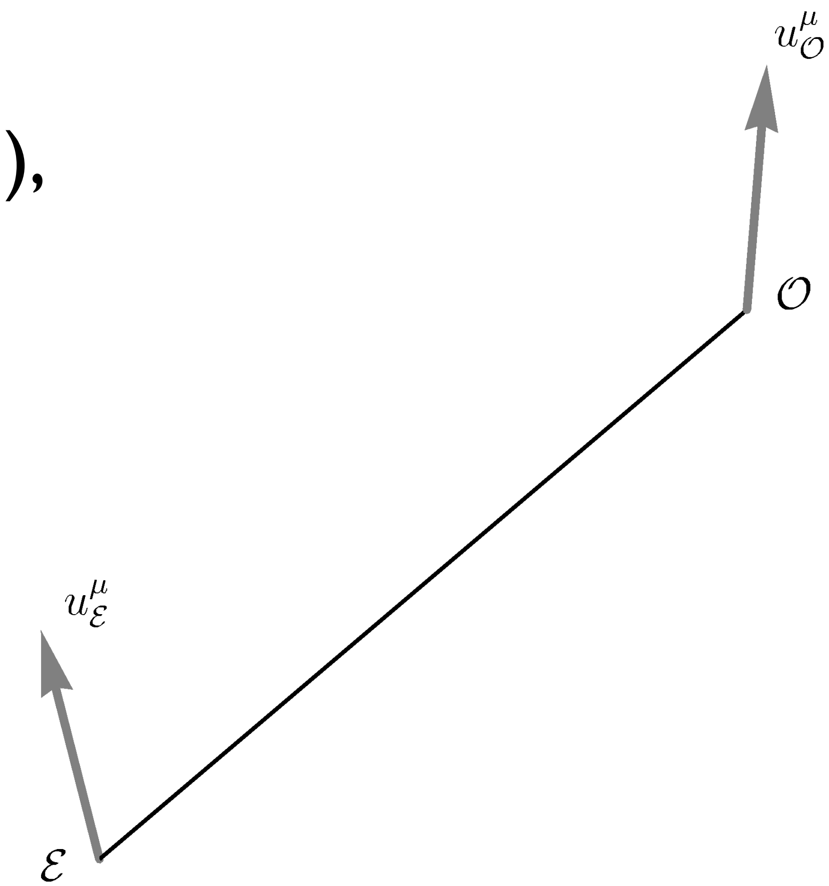

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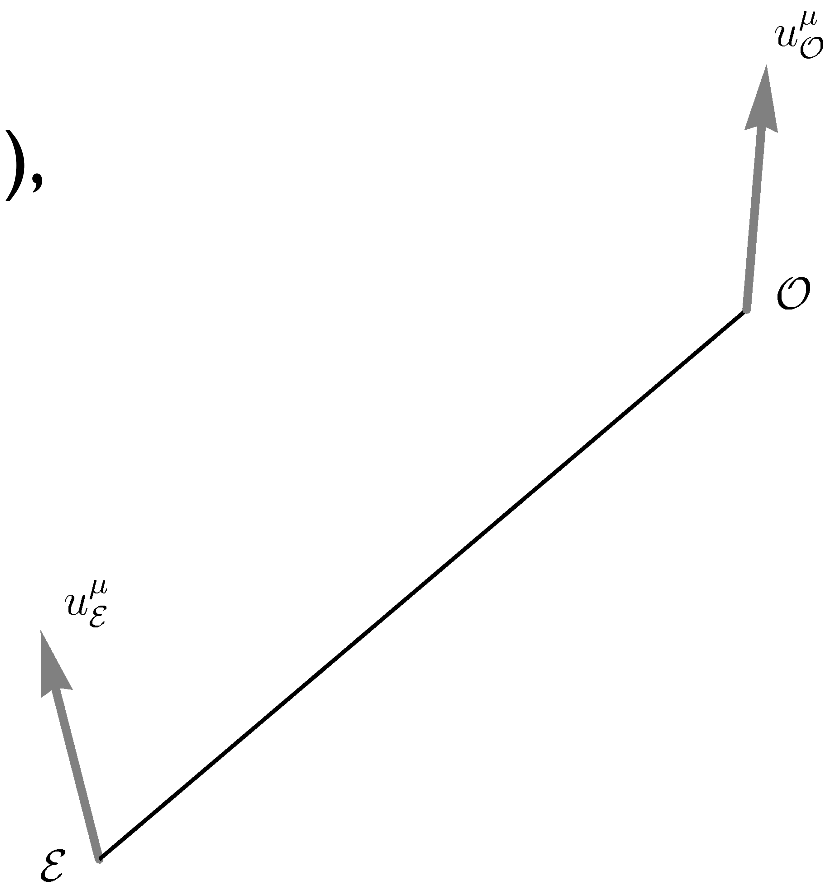

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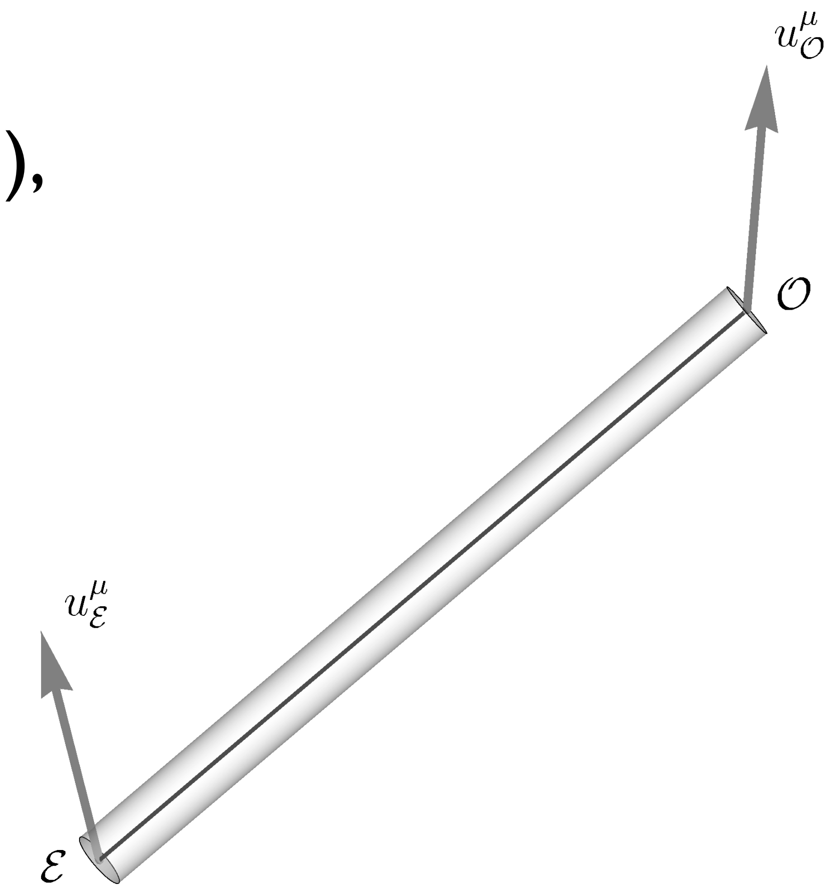

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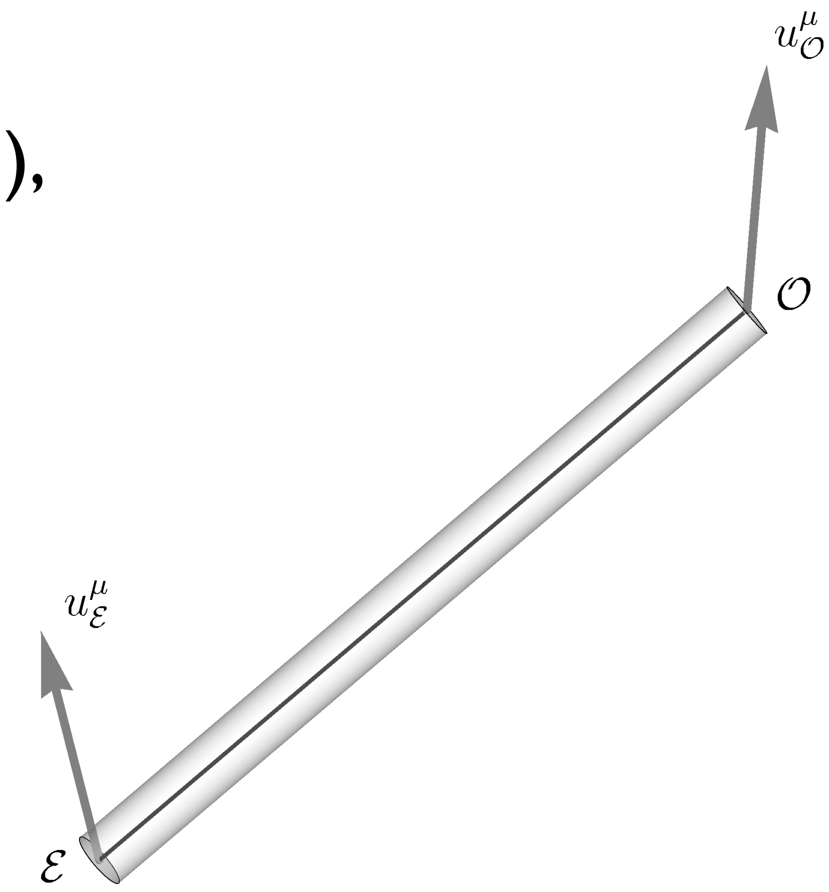

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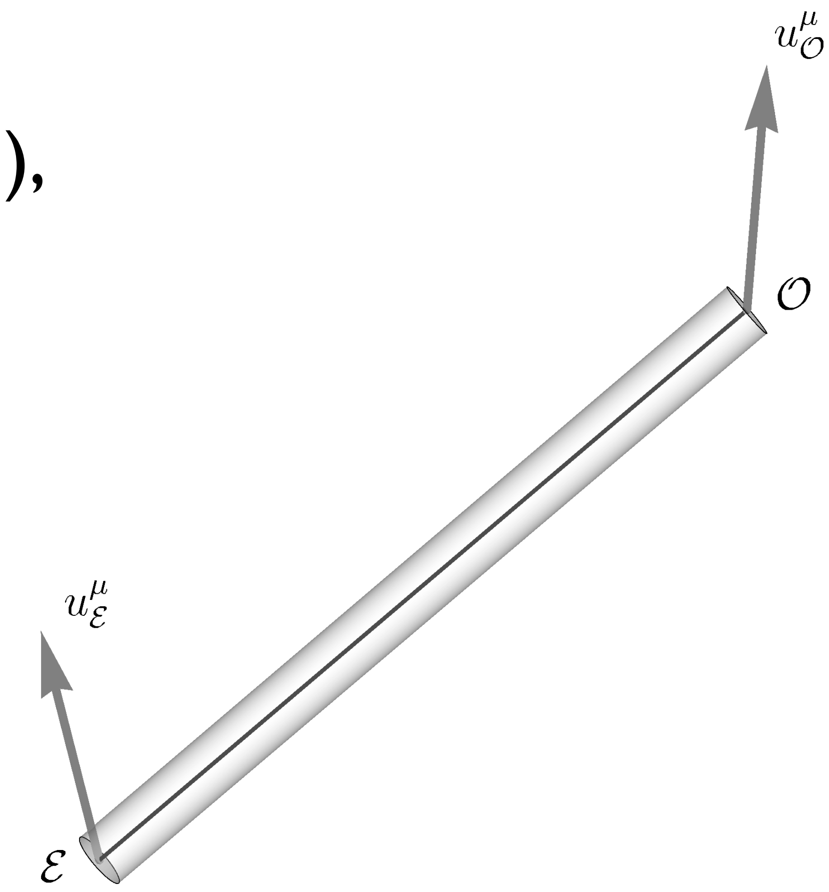

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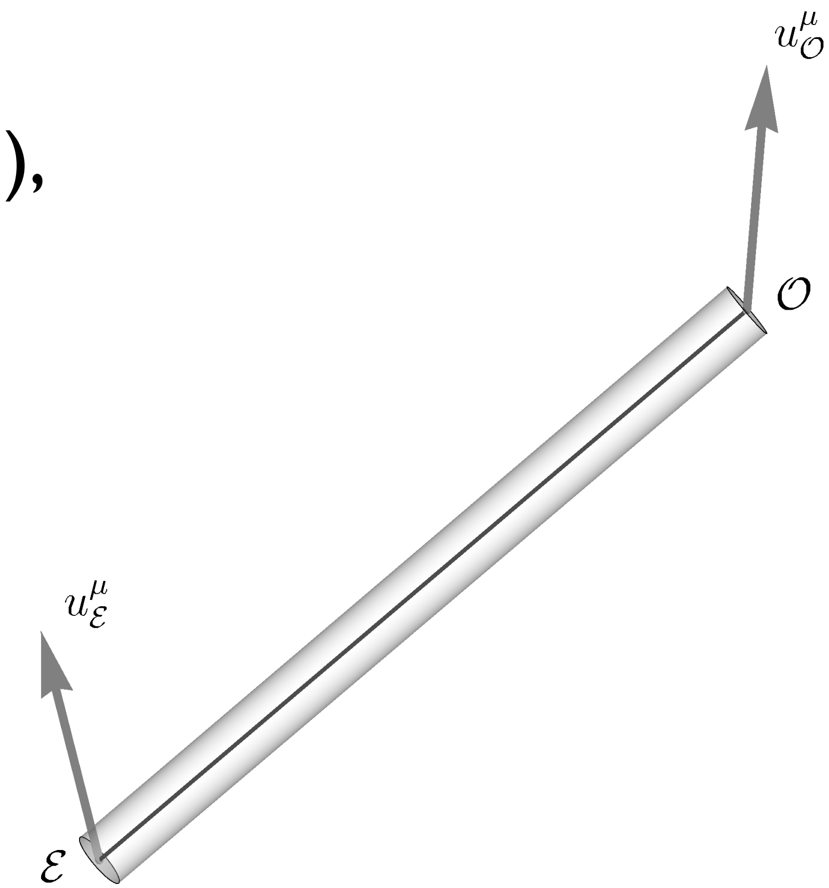

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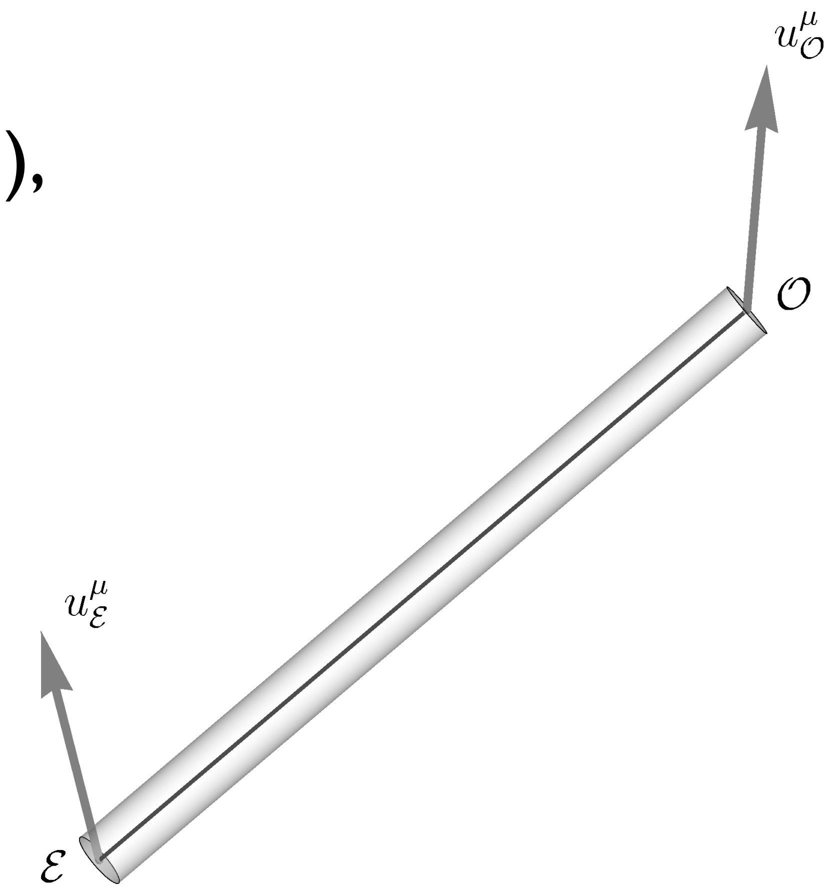

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- Based on the geodesic deviation equation (GDE)
- Exact (all GR effects, all spacetimes)
- Limitations: geometric optics, narrow beam
- Applications: numerics, gravitational lensing, (weak lensing…)

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Drift effects in GR - extending Sachs formalism

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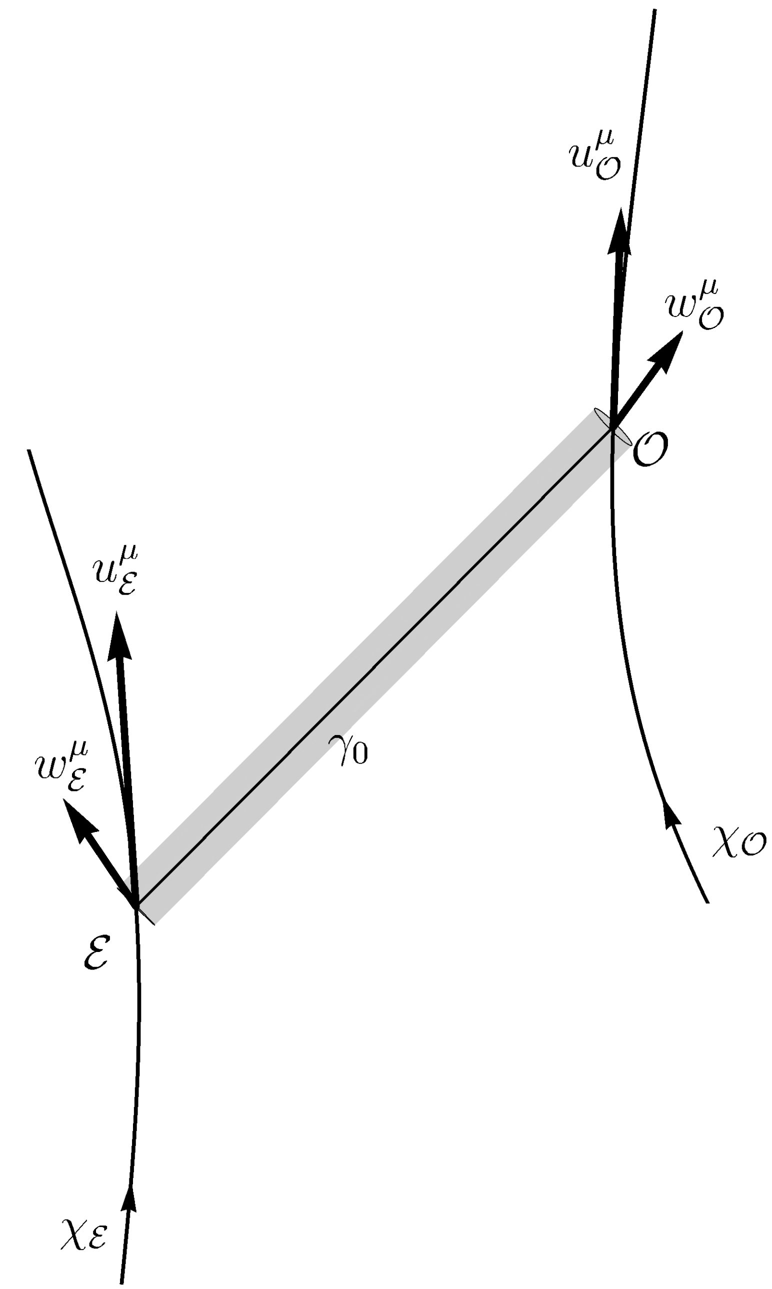

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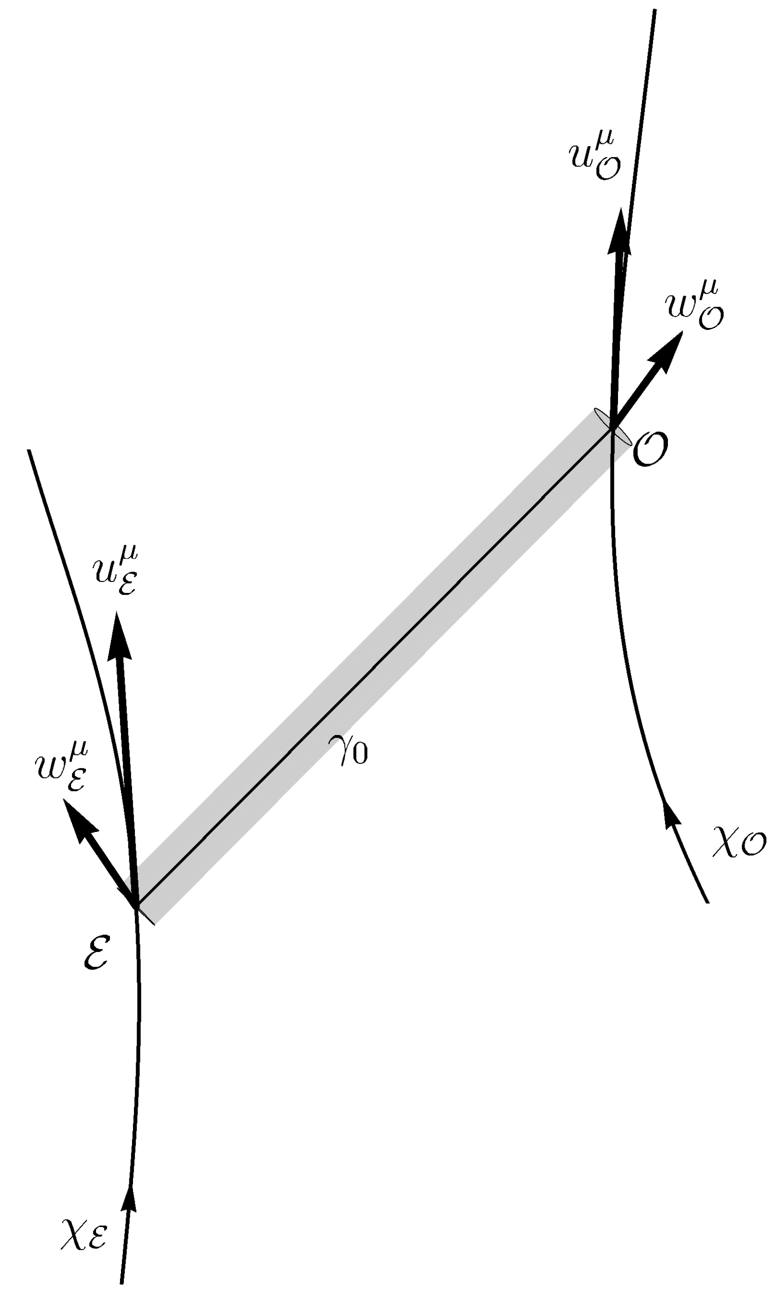

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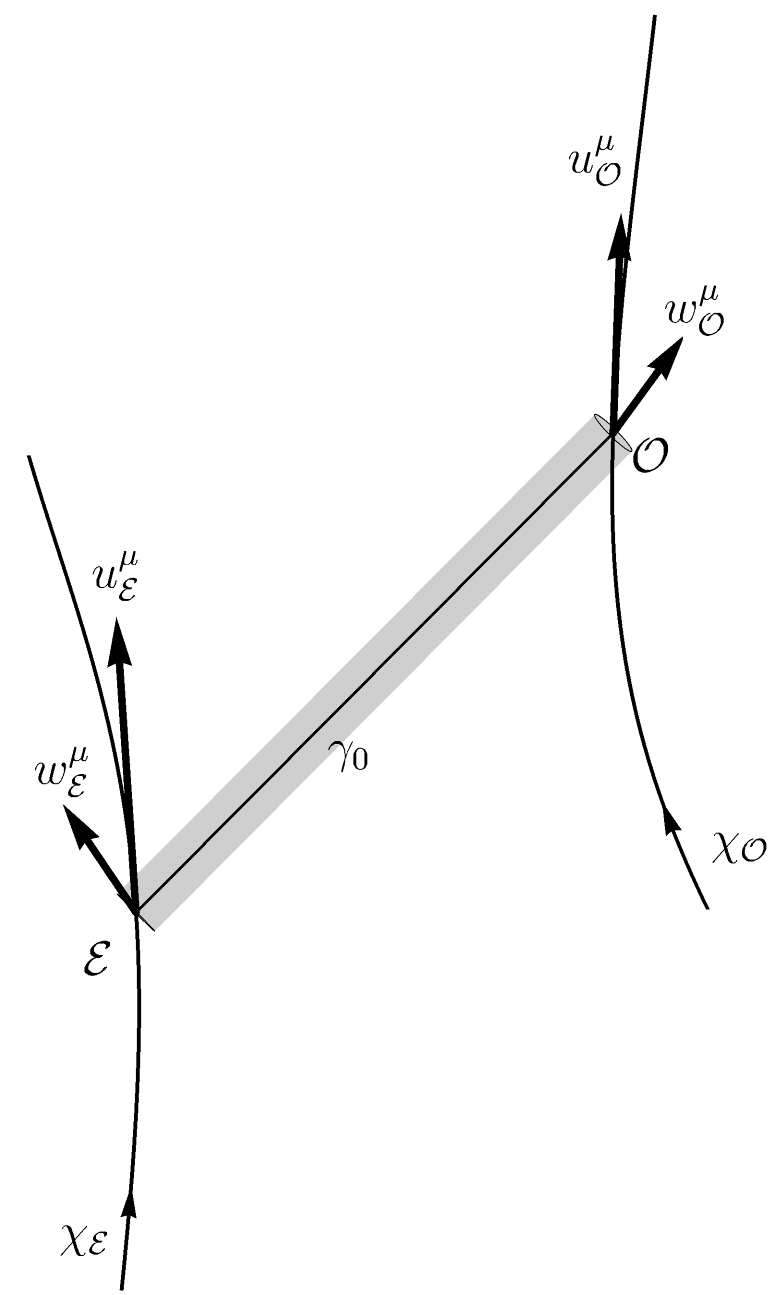

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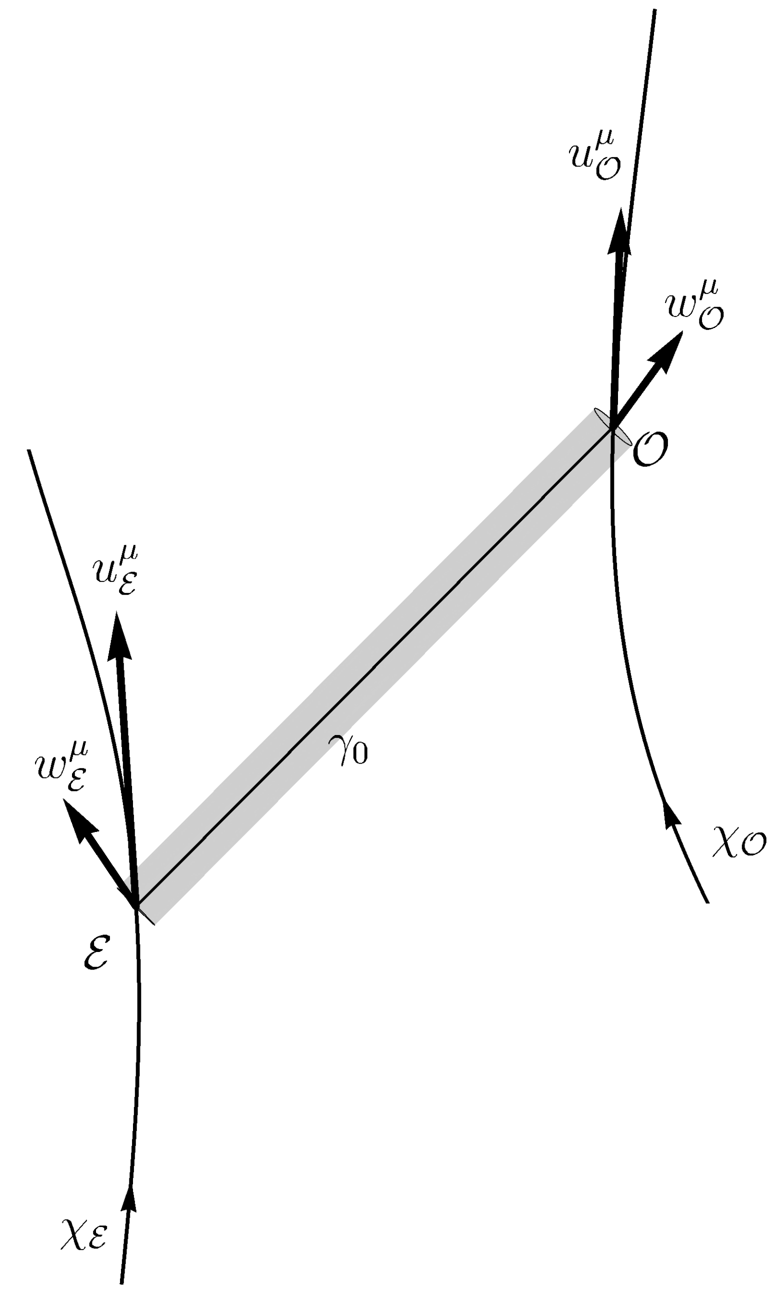
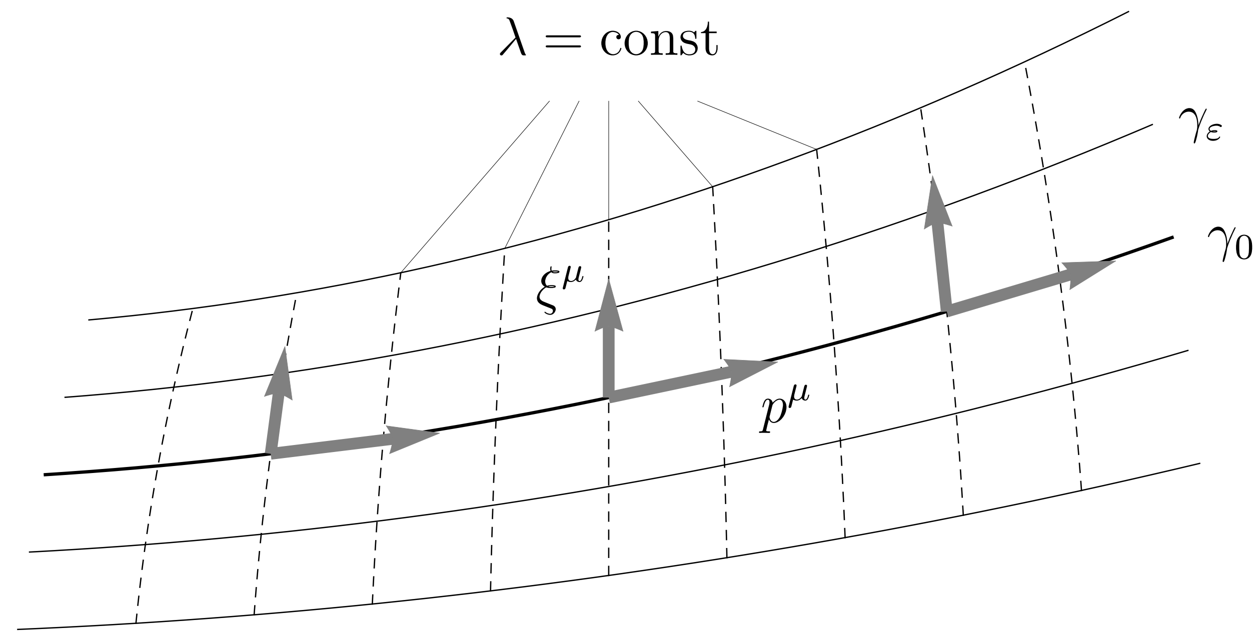


- Based on the geodesic deviation equation (GDE)
- Exact (all GR effects, all spacetimes)
- Local effects (observer, emitter) vs propagation effects

Geometry

Geodesic deviation equation

$$\mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$



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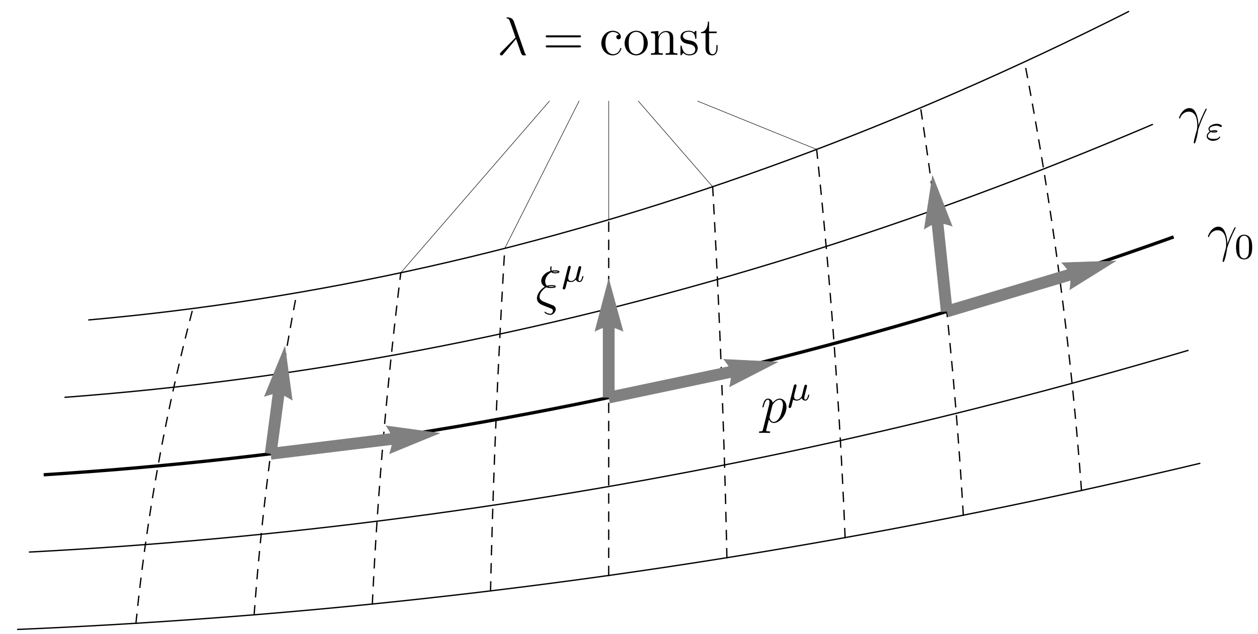

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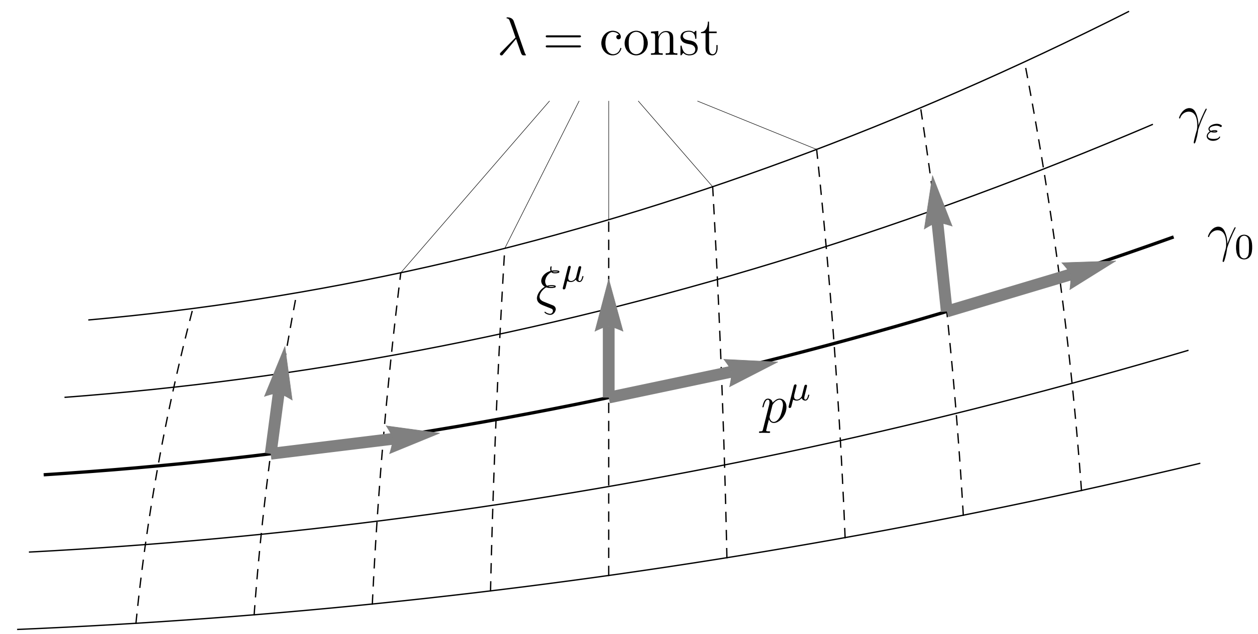

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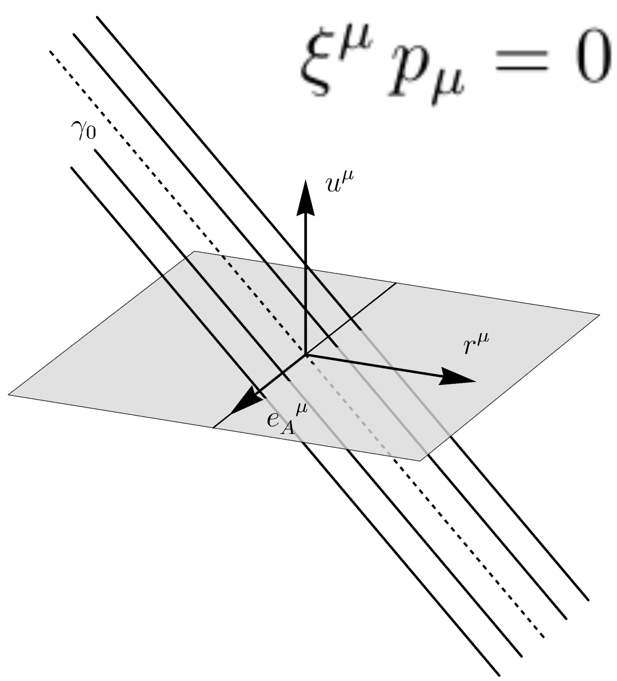

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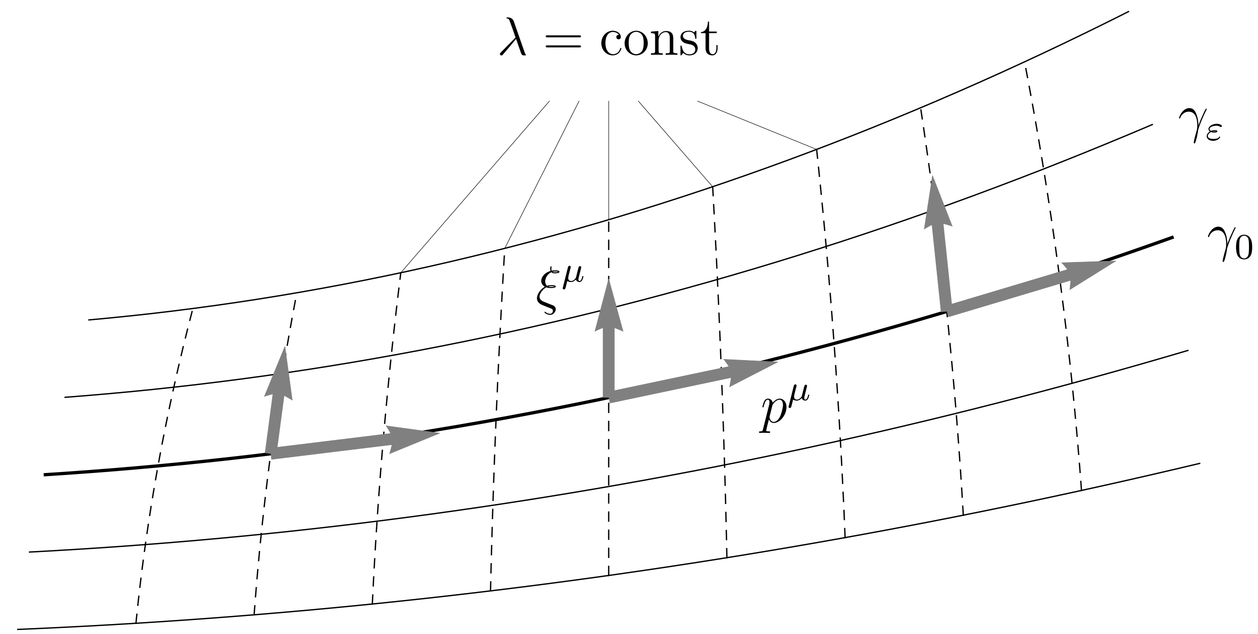

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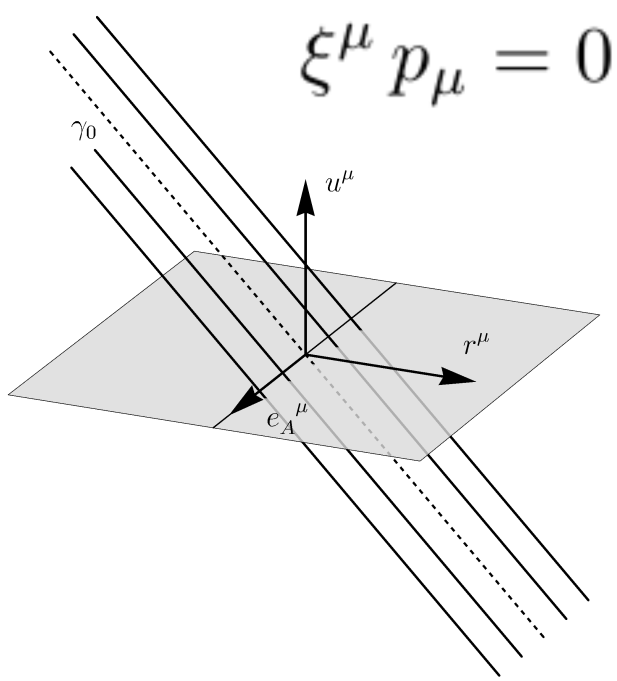
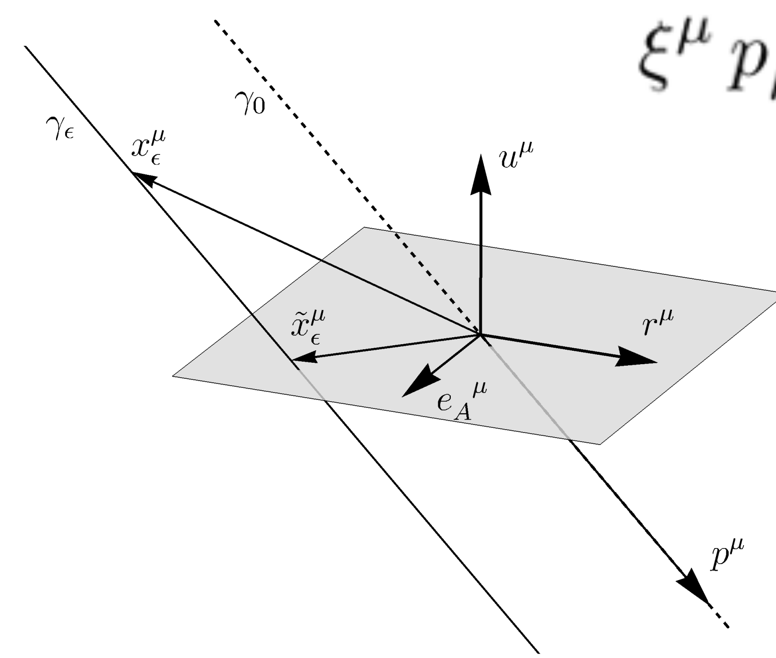


$$\xi^\mu\, p_\mu \neq 0$$



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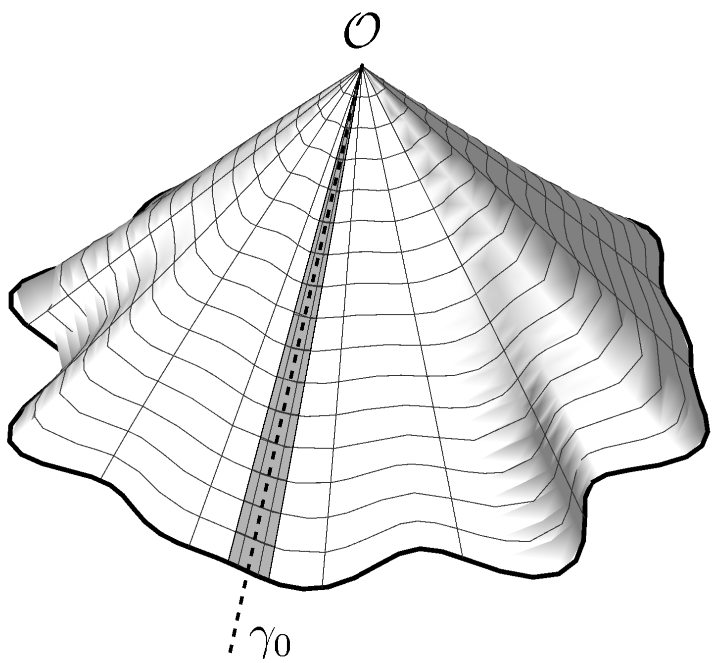
Jacobi matrix

$$\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\,\nabla_p \xi^A(\lambda_\mathcal{O})$$



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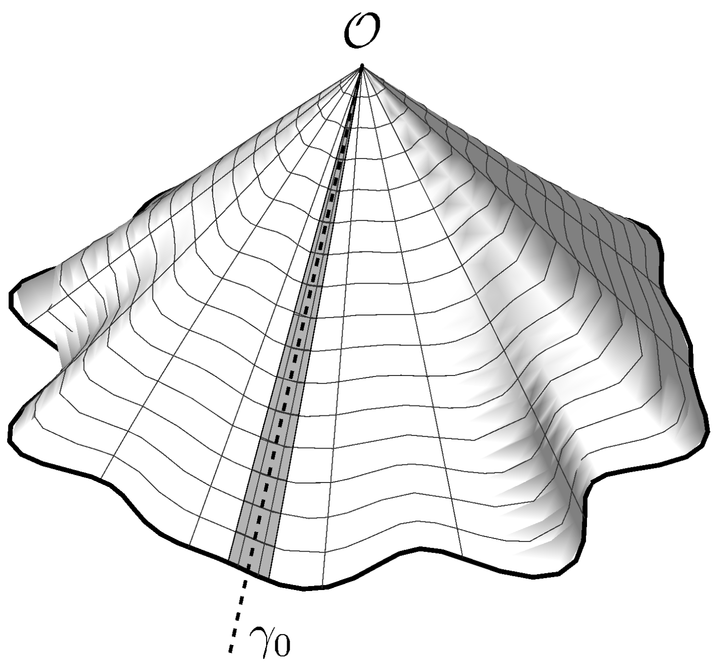
- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\mathcal{D}^A{}_B - R^A{}_{\nu\alpha C}\, p^\nu\, p^\alpha\, \mathcal{D}^C{}_B = 0$$

$$\begin{aligned} \mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) &= 0 \\ \frac{\mathrm{d}}{\mathrm{d}\lambda}\mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) &= \delta^A{}_B \end{aligned}$$



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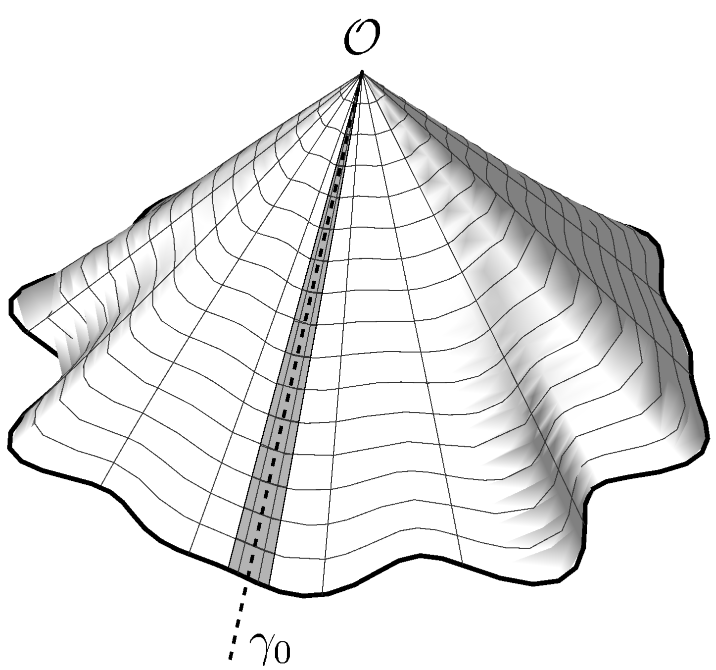

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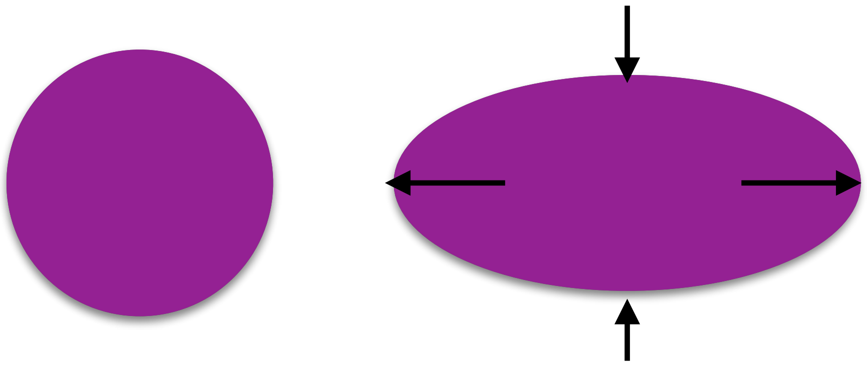
- gravitational lensing



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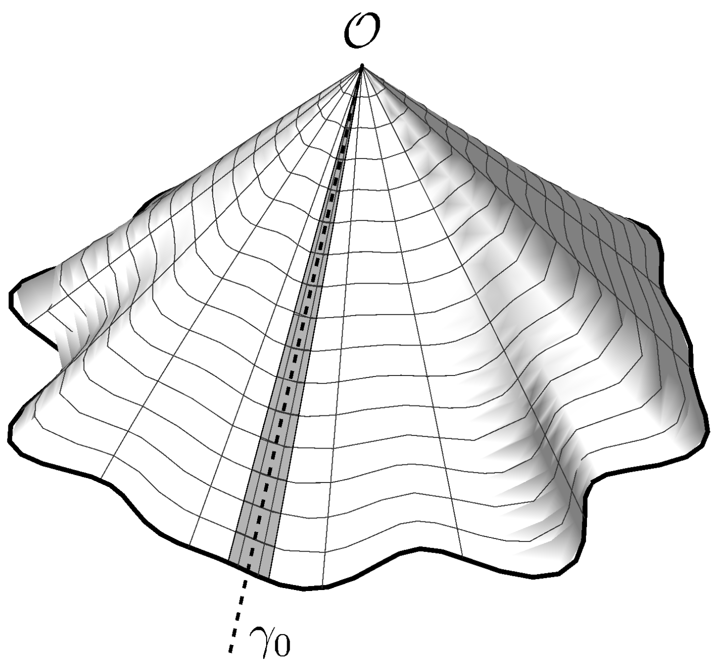
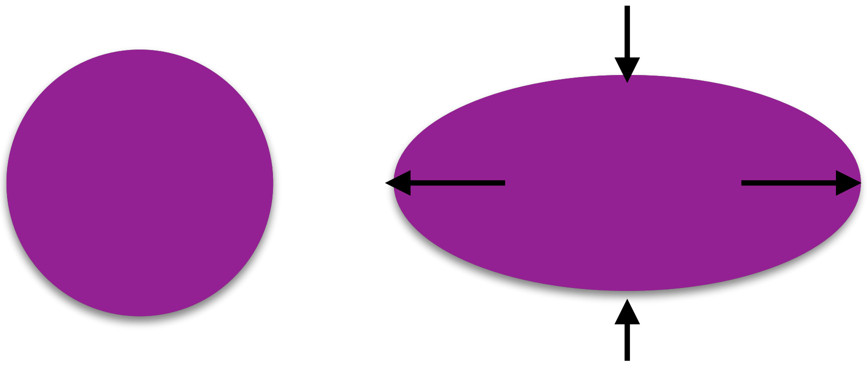
$$\xi^A(\lambda) = \mathcal{D}^A{}_B(\lambda)\,\nabla_p\xi^A(\lambda_\mathcal{O})$$



- given by ODE's

$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\mathcal{D}^A{}_B - R^A{}_{\nu\alpha C}\,p^\nu\,p^\alpha\,\mathcal{D}^C{}_B = 0$$
$$\mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) = 0$$
$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathcal{D}^A{}_B\,(\lambda_\mathcal{O}) = \delta^A{}_B$$

- gravitational lensing



- area, luminosity distances

$$D_{ang} = \left(p_\mu\,u^\mu_\mathcal{O}\right)\,\left|\det\mathcal{D}^A{}_B\,(\lambda_\mathcal{E})\right|^{1/2} \qquad D_{lum} = D_{ang}(1+z)^2$$

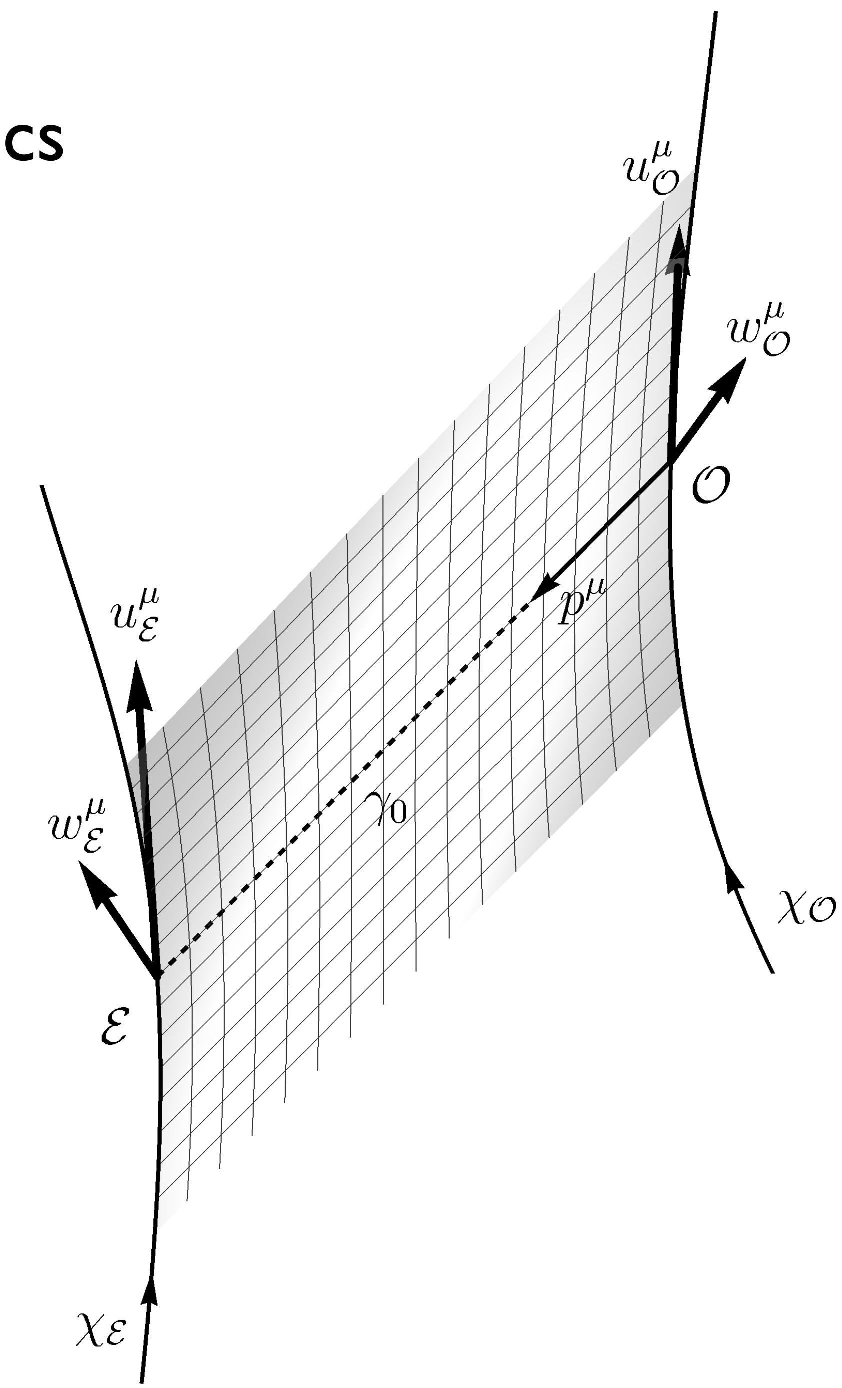
Geometry

Geometric setup

Geometry

Geometric setup

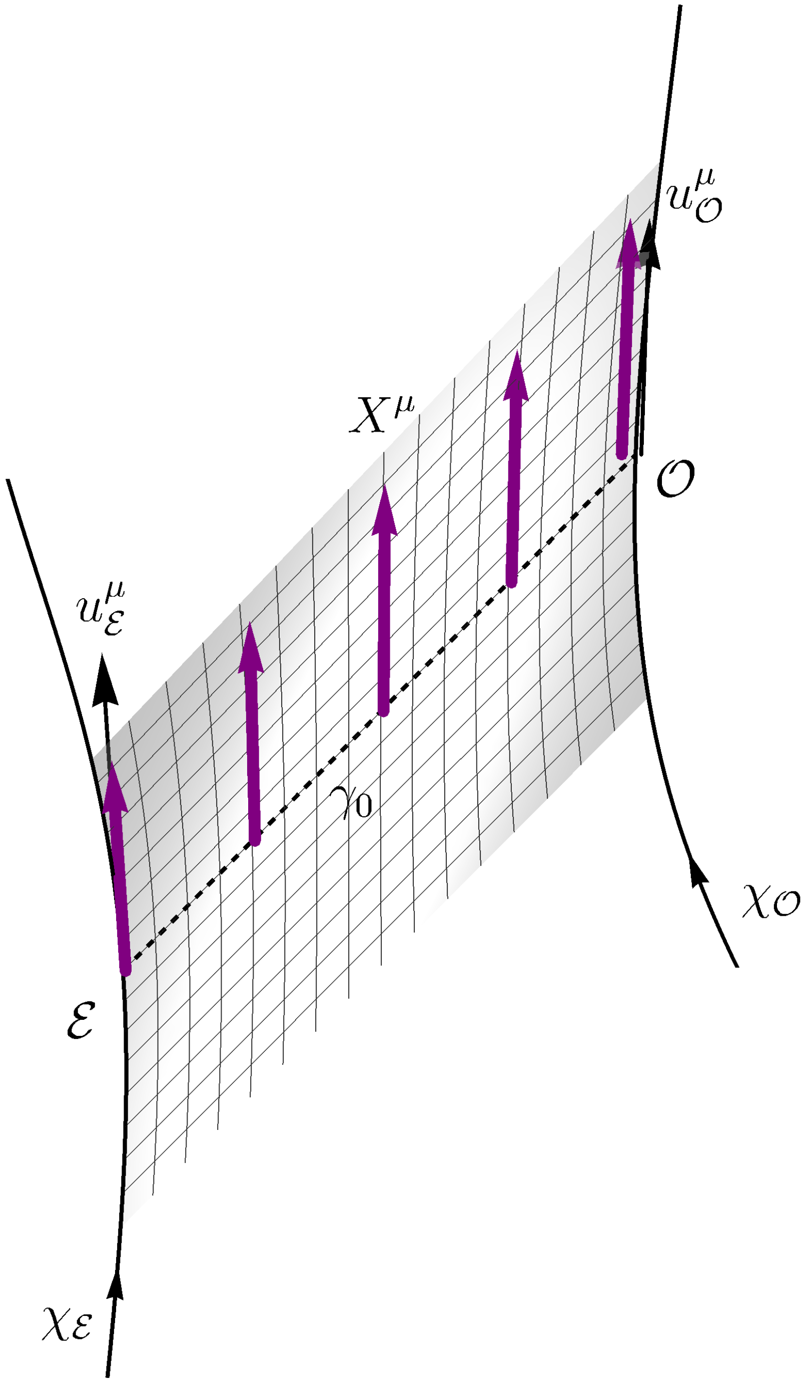
- null surface spanned by connecting null geodesics



Geometry

Geometric setup

- null surface spanned by connecting null geodes
- main tool: observation time vector $X^{\boldsymbol{\mu}}$



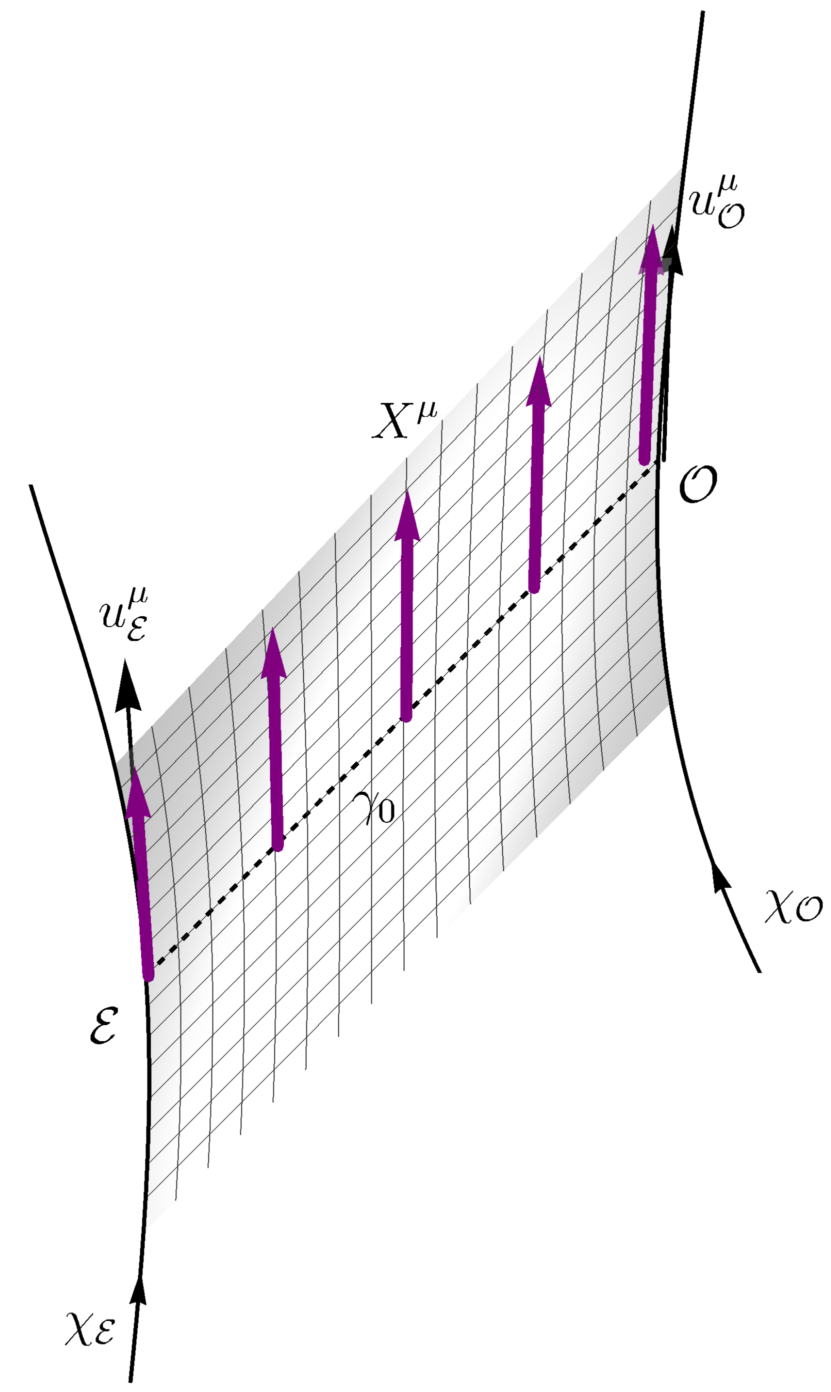
Geometry

Geometric setup

- null surface spanned by connecting null geodes
- main tool: observation time vector X^{μ}

$$\mathcal{G}[X]^{\mu} = 0$$

$$\begin{aligned} X^{\mu}(\lambda_{\mathcal{O}}) &= u^{\mu}_{\mathcal{O}} \\ X^{\mu}(\lambda_{\mathcal{E}}) &= \frac{1}{1+z}\, u^{\mu}_{\mathcal{E}} \end{aligned}$$



Geometry

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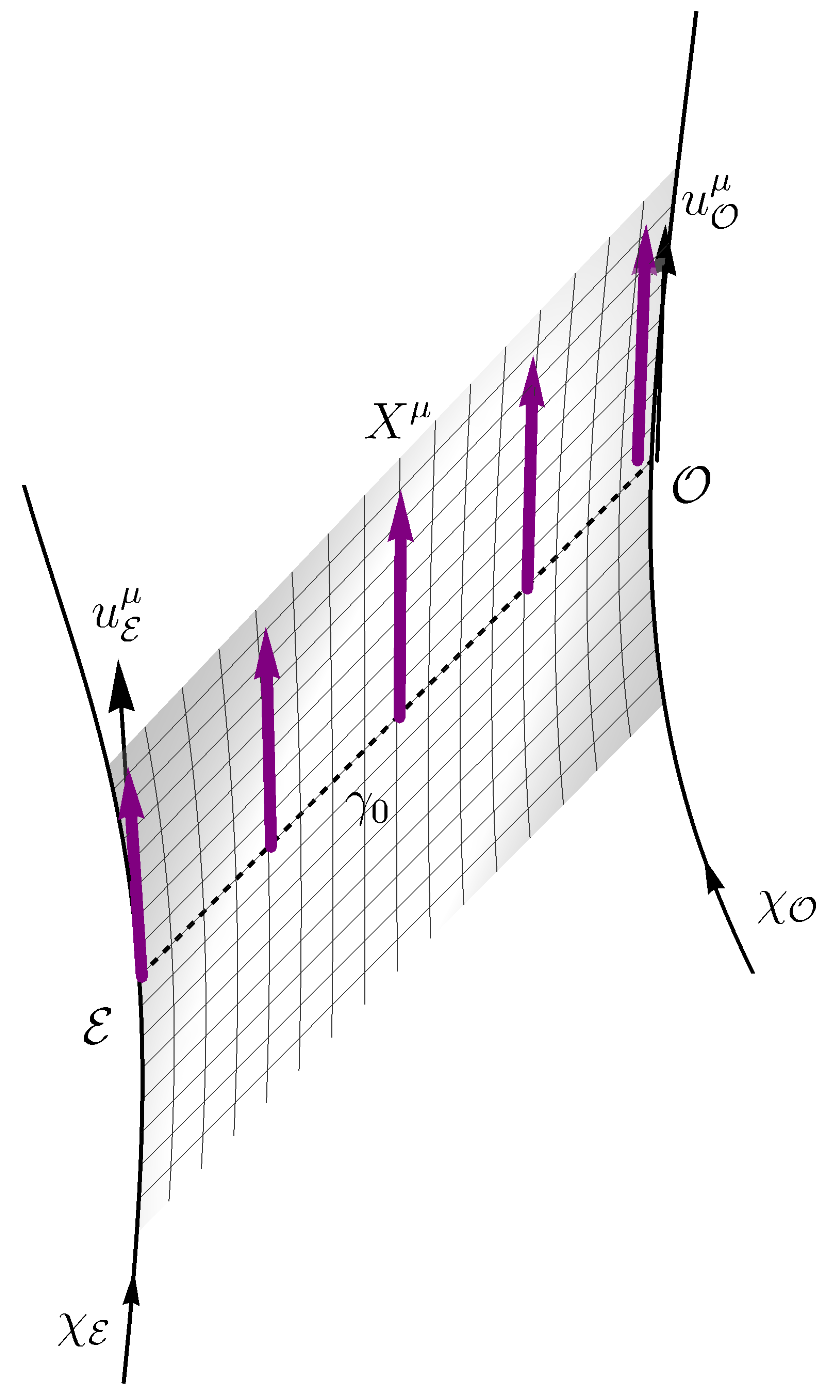
$$X^{\mu}(\lambda_{\mathcal{O}}) = u^{\mu}_{\mathcal{O}}$$

$$X^{\mu}(\lambda_{\mathcal{E}}) = \frac{1}{1+z}\, u^{\mu}_{\mathcal{E}}$$

- can be solved (up to $C \cdot p^{\mu}$) using $\mathcal{D}^{A}{}_{B}$



Geometry

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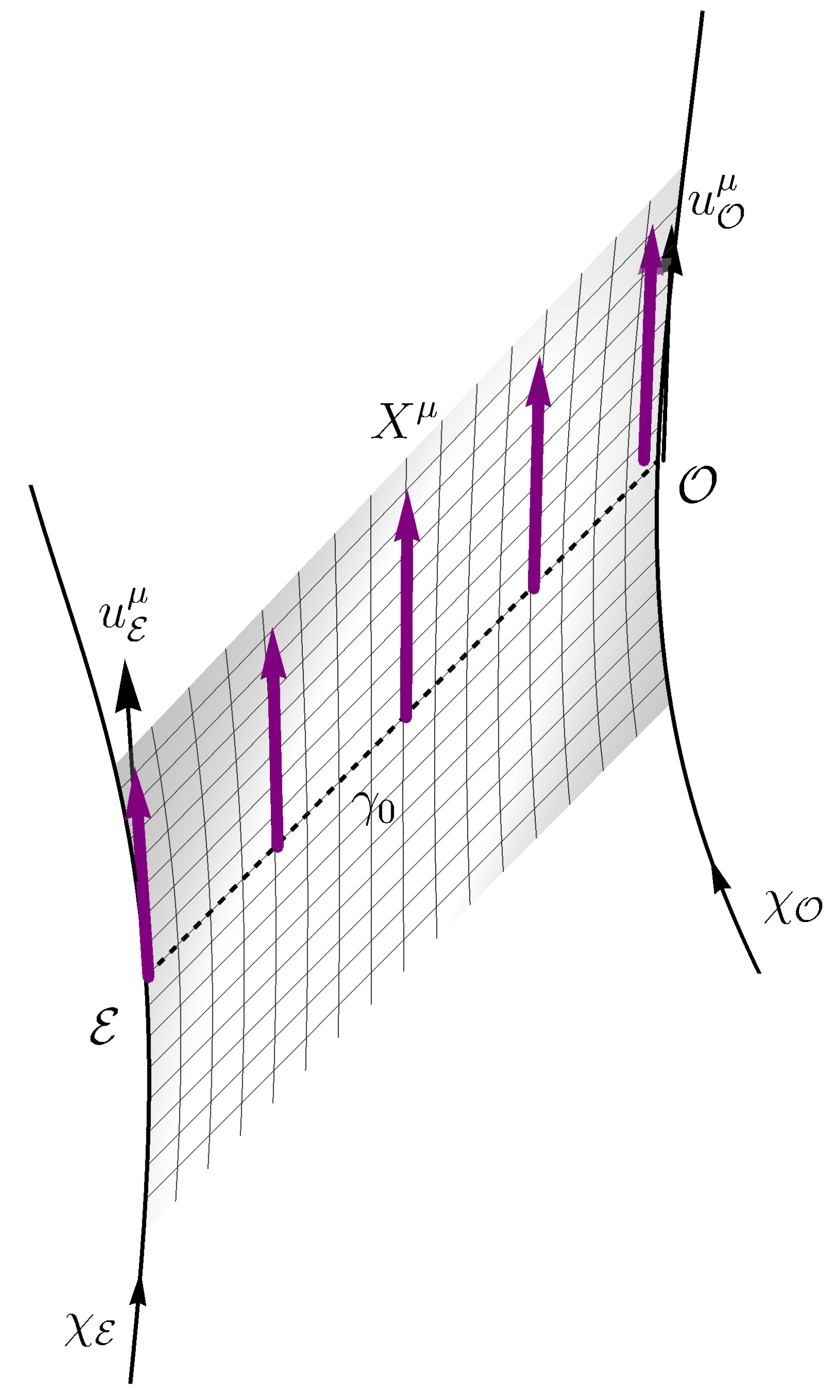
- can be solved (up to $C \cdot p^{\mu}$) using $\mathcal{D}^A{}_B$
- ∇_X gives the drift effects



Geometry

Geometric setup

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$$\mathcal{G}[X]^\mu = 0$$

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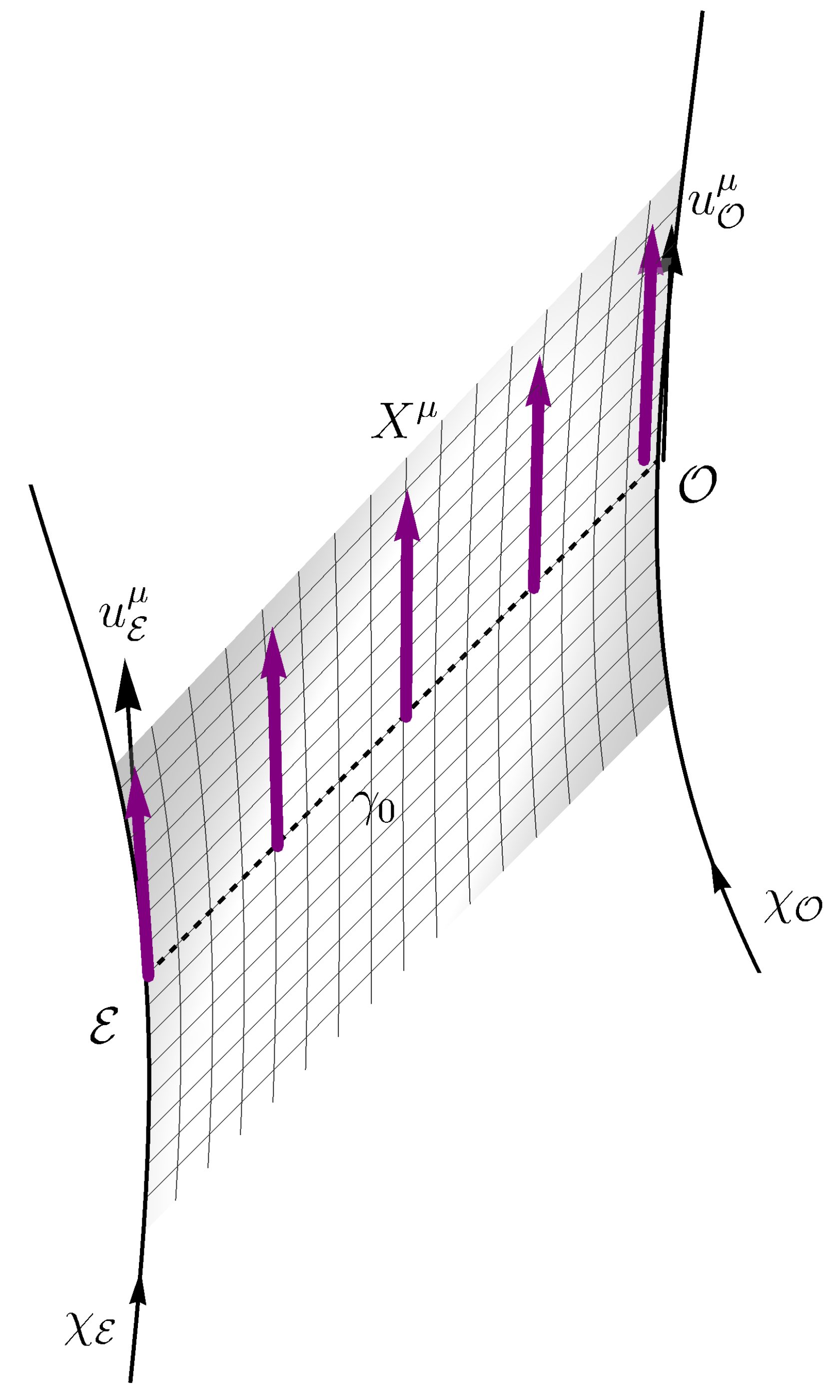
- can be solved (up to $C \cdot p^{\mu}$) using $\mathcal{D}^A{}_B$
- ∇_X gives the drift effects

$\nabla_X p^{\mu}$ - position drift



Geometry

Geometric setup

- null surface spanned by connecting null geodes
- main tool: observation time vector X^μ

$$\mathcal{G}[X]^\mu = 0$$

$$X^\mu(\lambda_\mathcal{O}) = u^\mu_\mathcal{O}$$

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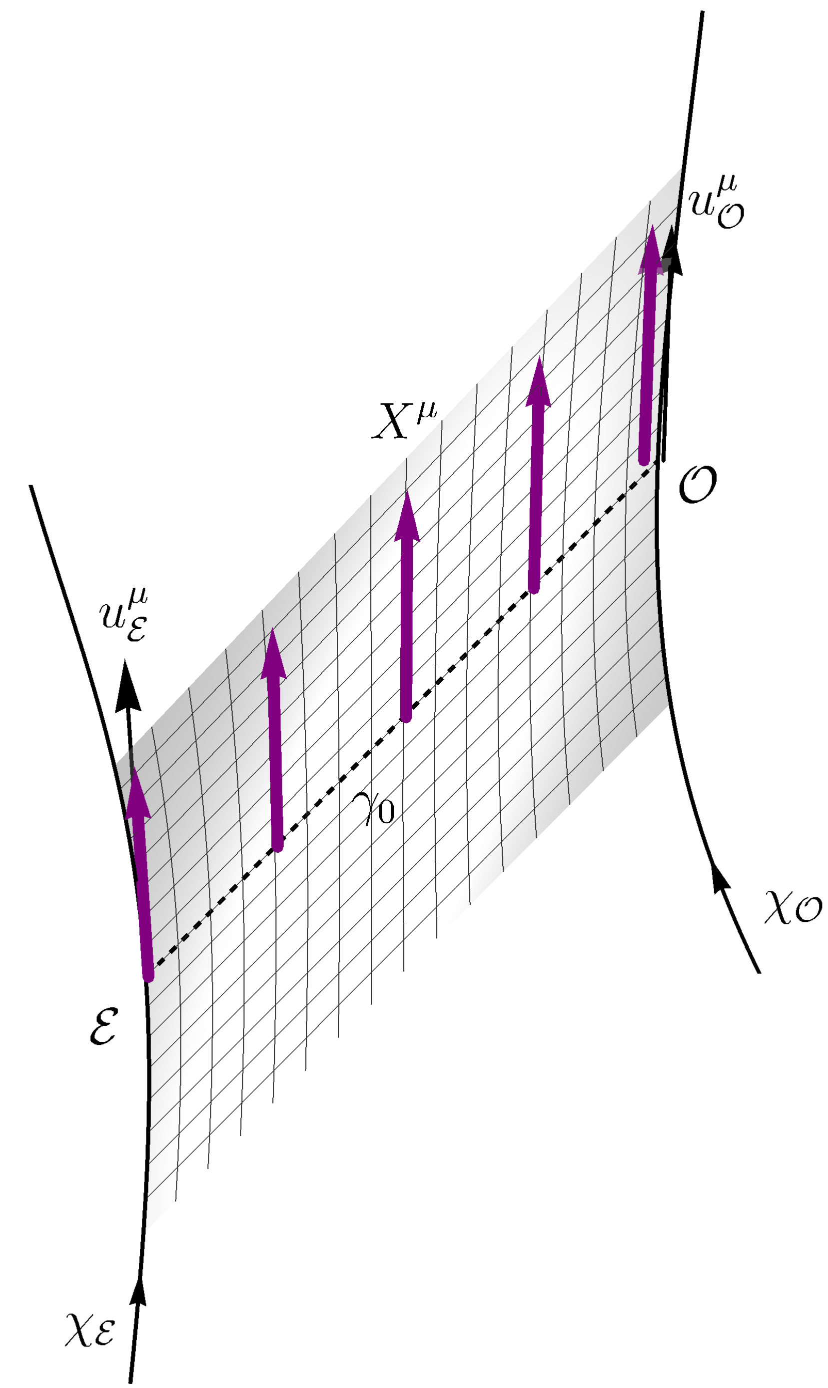
- can be solved (up to $C \cdot p^\mu$) using $\mathcal{D}^A{}_B$
- ∇_X gives the drift effects

  $\nabla_X p^\mu$ - position drift

  $\nabla_X (p_\mu u^\mu)$ - redshift drift



Geometry

Geometric setup

- null surface spanned by connecting null geodes
- main tool: observation time vector X^{μ}

$$\mathcal{G}[X]^{\mu} = 0$$

$$X^{\mu}(\lambda_{\mathcal{O}}) = u^{\mu}_{\mathcal{O}}$$

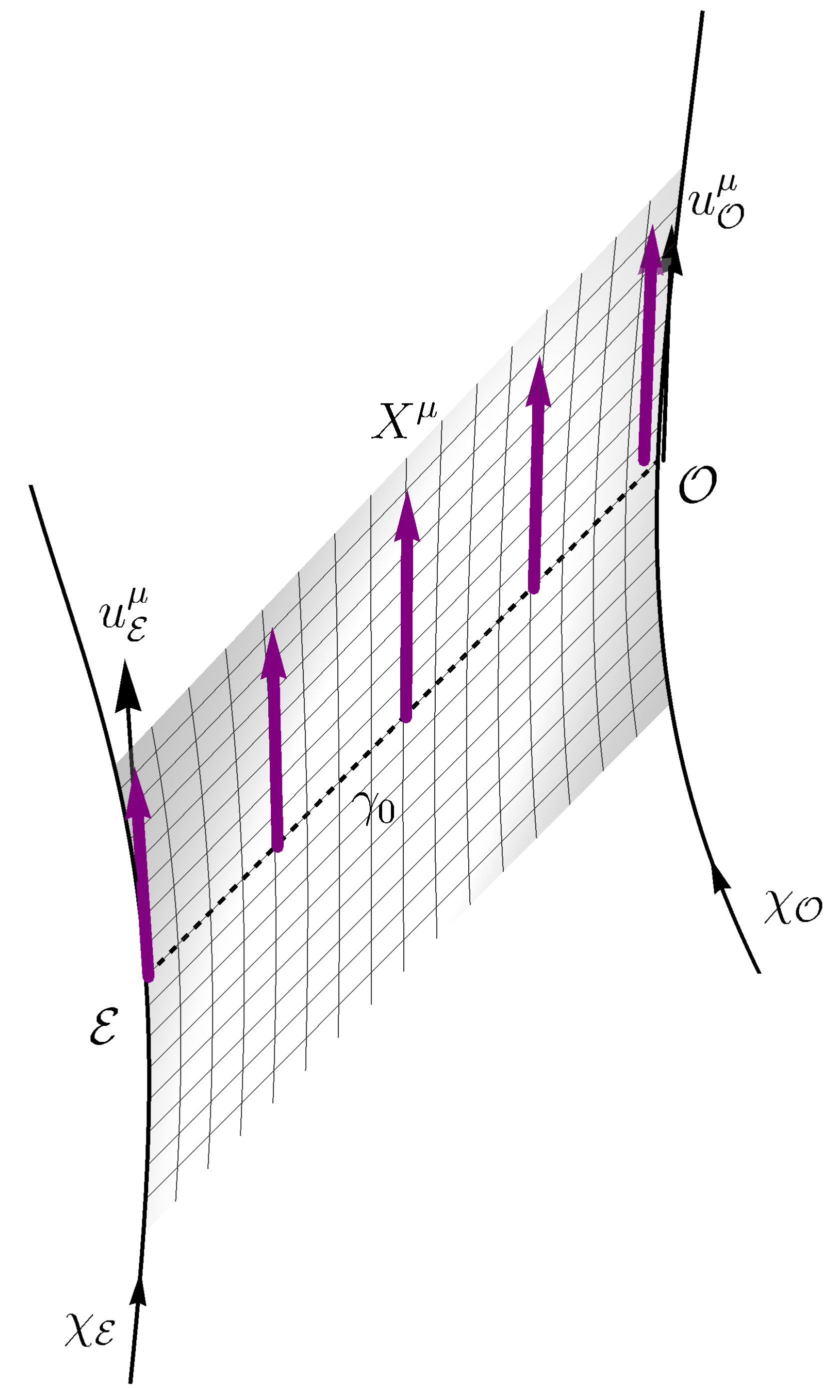
$$X^{\mu}(\lambda_{\mathcal{E}}) = \frac{1}{1+z}\, u^{\mu}_{\mathcal{E}}$$

- can be solved (up to $C \cdot p^{\mu}$) using $\mathcal{D}^{A}{}_{B}$
- ∇_X gives the drift effects

  $\nabla_X p^{\mu}$ - position drift

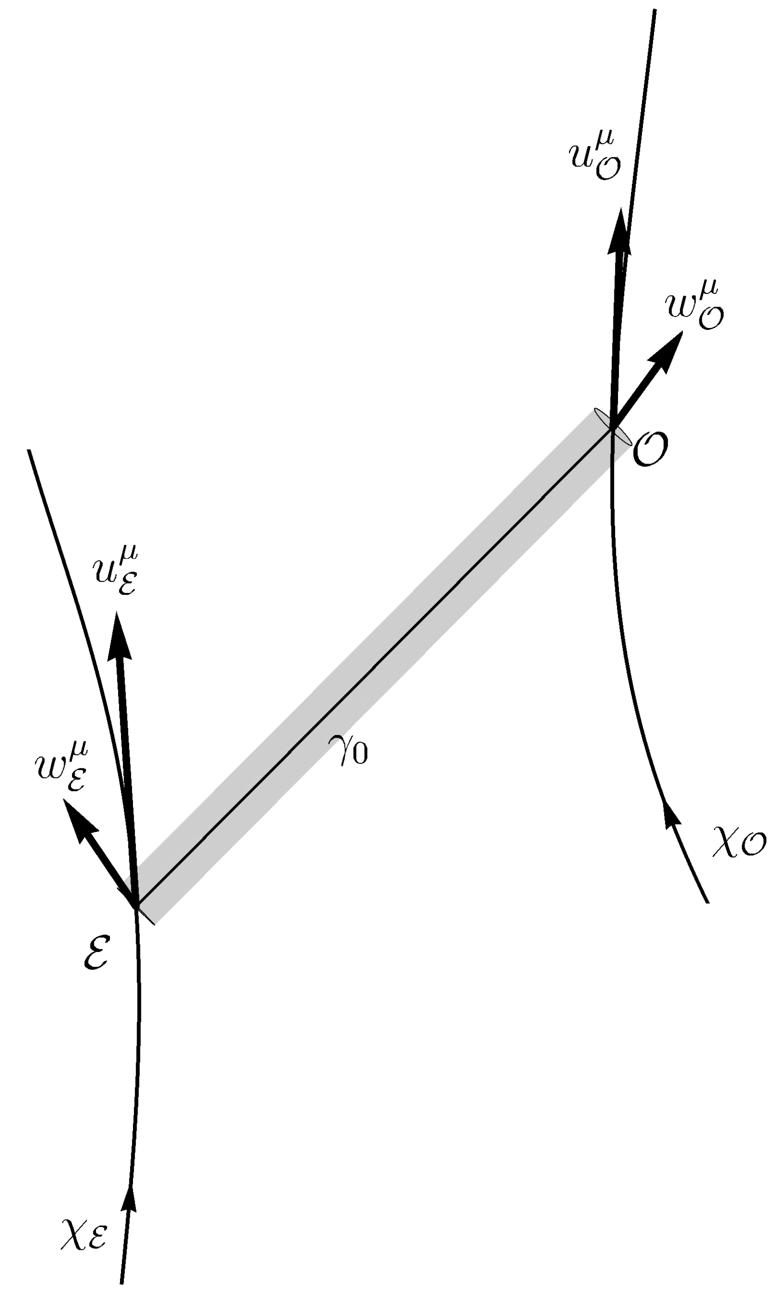
  $\nabla_X (p_{\mu} u^{\mu})$ - redshift drift

  $\nabla_X \mathcal{D}^{A}{}_{B}$ - distances drift



Drift effects

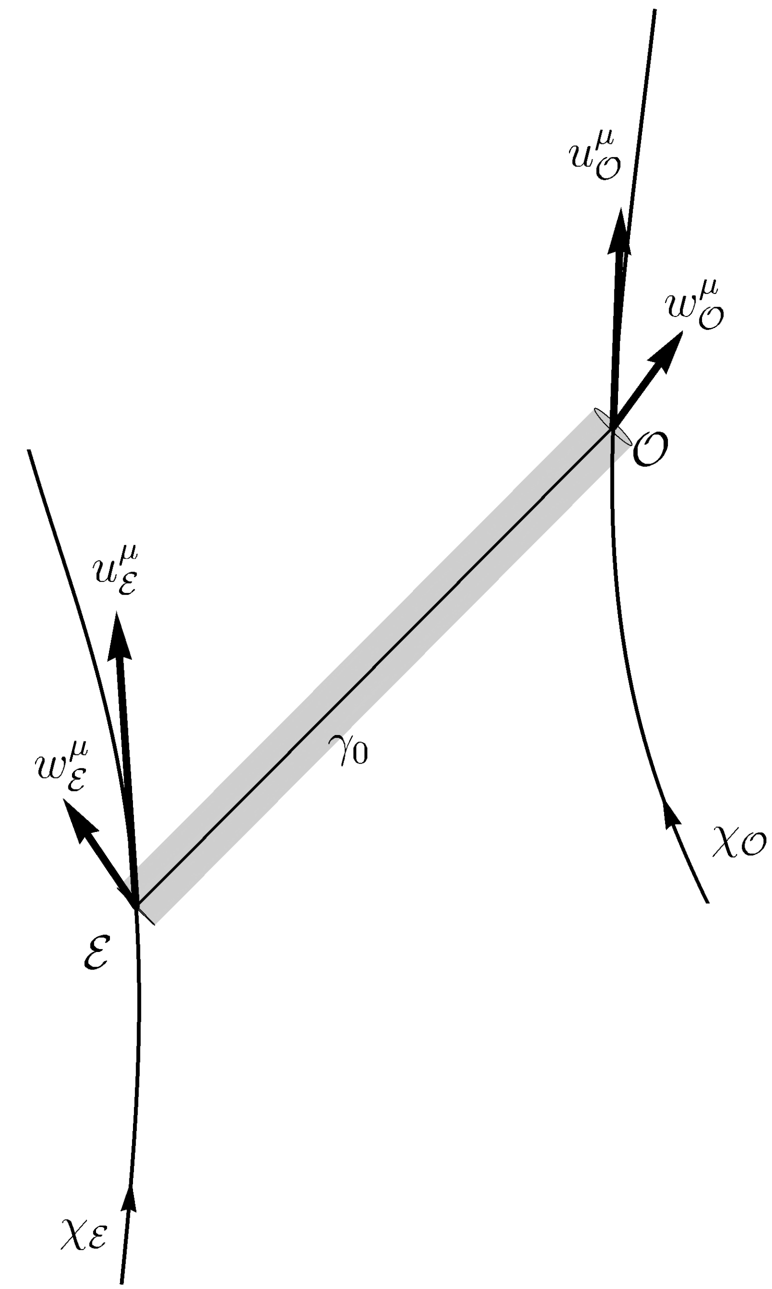
Position drift



Drift effects

Position drift

- parallel propagation of $\hat{u}_{\mathcal{O}}^{\mu}$

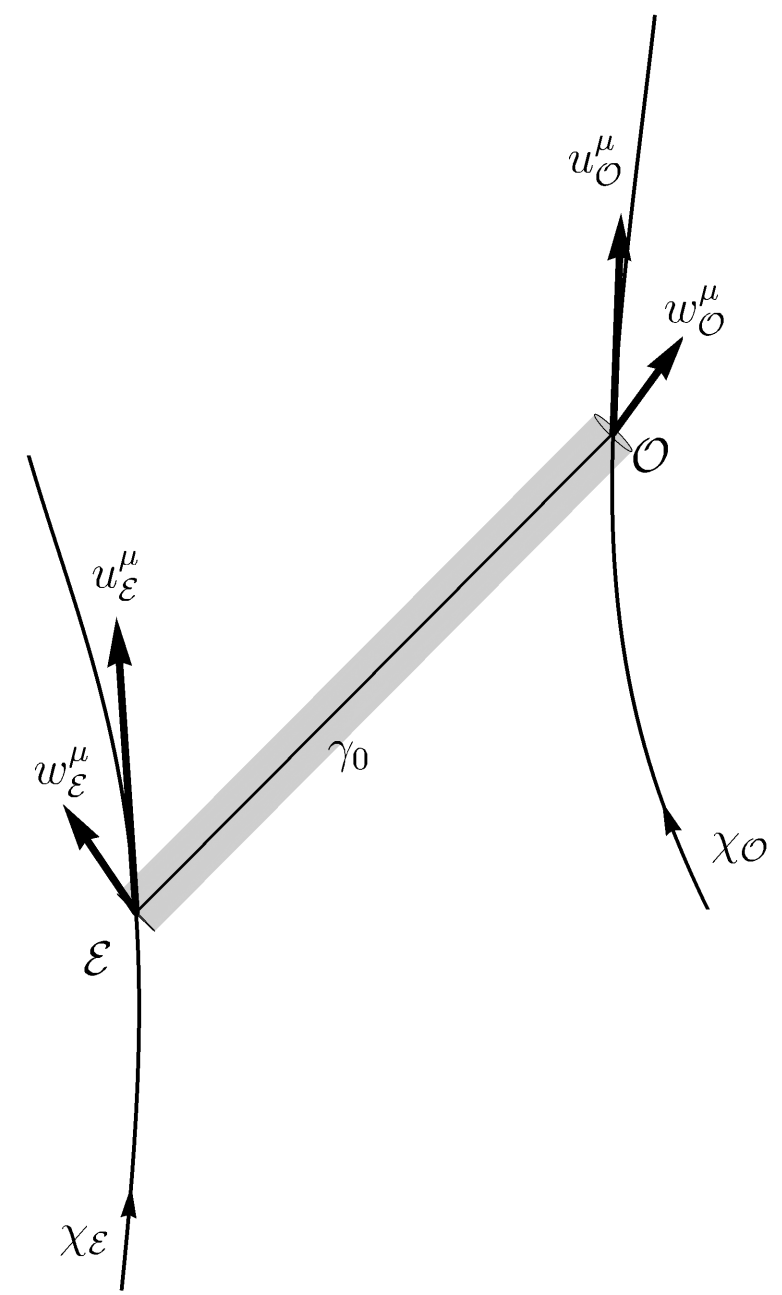


Drift effects

Position drift

- parallel propagation of $\hat{u}^{\mu}_{\mathcal{O}}$
- inhomogeneous (perpendicular) GDE

$$
\begin{aligned}
\mathcal{G}[m]^A &= R^A{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \hat{u}^\beta_{\mathcal{O}} \\
m^A(\lambda_{\mathcal{O}}) &= 0 \\
\nabla_p m^A(\lambda_{\mathcal{O}}) &= 0
\end{aligned}
$$



Drift effects

Position drift

- parallel propagation of $\hat{u}^\mu_{\mathcal{O}}$
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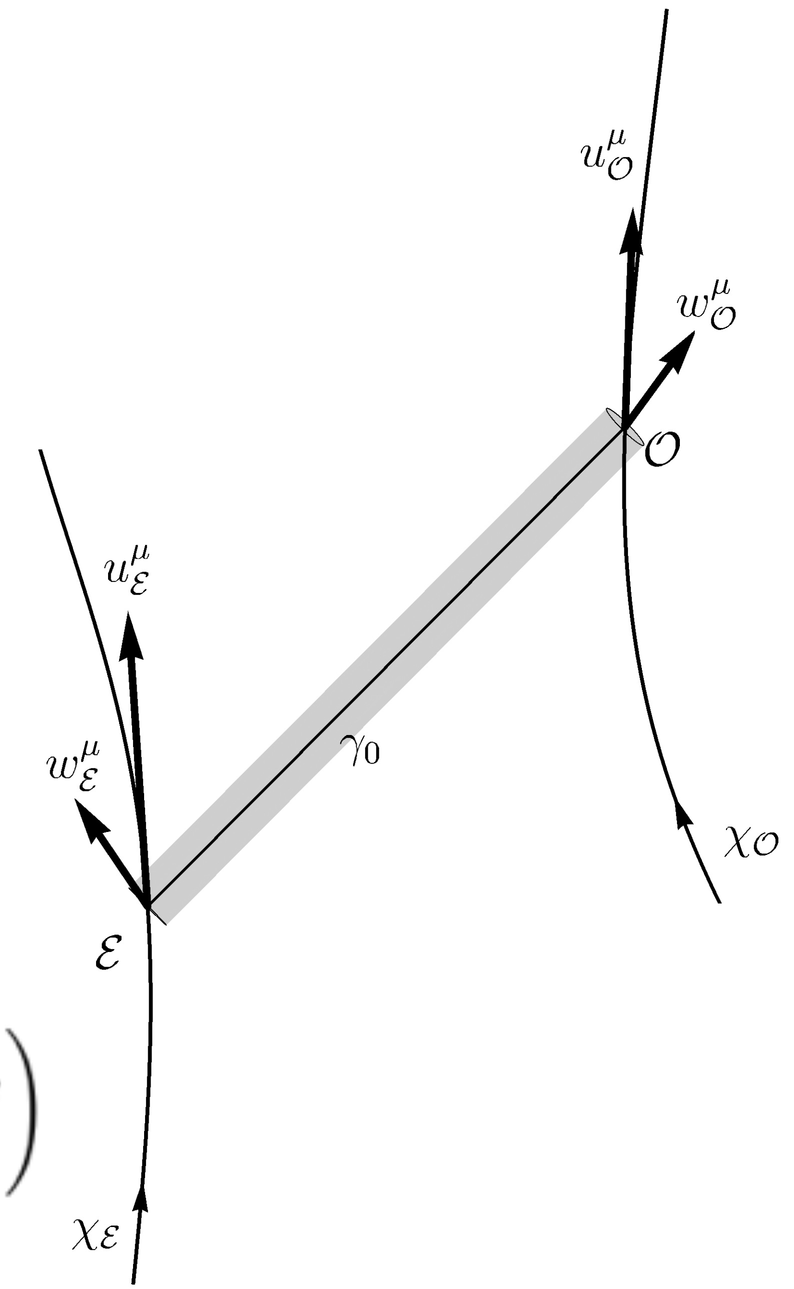
$$\begin{aligned} \mathcal{G}[m]^A &= R^A{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \hat{u}^\beta_{\mathcal{O}} \\ m^A(\lambda_{\mathcal{O}}) &= 0 \\ \nabla_p m^A(\lambda_{\mathcal{O}}) &= 0 \end{aligned}$$

- result

$$\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, \mathcal{D}^{-1}(\lambda_{\mathcal{E}})^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B \right)$$



Drift effects

Position drift

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- inhomogeneous (perpendicular) GDE

$$
\begin{aligned}
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m^A(\lambda_\mathcal{O}) &= 0 \\
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\end{aligned}
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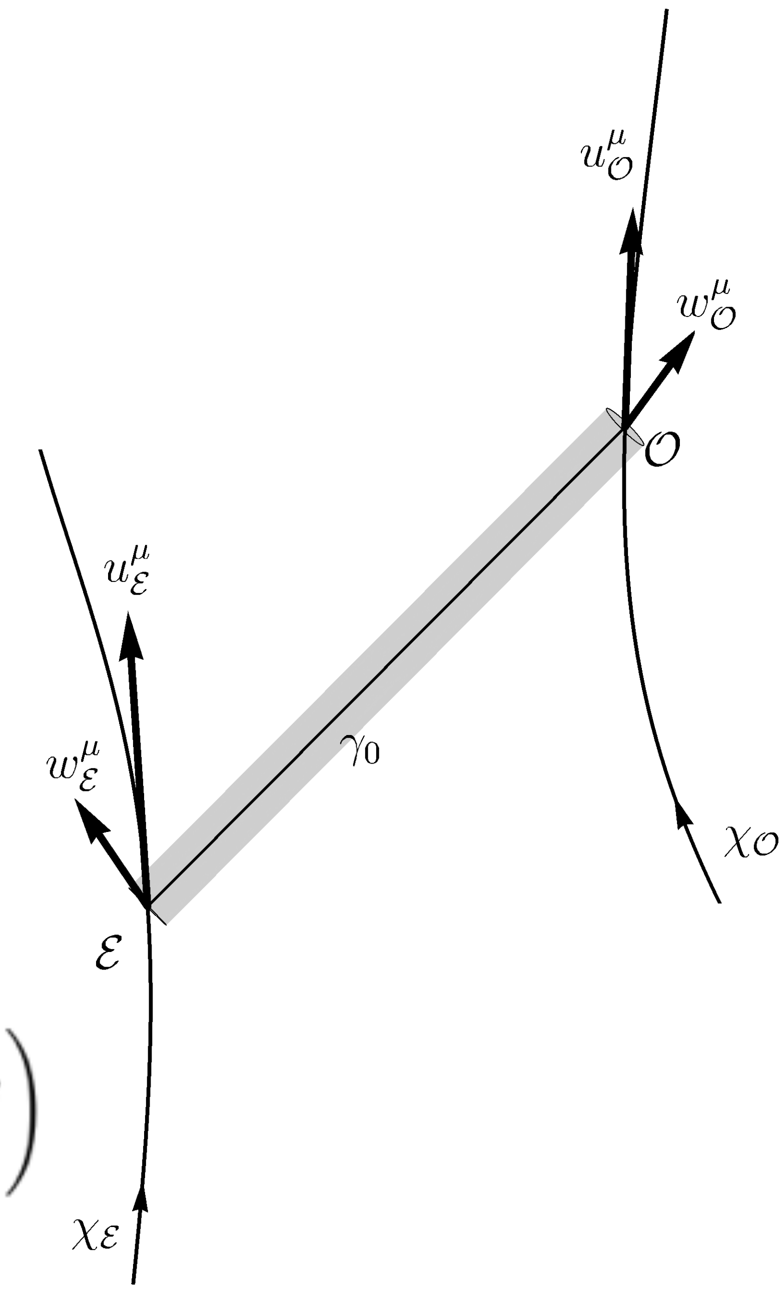
- result

$$
\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_\mathcal{O} + \frac{1}{p_\sigma\, u^\sigma_\mathcal{O}}\, \mathcal{D}^{-1}(\lambda_\mathcal{E})^A{}_B \left(\left(\frac{1}{1+z}\, u_\mathcal{E} - \hat{u}_\mathcal{O} \right)^B - m^B \right)
$$

Fermi-Walker derivative (observer)



Drift effects

Position drift

- parallel propagation of $\hat{u}^\mu_{\mathcal{O}}$
- inhomogeneous (perpendicular) GDE

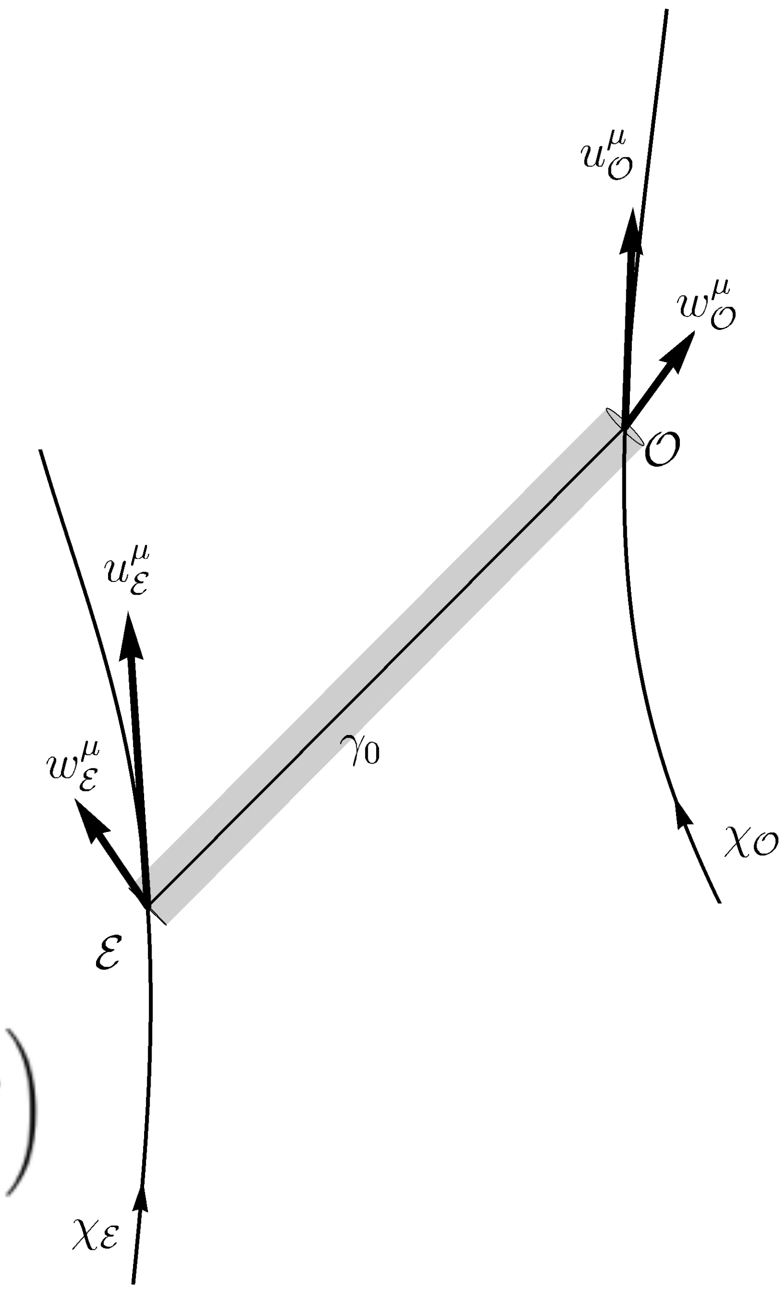
$$
\begin{aligned}
\mathcal{G}[m]^A &= R^A{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \hat{u}^\beta_{\mathcal{O}} \\
m^A(\lambda_{\mathcal{O}}) &= 0 \\
\nabla_p m^A(\lambda_{\mathcal{O}}) &= 0
\end{aligned}
$$

- result

$$
\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, \mathcal{D}^{-1}(\lambda_{\mathcal{E}})^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B \right)
$$

Fermi-Walker derivative (observer)

aberration



Drift effects

Position drift

- parallel propagation of $\hat{u}^\mu_{\mathcal{O}}$
- inhomogeneous (perpendicular) GDE

$$
\begin{aligned}
\mathcal{G}[m]^A &= R^A{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \hat{u}^\beta_{\mathcal{O}} \\
m^A(\lambda_{\mathcal{O}}) &= 0 \\
\nabla_p m^A(\lambda_{\mathcal{O}}) &= 0
\end{aligned}
$$

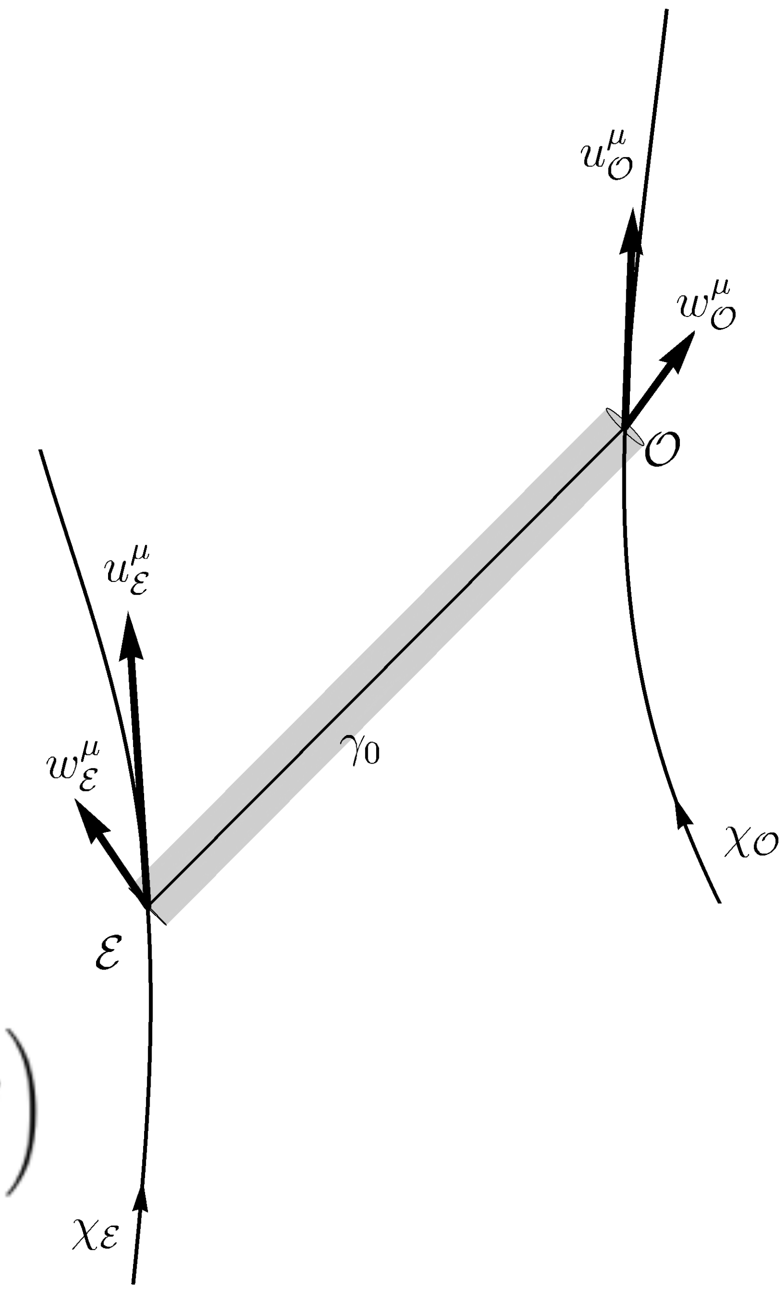
- result

4-velocity difference

$$
\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, \mathcal{D}^{-1}(\lambda_{\mathcal{E}})^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B \right)
$$

Fermi-Walker derivative (observer)

aberration



Drift effects

Position drift

- parallel propagation of $\hat{u}_{\mathcal{O}}^{\mu}$
- inhomogeneous (perpendicular) GDE

$$
\begin{aligned}
\mathcal{G}[m]^A &= R^A{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \hat{u}_{\mathcal{O}}^\beta \\
m^A(\lambda_{\mathcal{O}}) &= 0 \\
\nabla_p m^A(\lambda_{\mathcal{O}}) &= 0
\end{aligned}
$$

- result

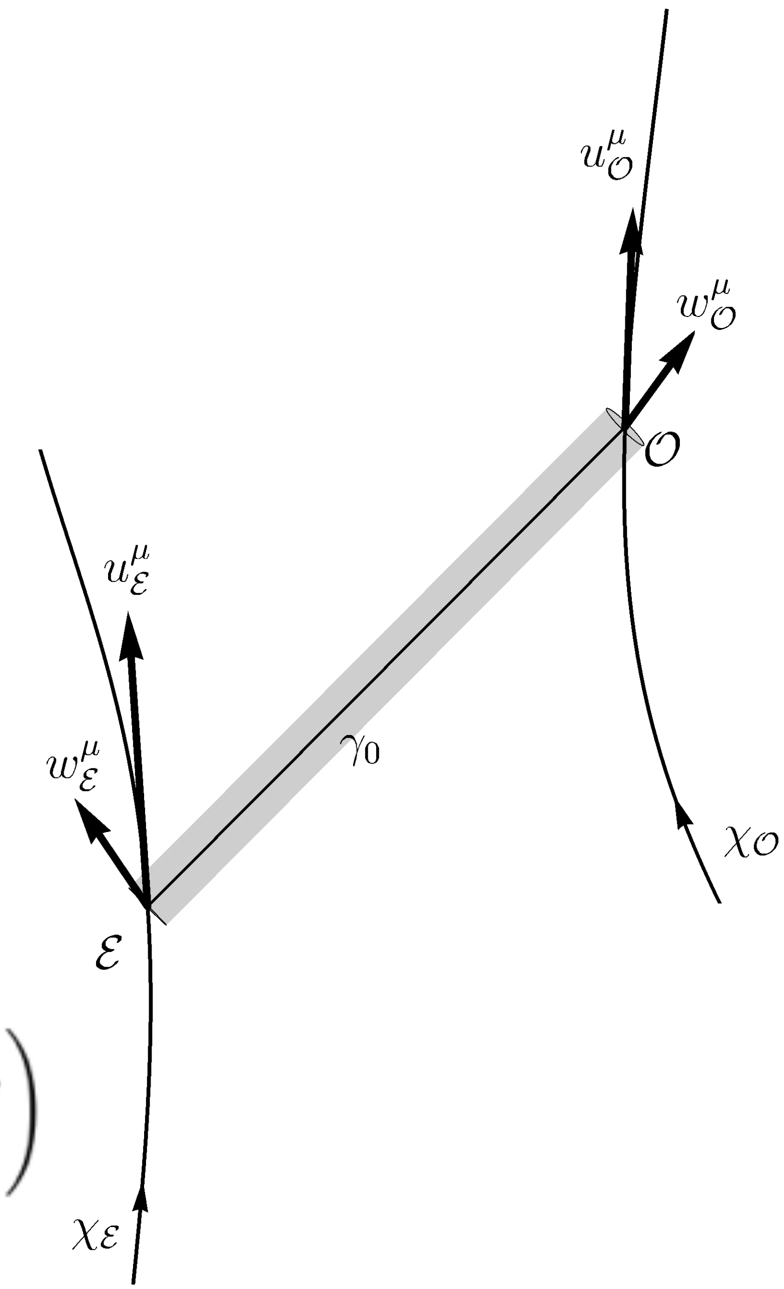
4-velocity difference

$$
\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w_{\mathcal{O}}^A + \frac{1}{p_\sigma\, u_{\mathcal{O}}^\sigma}\, \mathcal{D}^{-1}(\lambda_{\mathcal{E}})^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B \right)
$$

Fermi-Walker derivative (observer)

aberration

time-dep. light bending



Drift effects

Position drift

- parallel propagation of $\hat{u}^{\mu}_{\mathcal{O}}$
- inhomogeneous (perpendicular) GDE

$$
\begin{aligned}
\mathcal{G}[m]^A &= R^A{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \hat{u}^\beta_{\mathcal{O}} \\
m^A(\lambda_{\mathcal{O}}) &= 0 \\
\nabla_p m^A(\lambda_{\mathcal{O}}) &= 0
\end{aligned}
$$

- result

$$
\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_{\mathcal{O}} + \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\, \mathcal{D}^{-1}(\lambda_{\mathcal{E}})^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B \right)
$$

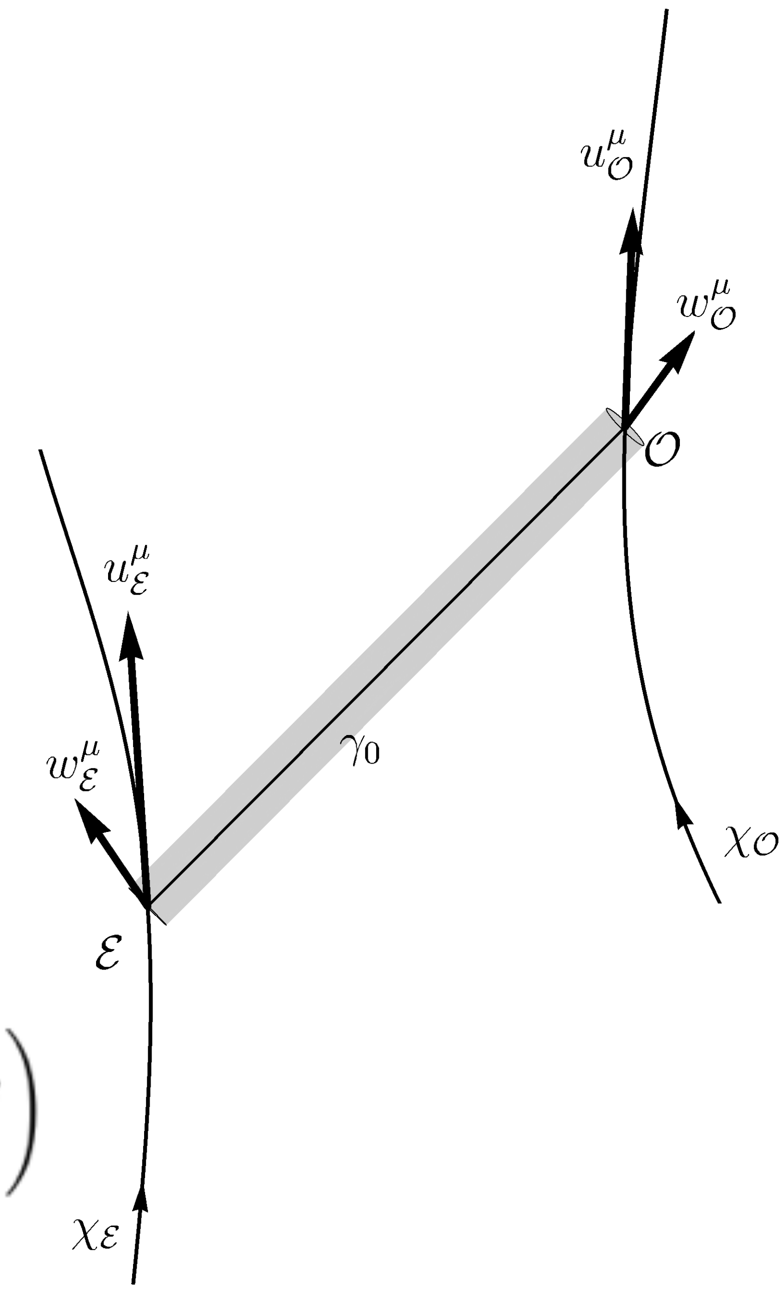
4-velocity difference

Fermi-Walker derivative (observer)

aberration

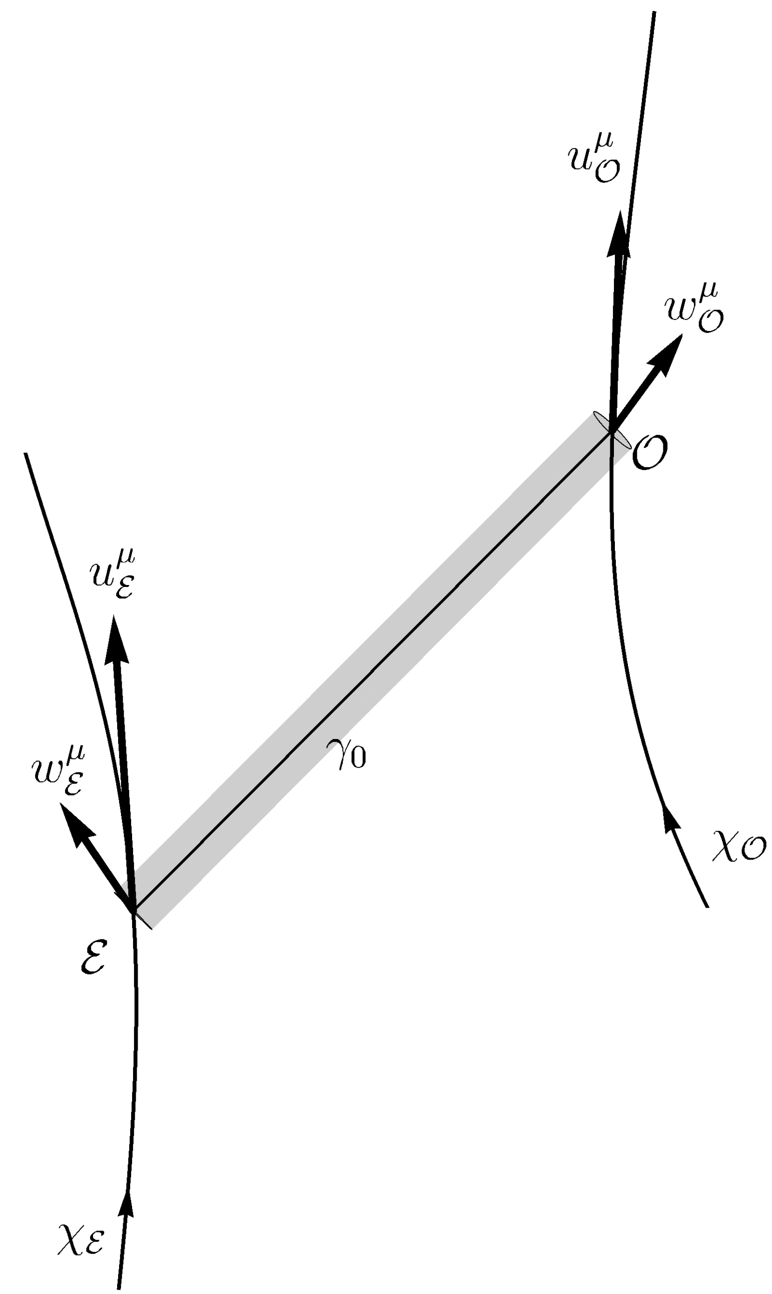
lensing

time-dep. light bending



Drift effects

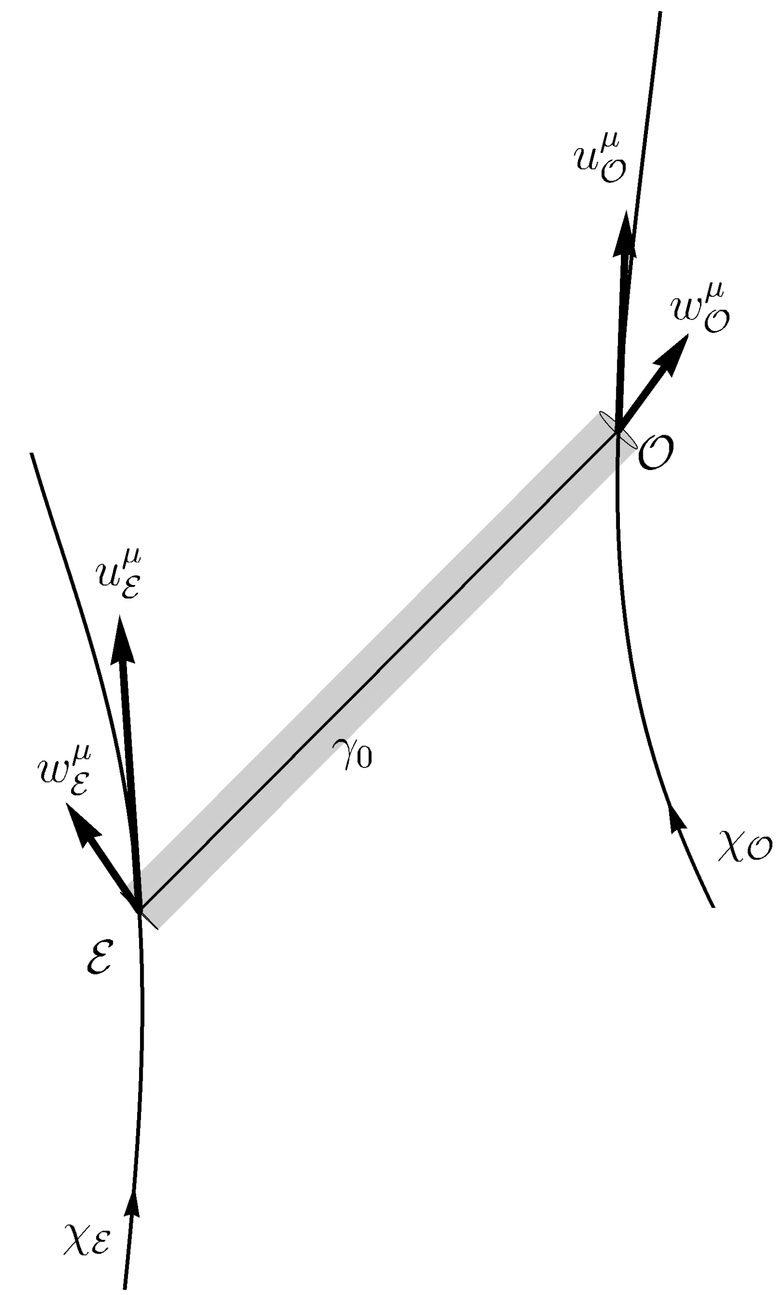
Redshift drift



Drift effects

Redshift drift

- parallel propagation of $\hat{w}^{\mu}_{\mathcal{E}}$, $\hat{u}^{\beta}_{\mathcal{E}}$

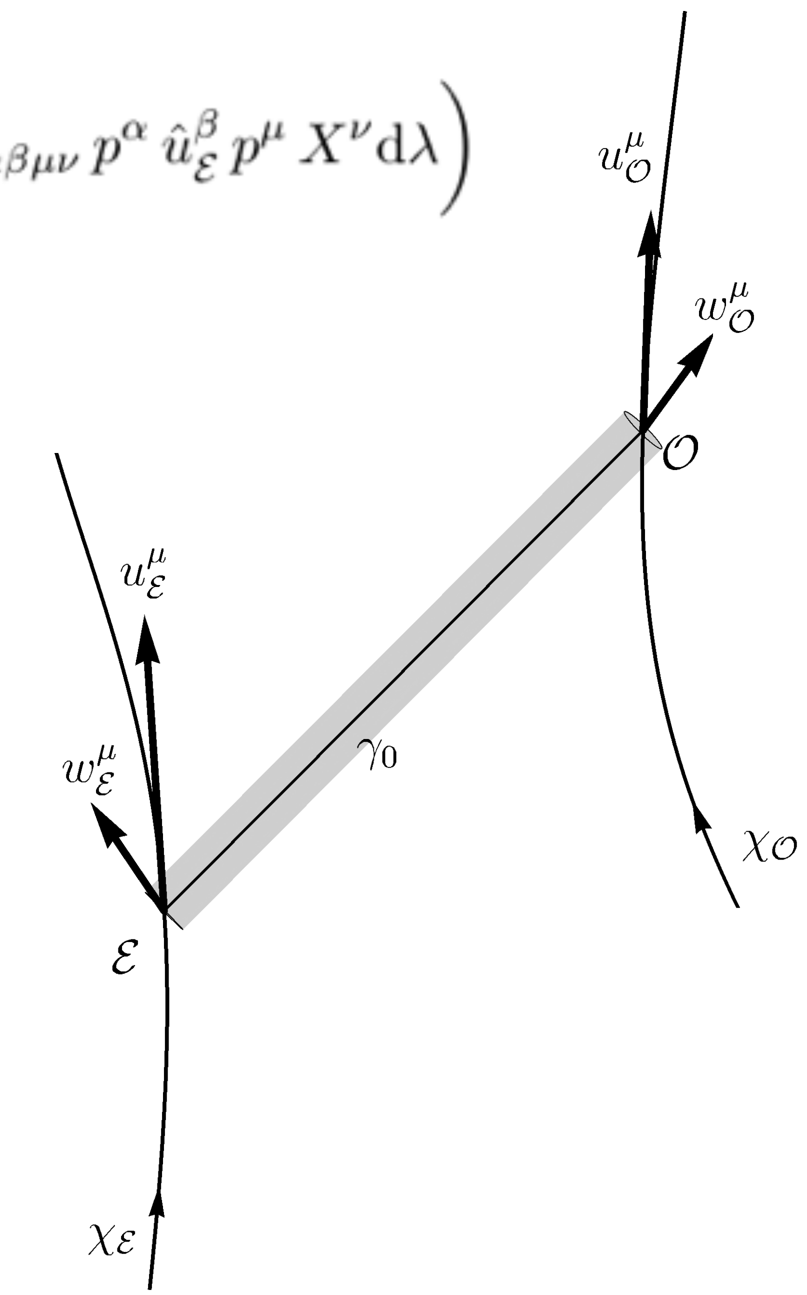


Drift effects

Redshift drift

- parallel propagation of $\hat{w}^{\mu}_{\mathcal{E}}$, $\hat{u}^{\beta}_{\mathcal{E}}$

$$\nabla_X \ln(1+z) = \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\left(\left(\frac{1}{(1+z)^2}\hat{w}^{\mu}_{\mathcal{E}} - w^{\mu}_{\mathcal{O}}\right)p_\mu + \frac{1}{1+z}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} R_{\alpha\beta\mu\nu}\, p^\alpha\, \hat{u}^{\beta}_{\mathcal{E}}\, p^\mu\, X^\nu \mathrm{d}\lambda\right)$$
$$+\left(\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A - w^A_{\mathcal{O}}\right)\left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)_A \Bigg|_{\lambda=\lambda_{\mathcal{O}}}$$



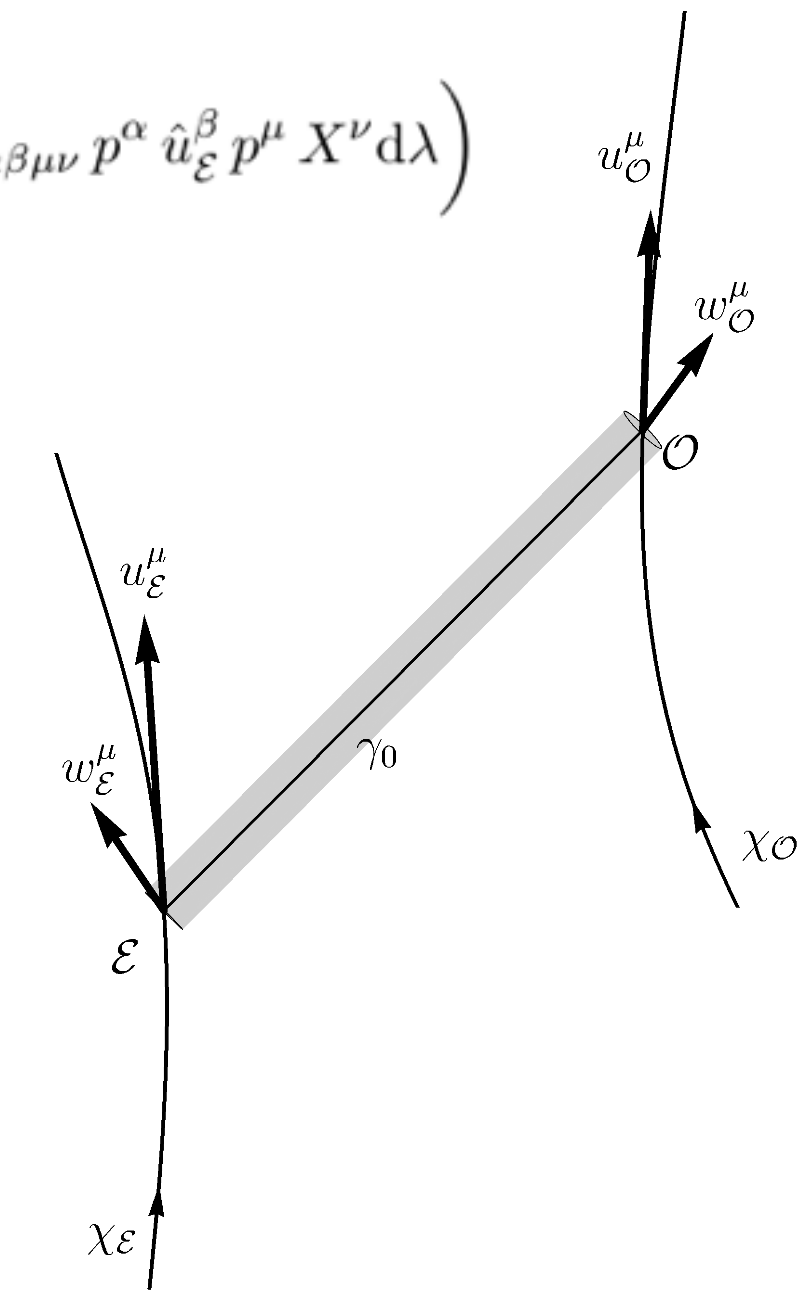
Drift effects

Redshift drift

- parallel propagation of $\hat{w}^{\mu}_{\mathcal{E}}$, $\hat{u}^{\beta}_{\mathcal{E}}$

line-of-sight
4-acceleration difference

$$\nabla_X \ln(1+z) = \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\left(\left(\frac{1}{(1+z)^2}\hat{w}^{\mu}_{\mathcal{E}} - w^{\mu}_{\mathcal{O}}\right) p_\mu + \frac{1}{1+z}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} R_{\alpha\beta\mu\nu}\, p^\alpha\, \hat{u}^{\beta}_{\mathcal{E}}\, p^\mu\, X^\nu \mathrm{d}\lambda\right) + \left(\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A - w^A_{\mathcal{O}}\right)\left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)_A\Bigg|_{\lambda=\lambda_{\mathcal{O}}}$$



Drift effects

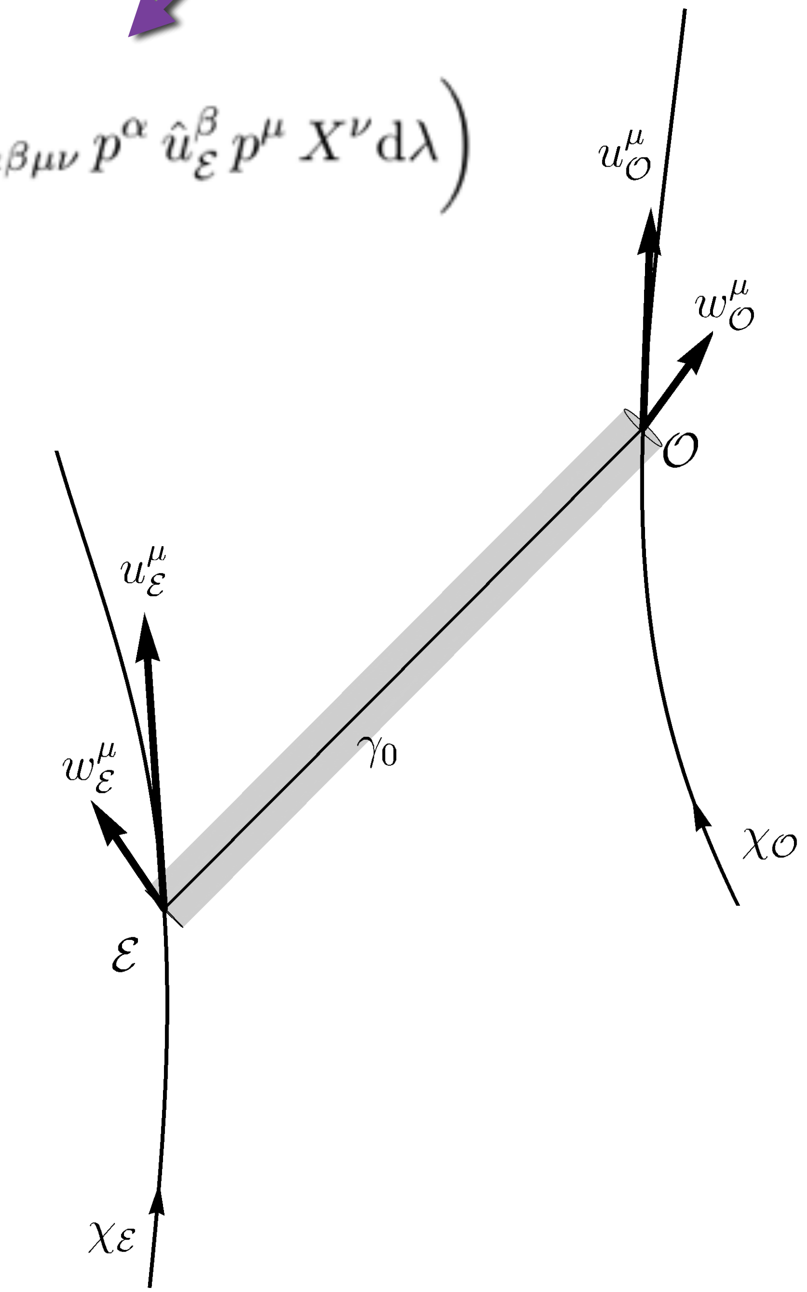
Redshift drift

- parallel propagation of $\hat{w}^\mu_{\mathcal{E}}$, $\hat{u}^\beta_{\mathcal{E}}$

line-of-sight 4-acceleration difference

integral of Riemann

$$\nabla_X \ln(1+z) = \frac{1}{p_\sigma\, u^\sigma_{\mathcal{O}}}\left(\left(\frac{1}{(1+z)^2}\hat{w}^\mu_{\mathcal{E}} - w^\mu_{\mathcal{O}}\right) p_\mu + \frac{1}{1+z}\int_{\lambda_{\mathcal{E}}}^{\lambda_{\mathcal{O}}} R_{\alpha\beta\mu\nu}\, p^\alpha\, \hat{u}^\beta_{\mathcal{E}}\, p^\mu\, X^\nu \mathrm{d}\lambda\right) + \left(\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A - w^A_{\mathcal{O}}\right)\left(\frac{1}{1+z}\hat{u}_{\mathcal{E}} - u_{\mathcal{O}}\right)_A \Bigg|_{\lambda=\lambda_{\mathcal{O}}}$$



Drift effects

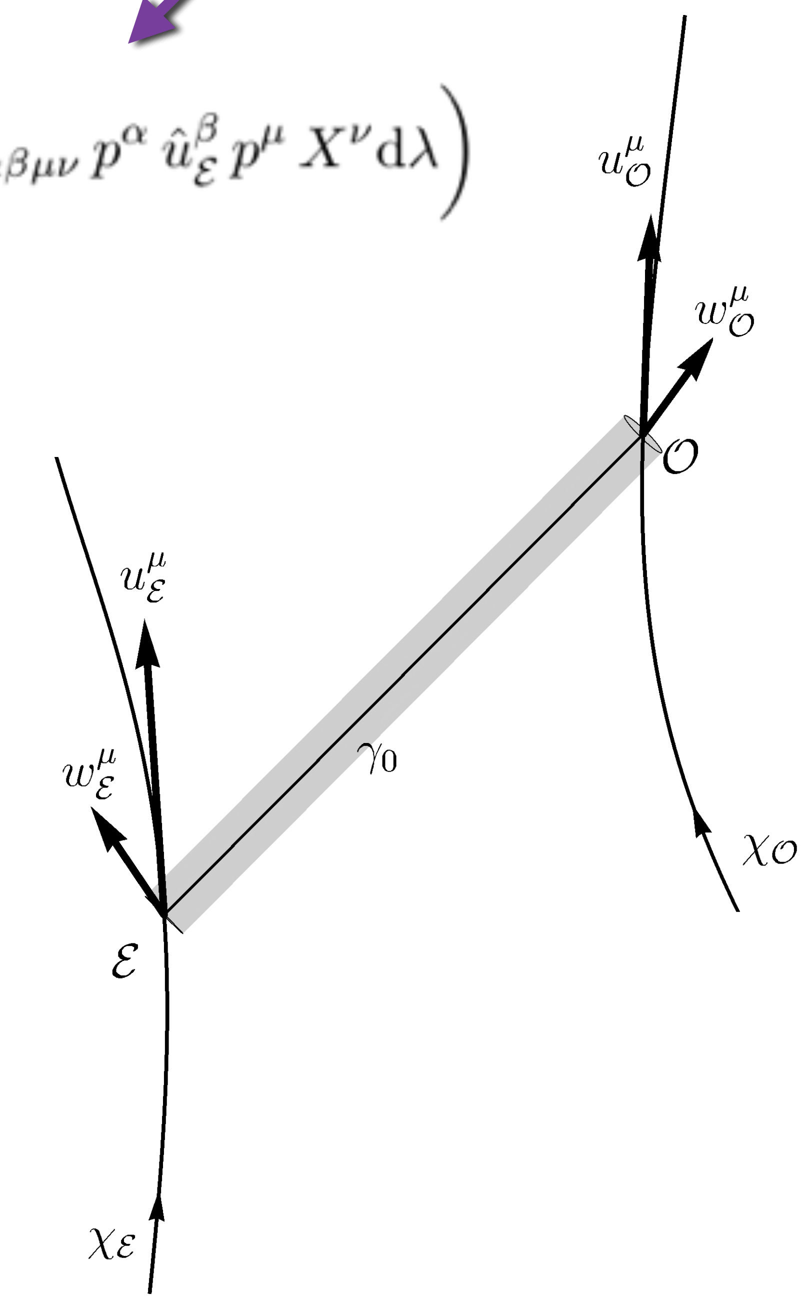
Redshift drift

- parallel propagation of $\hat{w}^\mu_\mathcal{E}$, $\hat{u}^\beta_\mathcal{E}$

line-of-sight 4-acceleration difference

integral of Riemann

$$\begin{aligned}\nabla_X \ln(1+z) \;&=\; \frac{1}{p_\sigma\, u^\sigma_\mathcal{O}}\left(\left(\frac{1}{(1+z)^2}\hat{w}^\mu_\mathcal{E} - w^\mu_\mathcal{O}\right)p_\mu + \frac{1}{1+z}\int_{\lambda_\mathcal{E}}^{\lambda_\mathcal{O}} R_{\alpha\beta\mu\nu}\, p^\alpha\, \hat{u}^\beta_\mathcal{E}\, p^\mu\, X^\nu \mathrm{d}\lambda\right)\\ &+ \left(\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A - w^A_\mathcal{O}\right)\left(\frac{1}{1+z}\hat{u}_\mathcal{E} - u_\mathcal{O}\right)_A \Bigg|_{\lambda=\lambda_\mathcal{O}}\end{aligned}$$



$$\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_\mathcal{O} + \frac{1}{p_\sigma\, u^\sigma_\mathcal{O}}\, \mathcal{D}^{-1}(\lambda_\mathcal{E})^A{}_B \left(\left(\frac{1}{1+z}u_\mathcal{E} - \hat{u}_\mathcal{O}\right)^B - m^B\right)$$

Drift effects

Redshift drift

- parallel propagation of $\hat{w}^\mu_\mathcal{E}$, $\hat{u}^\beta_\mathcal{E}$

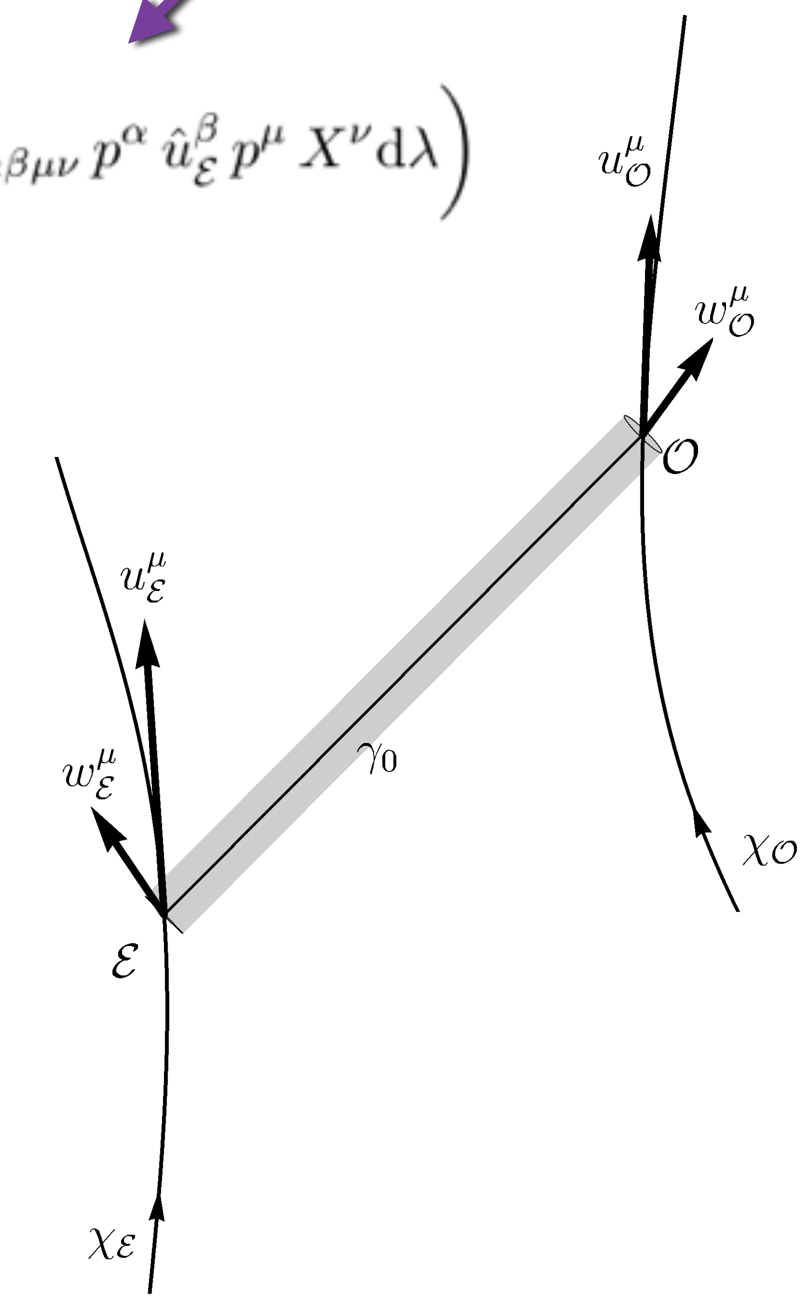
line-of-sight 4-acceleration difference

integral of Riemann

$$\nabla_X \ln(1+z) = \frac{1}{p_\sigma\, u^\sigma_\mathcal{O}}\left(\left(\frac{1}{(1+z)^2}\hat{w}^\mu_\mathcal{E} - w^\mu_\mathcal{O}\right) p_\mu + \frac{1}{1+z}\int_{\lambda_\mathcal{E}}^{\lambda_\mathcal{O}} R_{\alpha\beta\mu\nu}\, p^\alpha\, \hat{u}^\beta_\mathcal{E}\, p^\mu\, X^\nu \mathrm{d}\lambda\right)$$
$$+ \left(\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A - w^A_\mathcal{O}\right)\left(\frac{1}{1+z}\hat{u}_\mathcal{E} - u_\mathcal{O}\right)_A\Bigg|_{\lambda=\lambda_\mathcal{O}}$$

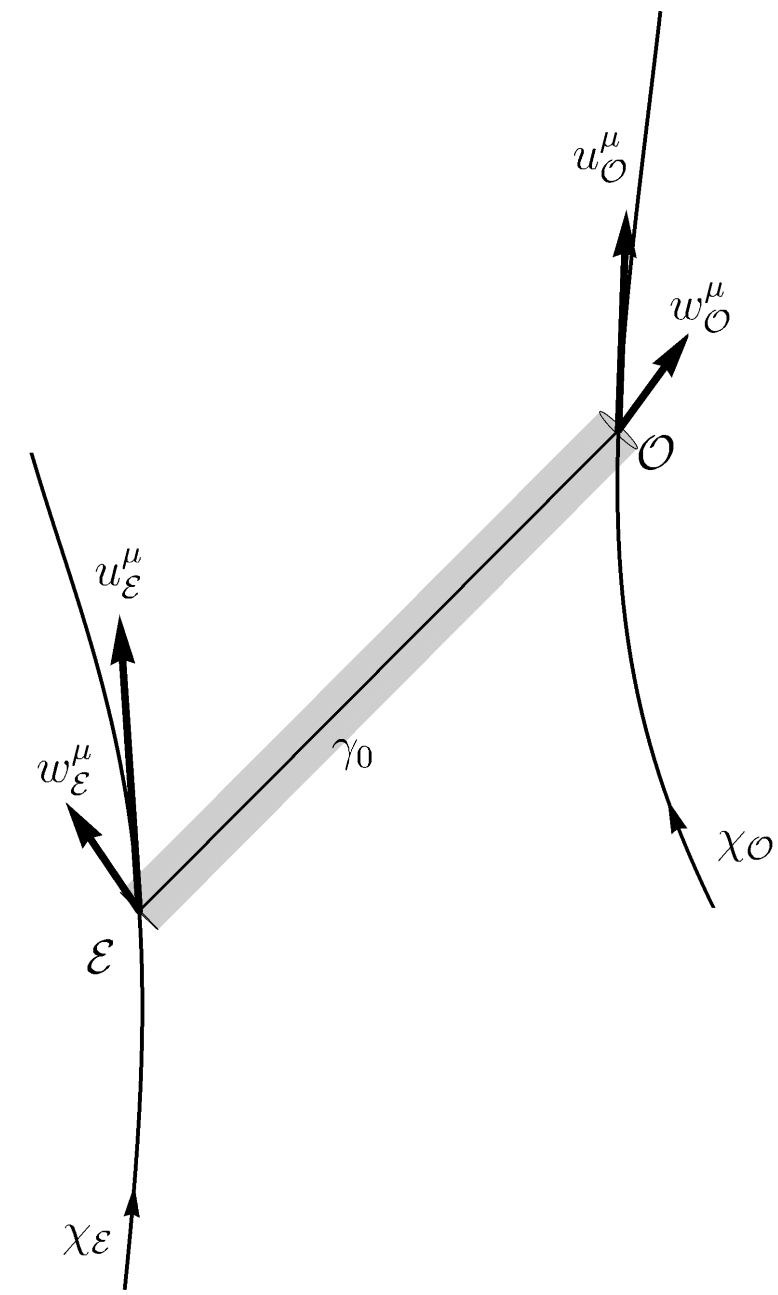
transversal 4-velocity difference, Jacobi matrix

$$\frac{\mathrm{D}^{F-W}}{\mathrm{d}\tau} r^A = w^A_\mathcal{O} + \frac{1}{p_\sigma\, u^\sigma_\mathcal{O}} \mathcal{D}^{-1}(\lambda_\mathcal{E})^A{}_B\left(\left(\frac{1}{1+z}u_\mathcal{E} - \hat{u}_\mathcal{O}\right)^B - m^B\right)$$



Drift effects

Jacobi matrix drift

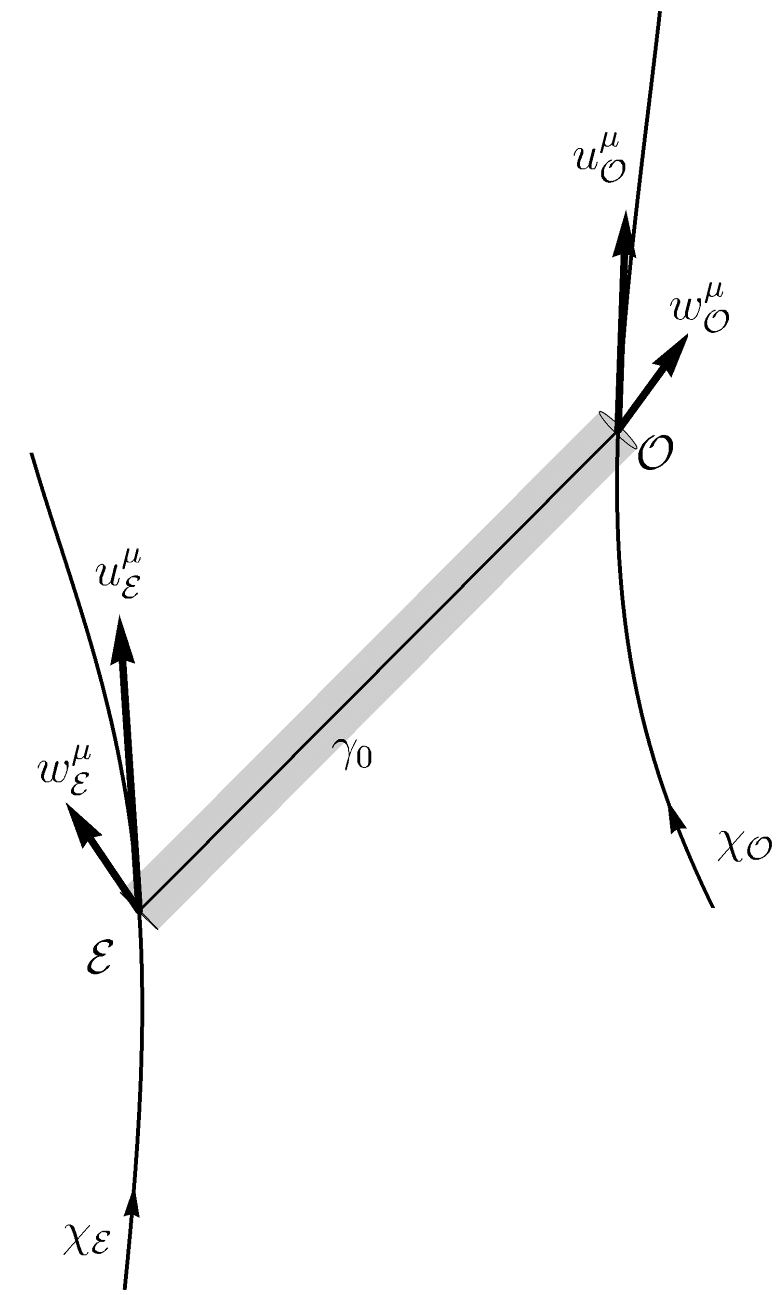


Drift effects

Jacobi matrix drift

- need to differentiate the GDE

$$\mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$

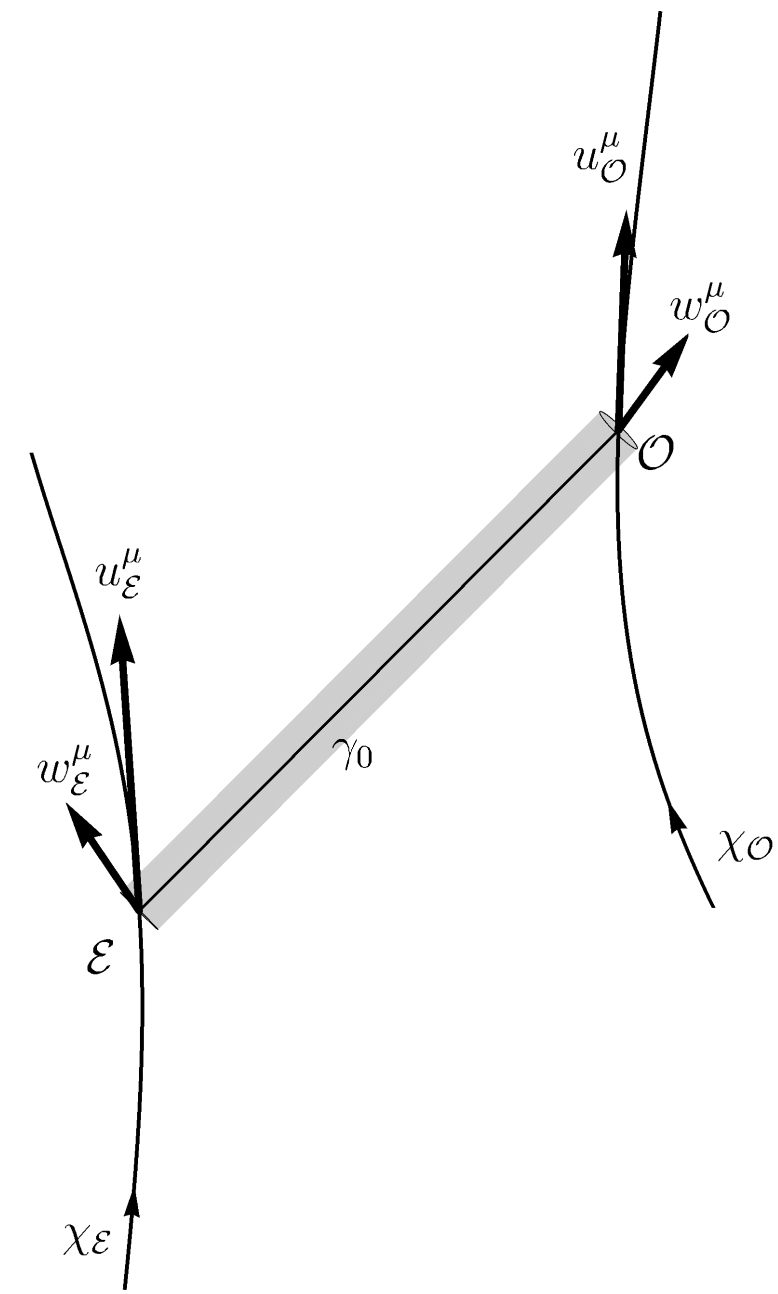


Drift effects

Jacobi matrix drift

- need to differentiate the GDE

$$\nabla_X \Big| \qquad \mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$



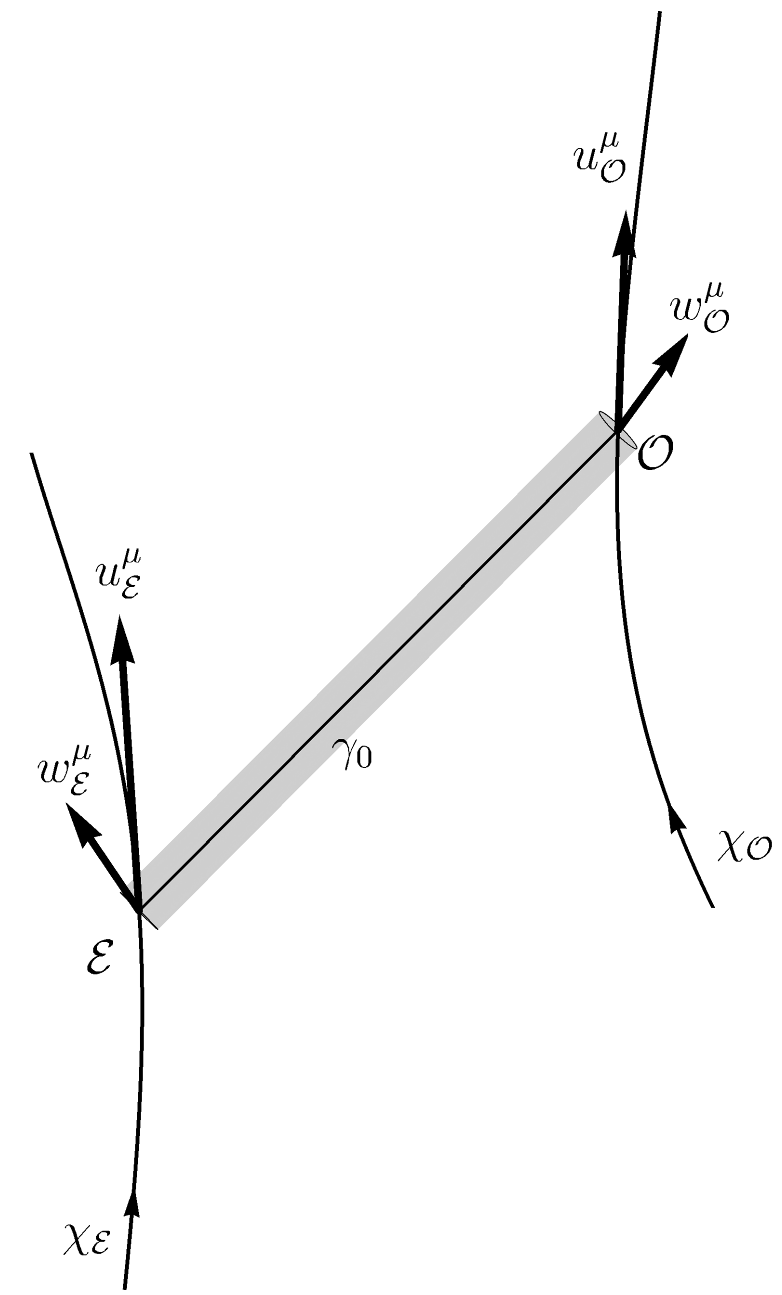
Drift effects

Jacobi matrix drift

- need to differentiate the GDE

$$\nabla_X \Big| \qquad \mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$

- inhomogeneous GDE



Drift effects

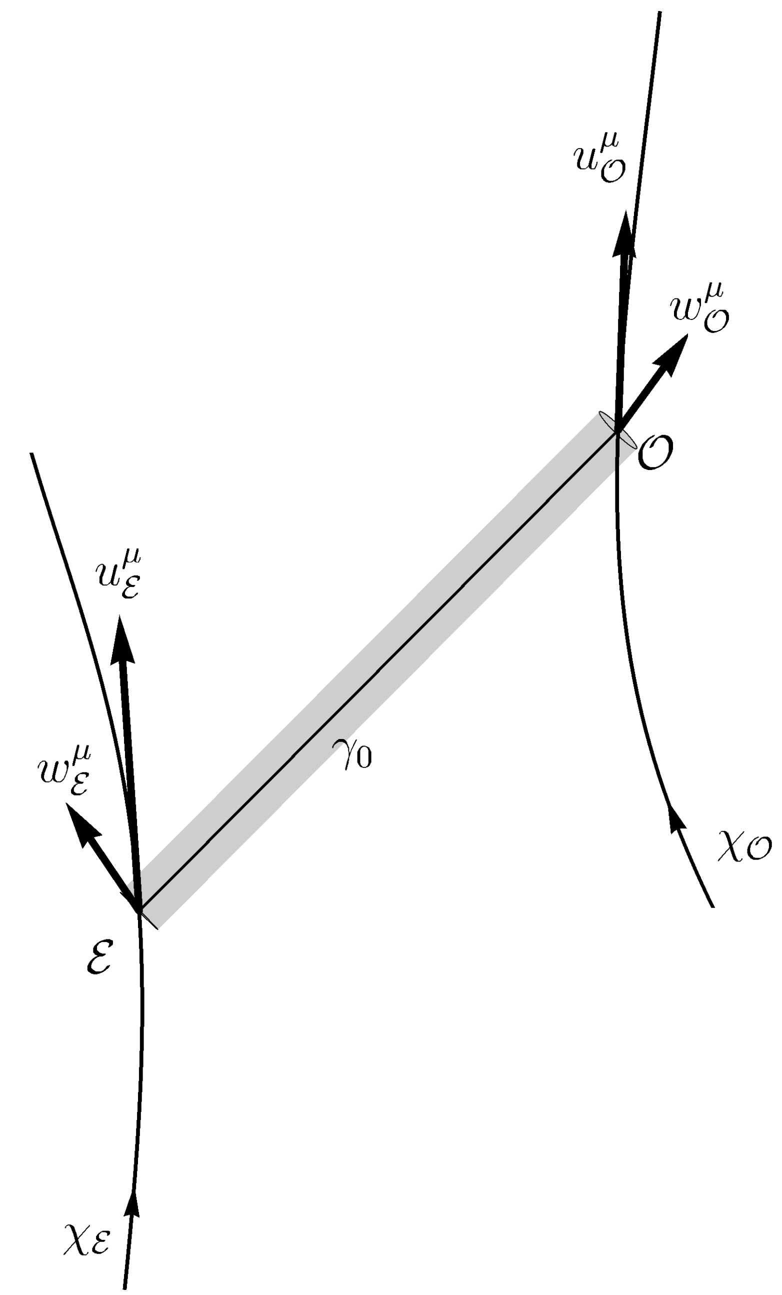
Jacobi matrix drift

- need to differentiate the GDE

$$\nabla_X \Big| \qquad \mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$

- inhomogeneous GDE

$$\nabla_p \nabla_p \left(\nabla_X \xi^\mu\right) - R^\mu{}_{\alpha\beta\nu}\, p^\alpha\, p^\beta\, \nabla_X \xi^\nu \quad = \quad \mathcal{M}^\mu{}_\nu\, \xi^\nu + \mathcal{N}^\mu{}_\nu\, \nabla_p \xi^\nu$$



Drift effects

Jacobi matrix drift

- need to differentiate the GDE

$$\nabla_X \Big| \qquad \mathcal{G}[\xi]^\mu \equiv \nabla_p \nabla_p \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, p^\nu\, p^\alpha\, \xi^\beta = 0$$

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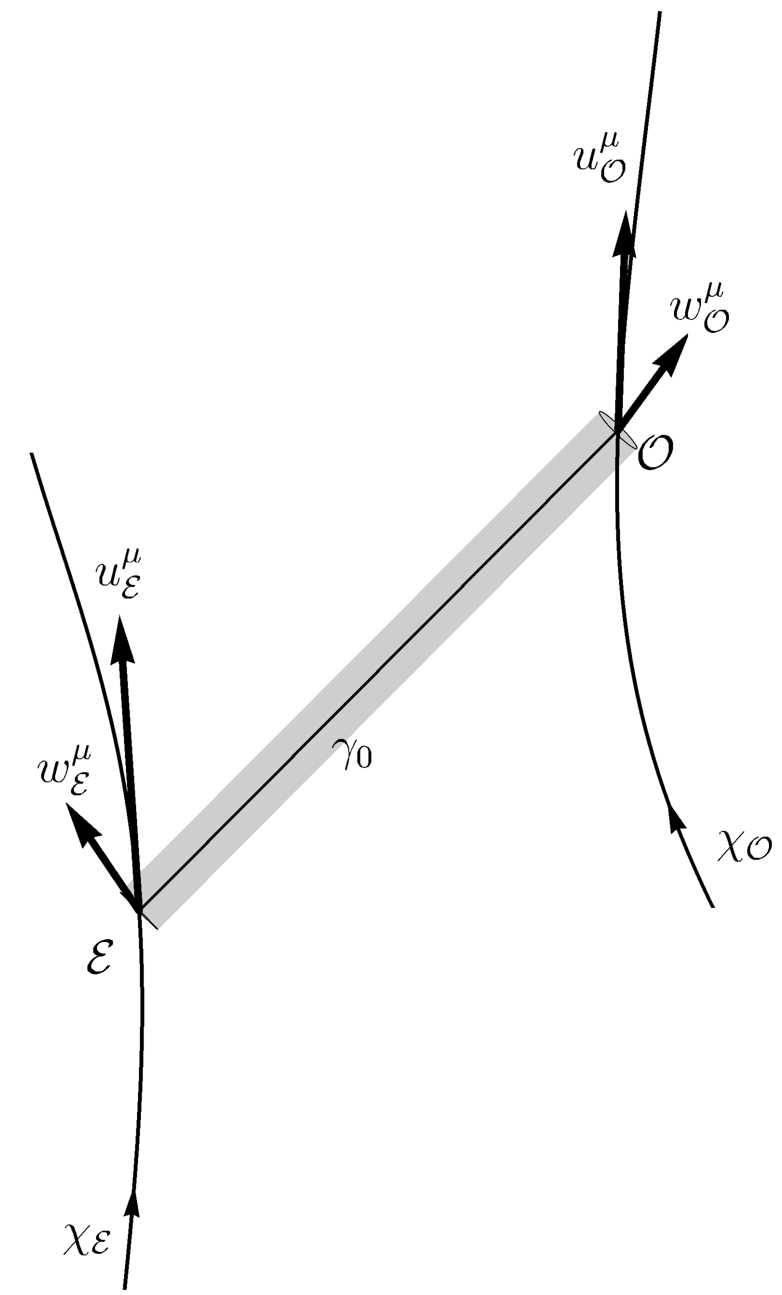
$$\begin{aligned} \mathcal{M}^\mu{}_\nu &= -\nabla_\alpha R^\mu{}_{\nu\rho\sigma}\, p^\alpha\, X^\rho\, p^\sigma + \nabla_\alpha R^\mu{}_{\beta\rho\nu}\, X^\alpha\, p^\beta\, p^\rho + 2R^\mu{}_{\beta\nu\sigma}\, \nabla_p X^\beta\, p^\sigma \\ \mathcal{N}^\mu{}_\nu &= -2R^\mu{}_{\nu\rho\sigma}\, X^\rho\, p^\sigma \end{aligned}$$



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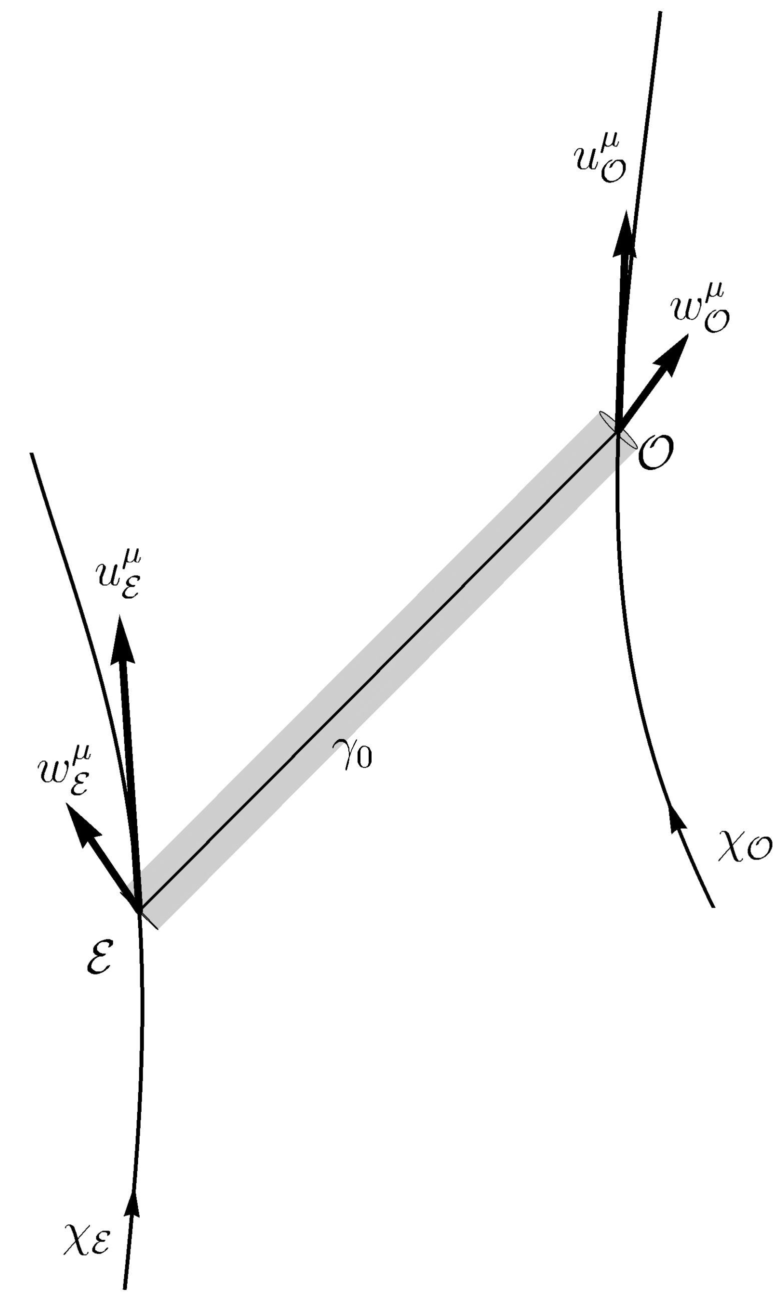

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f_A{}^\mu(\lambda_\mathcal{O}) &= 0 \\
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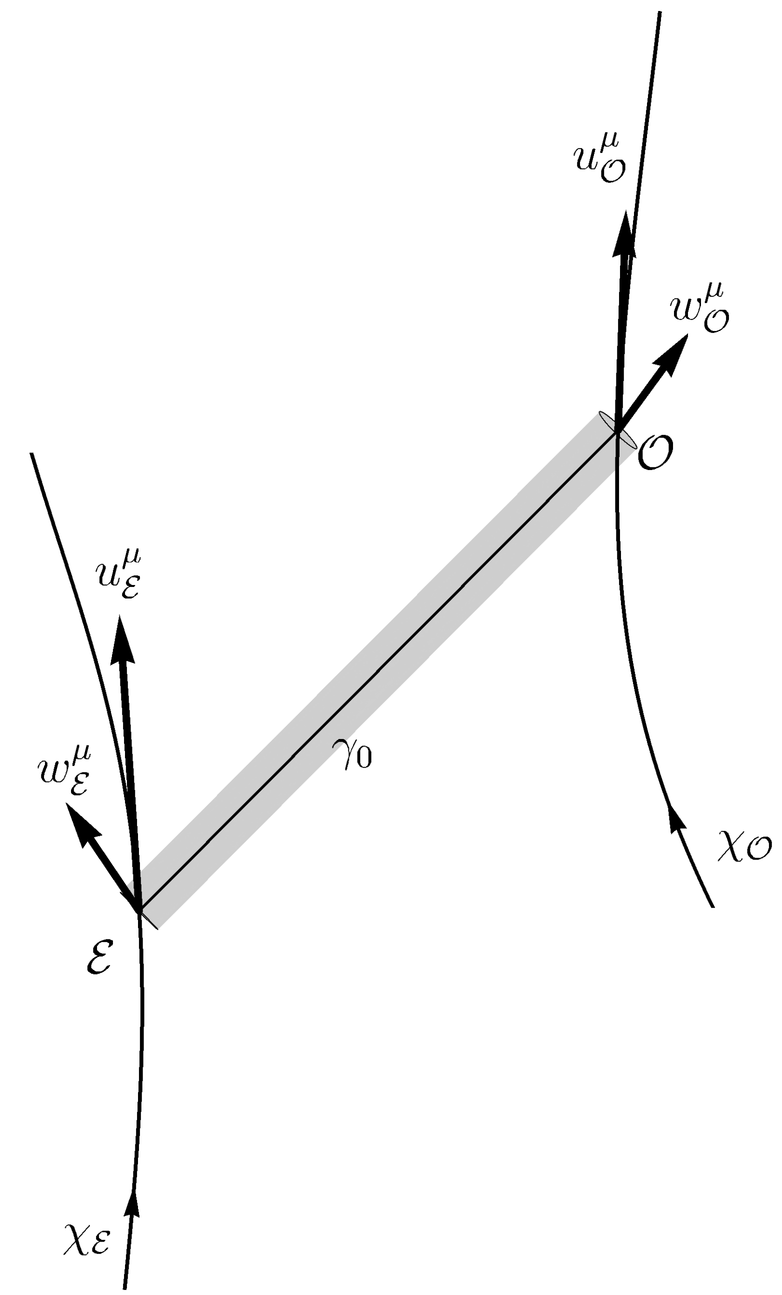

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- derivatives wrt X^μ

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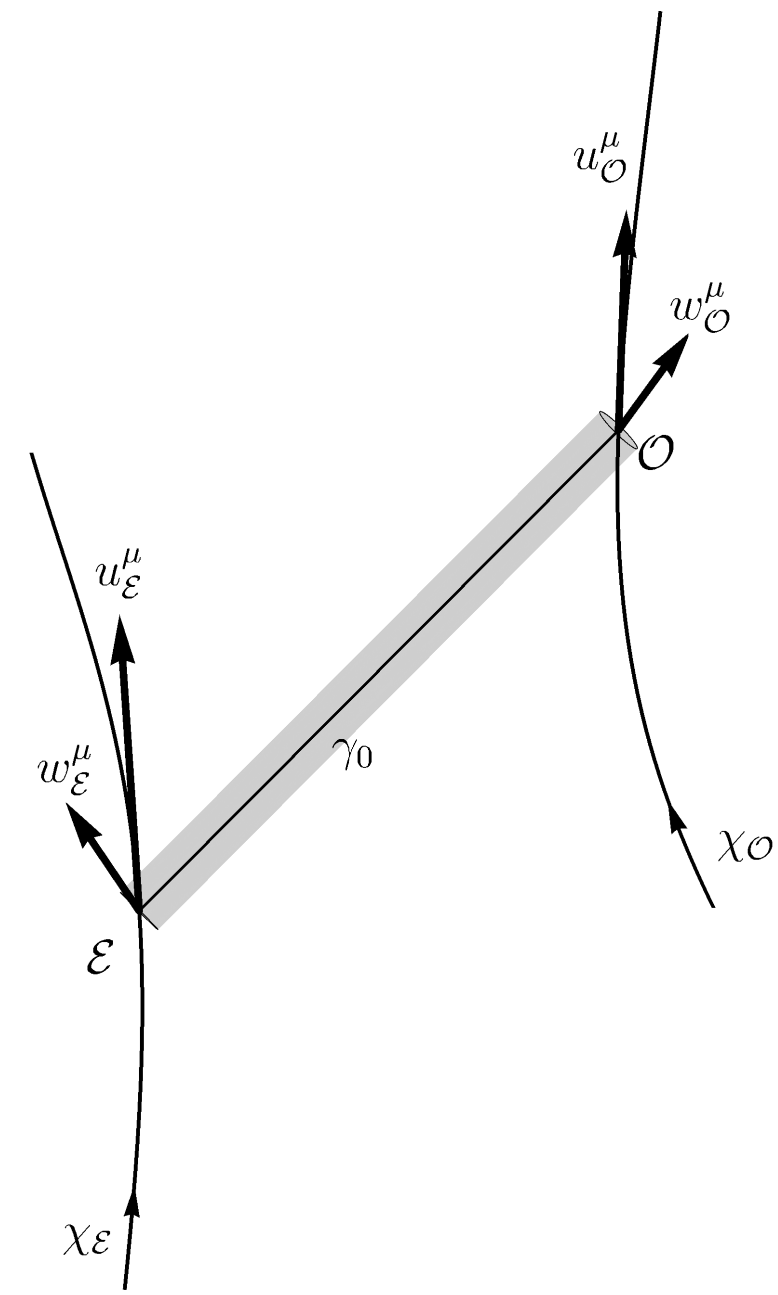
+ initial data at O



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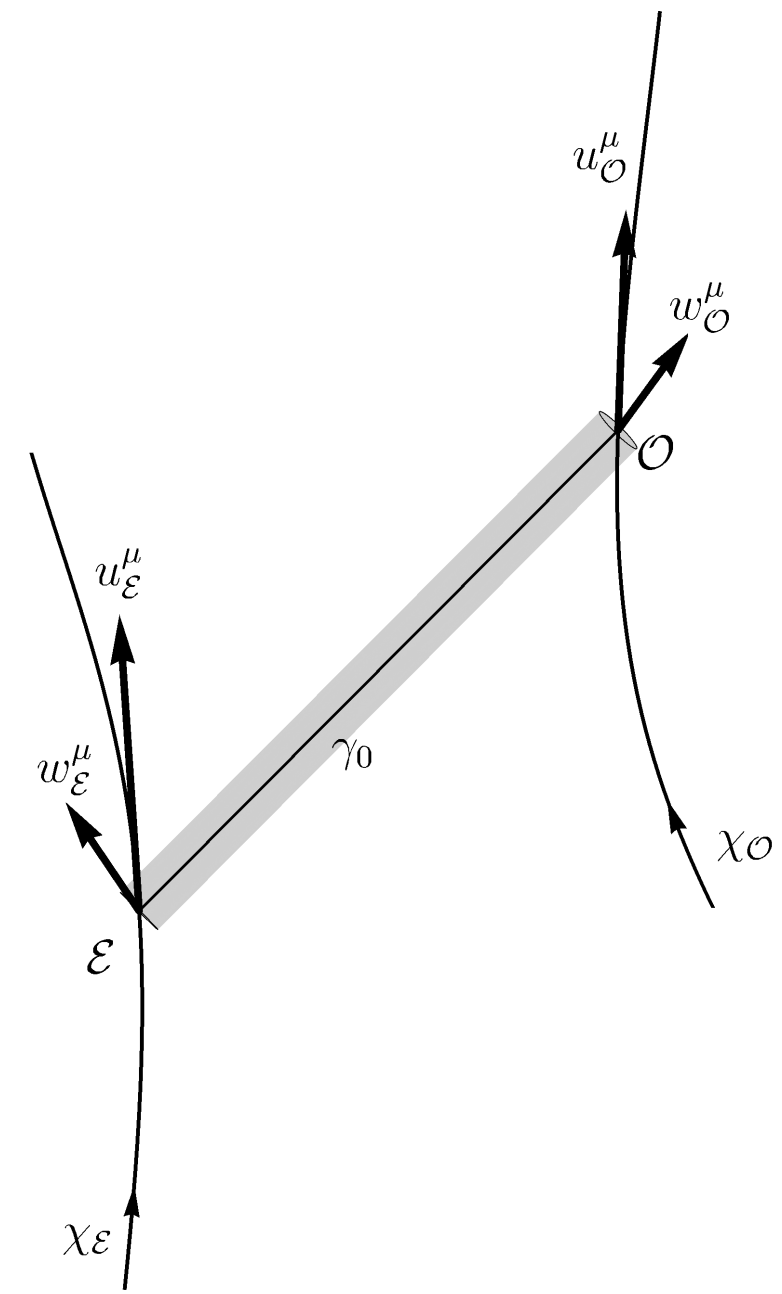
$$\nabla_X \mathcal{D}^A{}_B = \left(\nabla_X f_B\right)^A + \psi^A{}_C\, \left(f_B\right)^C - \nabla_p X^A\, \left(f_B\right)^p$$

antisymmertic, irrelevant



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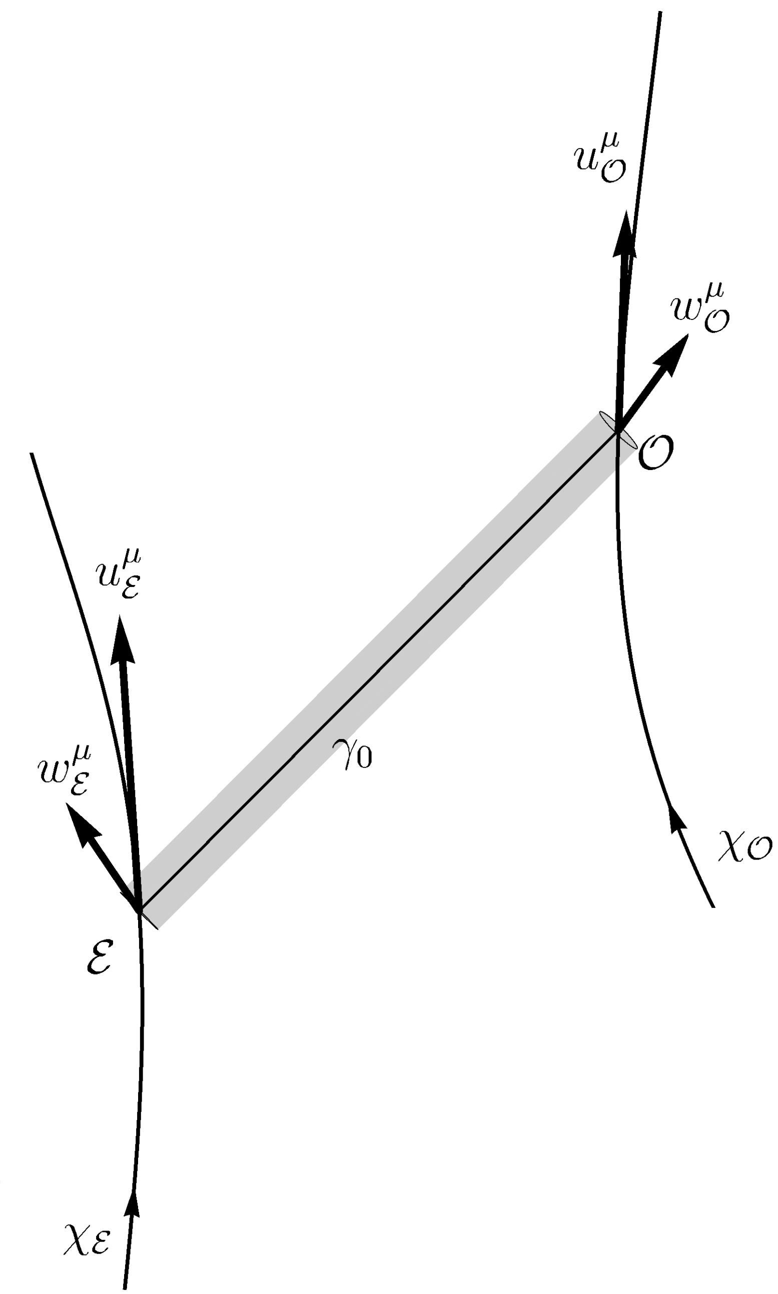

$$
\begin{aligned}
\nabla_X \ln D_{area} &= \nabla_p X^p(\lambda_\mathcal{O}) + \frac{p_\mu\, w^\mu_\mathcal{O}}{p_\nu\, u^\nu_\mathcal{O}} \\
&+ \frac{1}{2}\left(\left(\nabla_X f_B\right)^A(\lambda_\mathcal{E}) - \left(f_B\right)^p(\lambda_\mathcal{E})\, \nabla_p X^A(\lambda_\mathcal{E})\right) \mathcal{D}^{-1}(\lambda_\mathcal{E})^B{}_A
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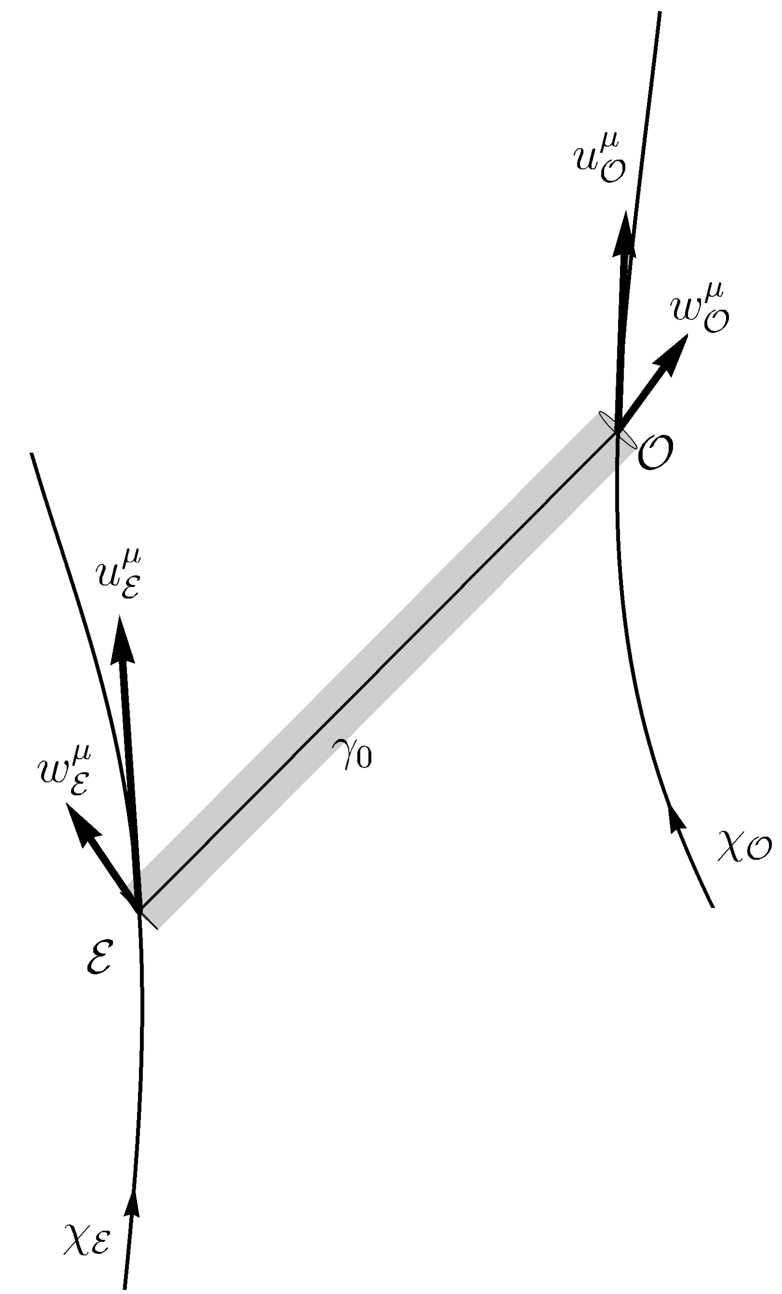
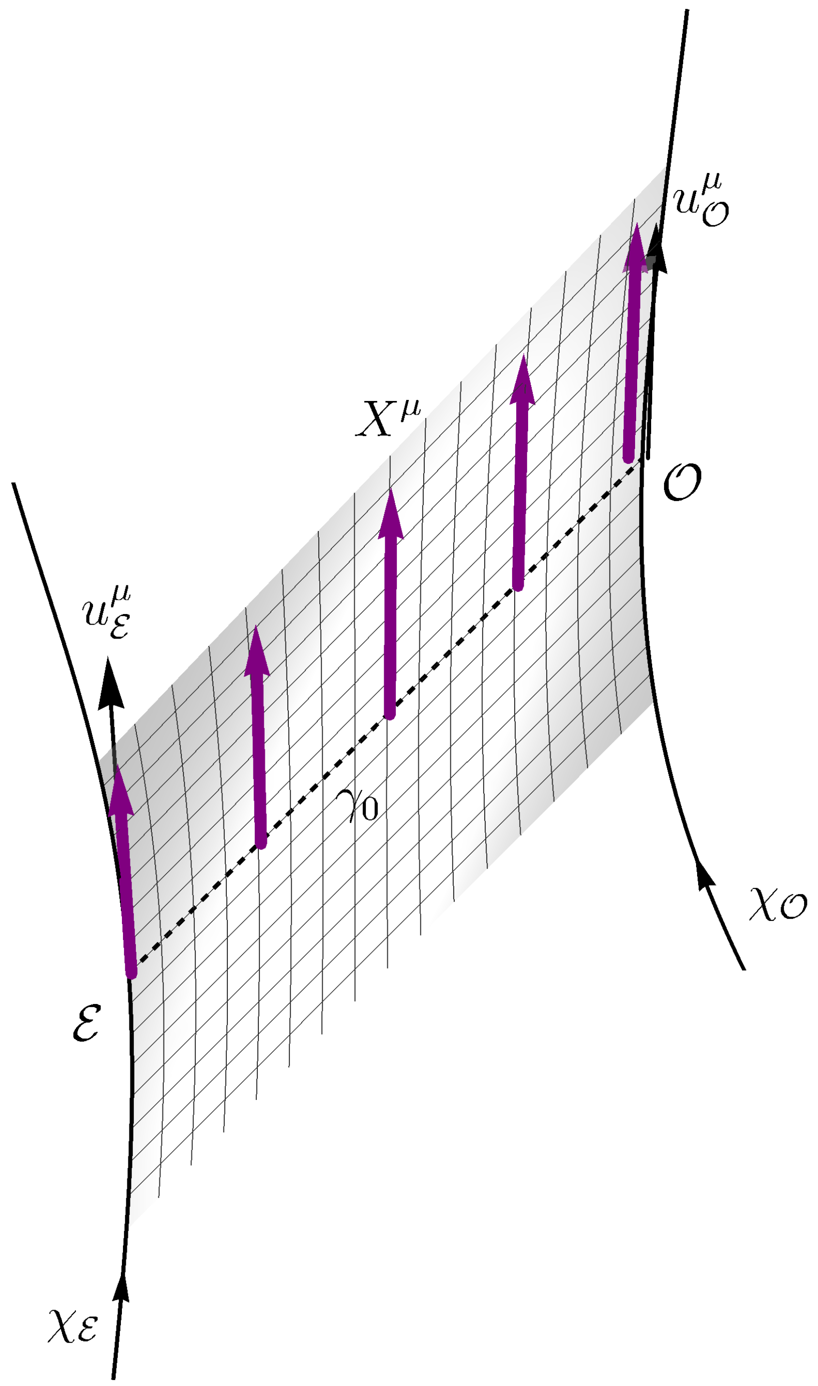
$$\nabla_X \ln D_{lum} = \nabla_X \ln D_{area} + 2\nabla_X \ln(1+z)$$



Drift effects

Remarks



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Remarks

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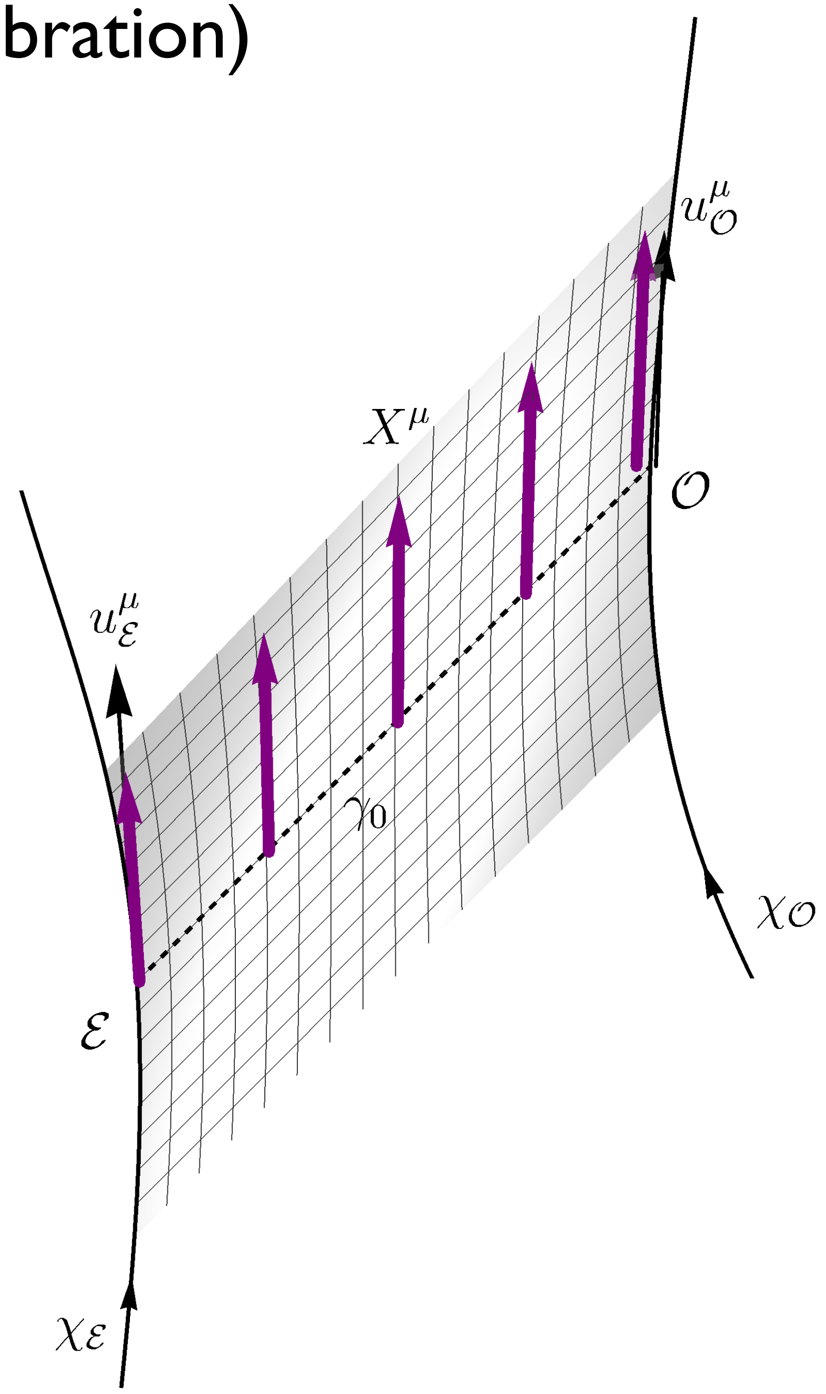

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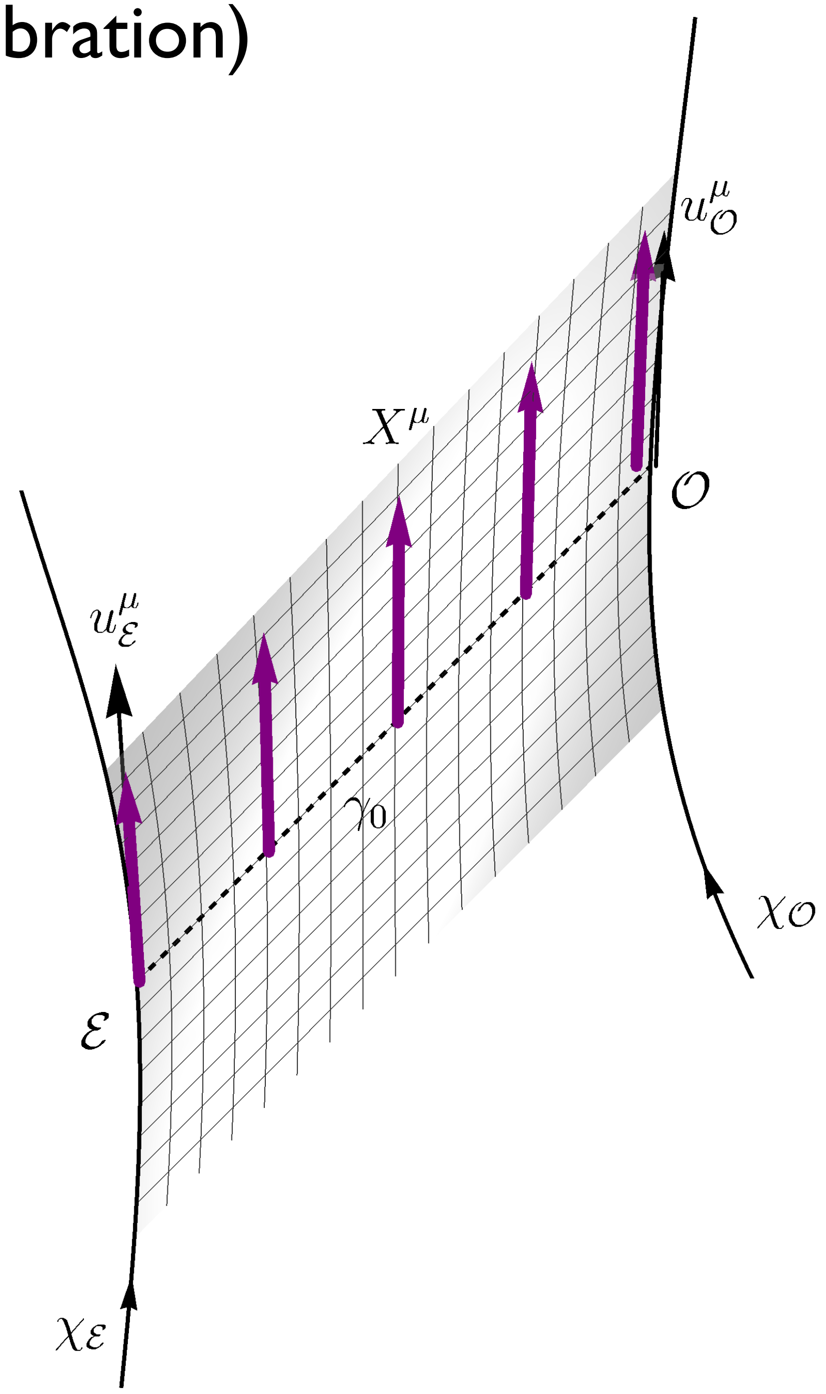

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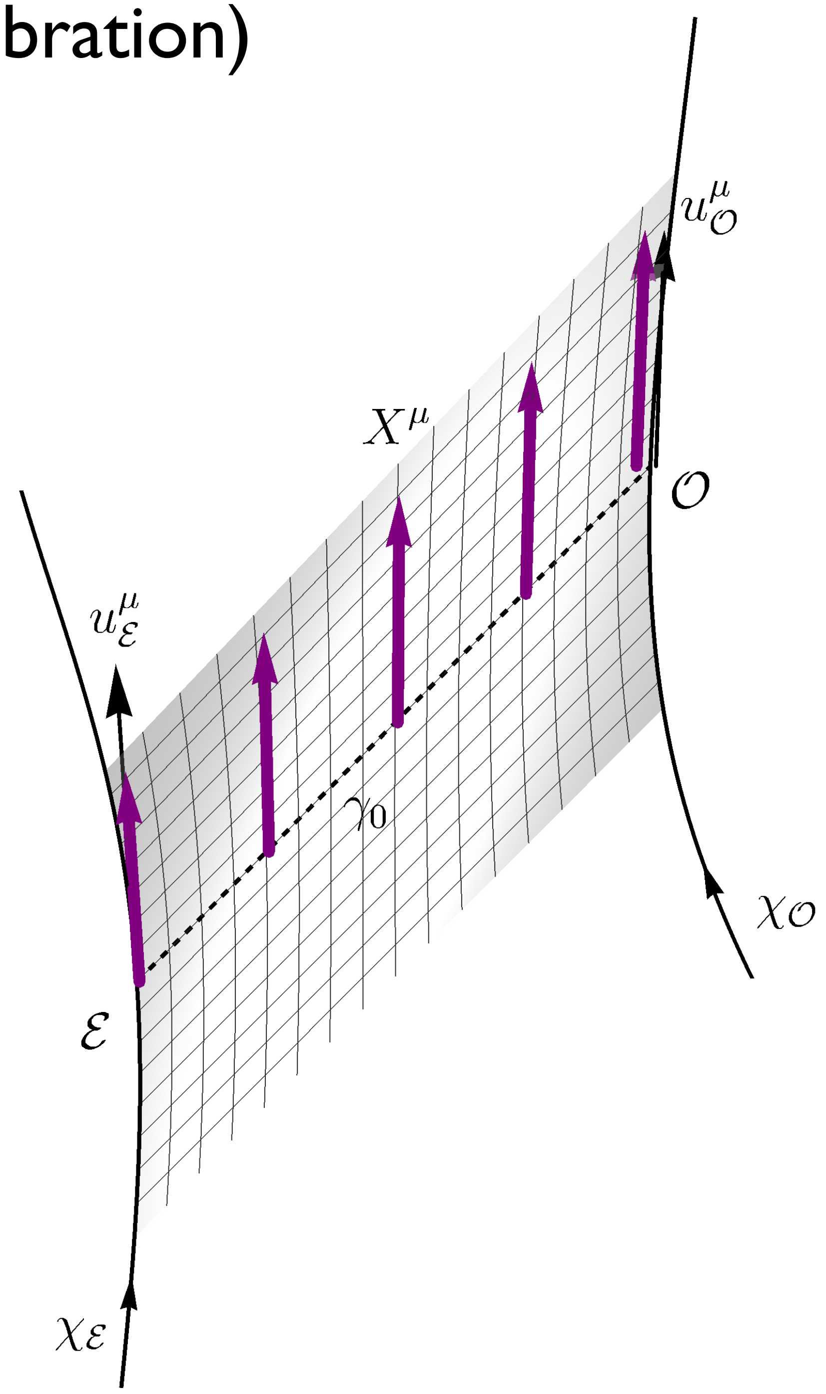

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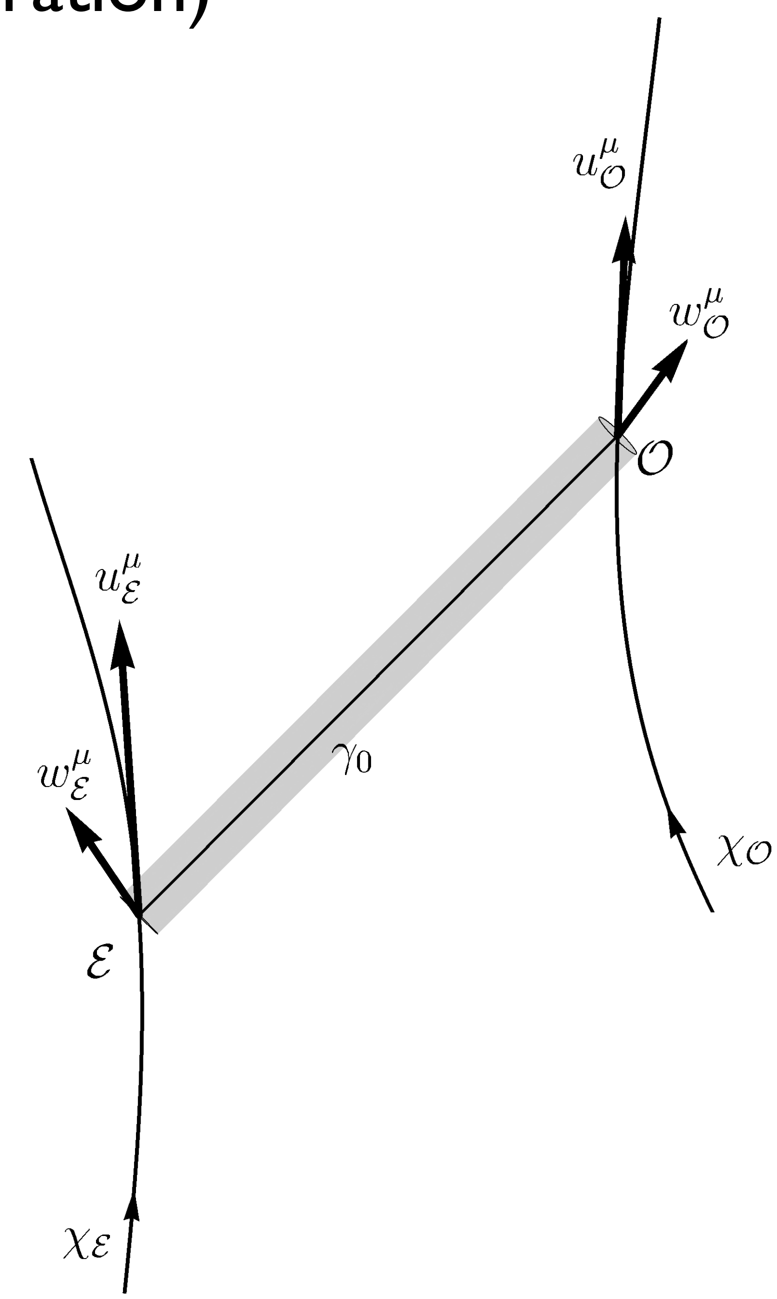

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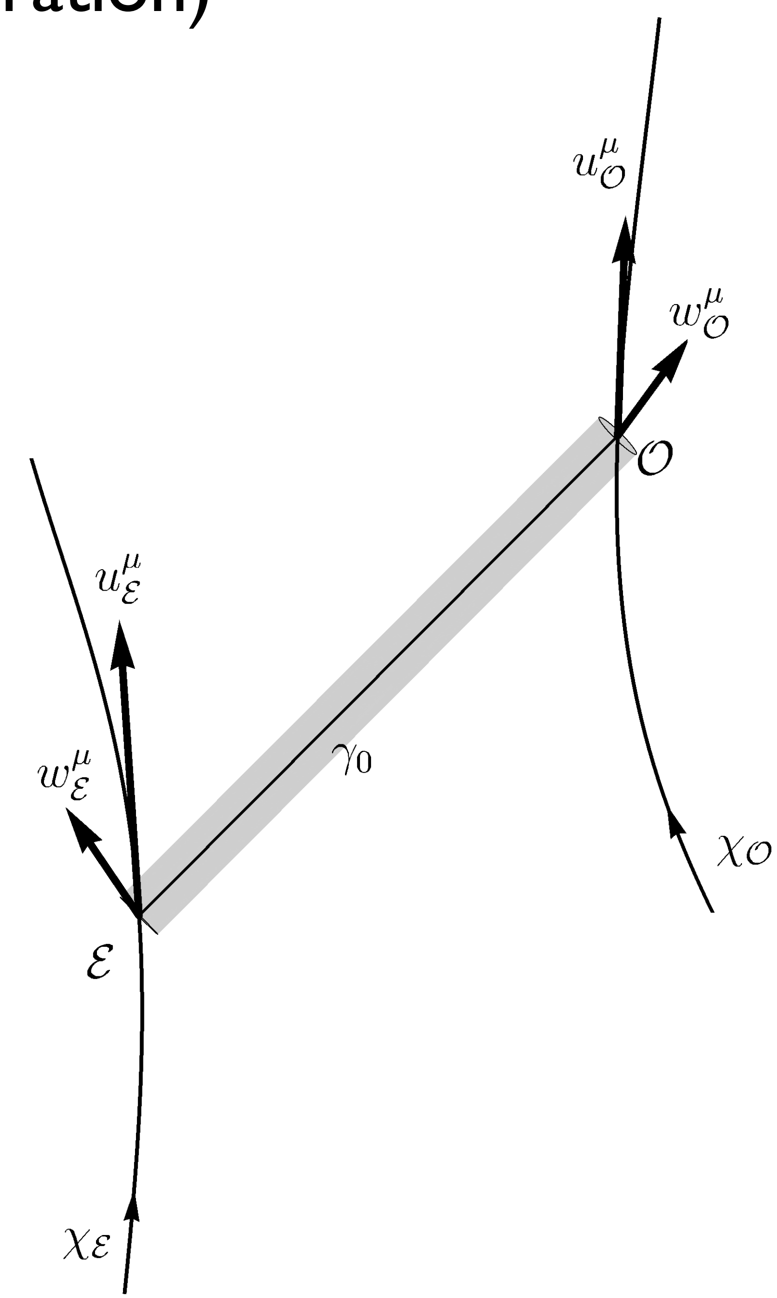

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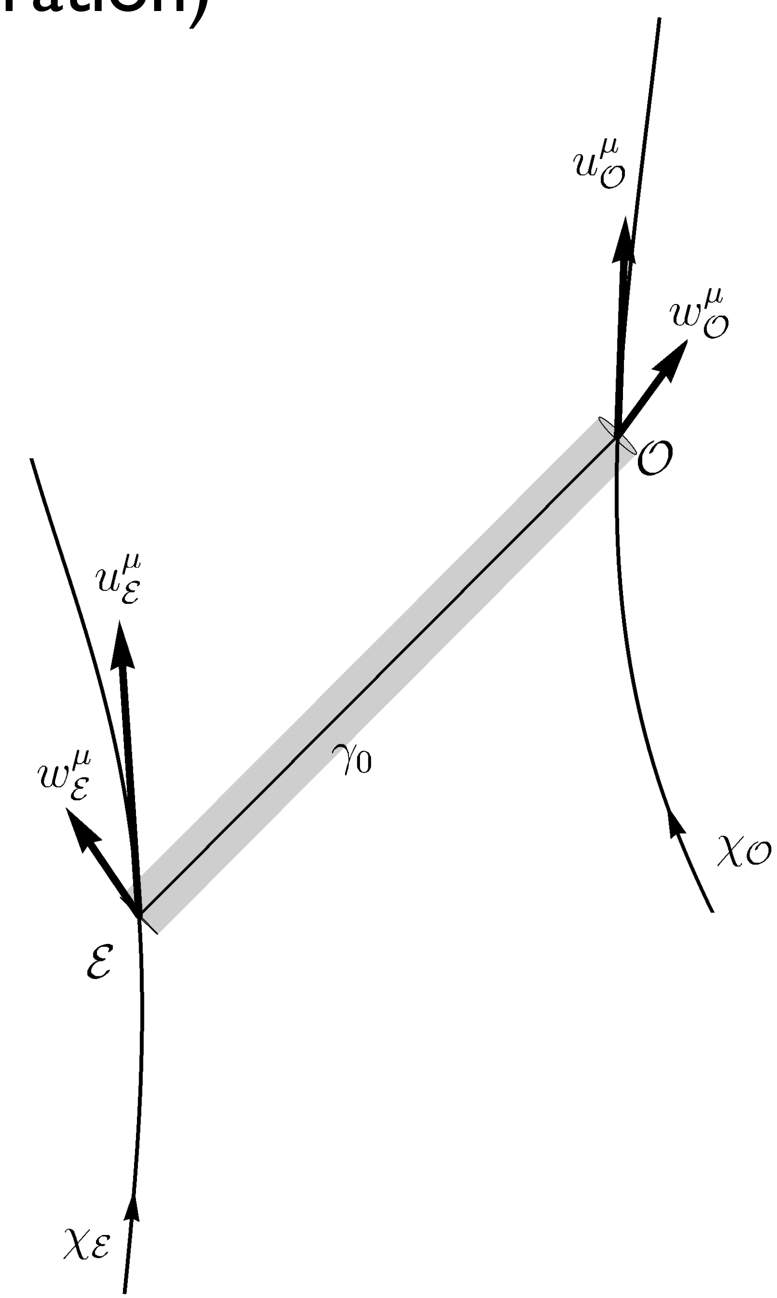

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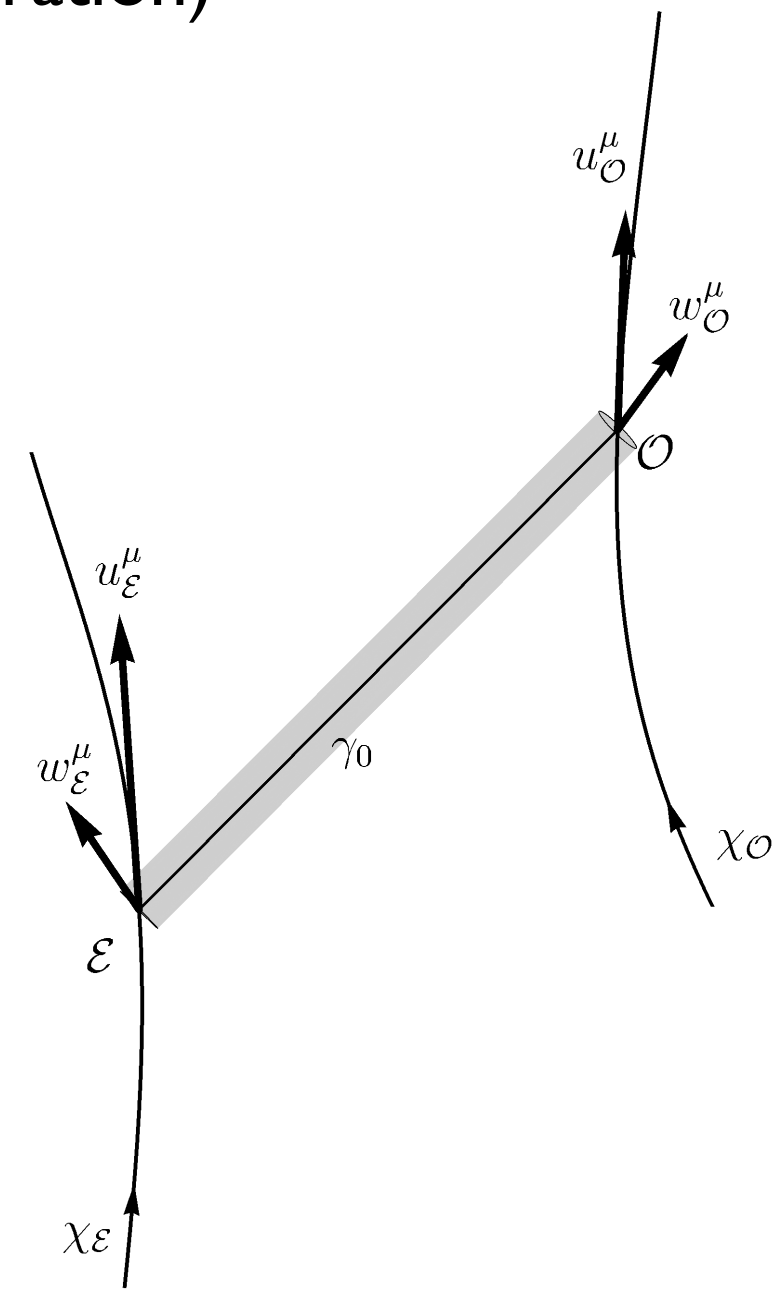

Applications

Extension of the Sachs formalism

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to be published soon