A possible connection of blueshifts in the Lemaitre - Tolman and Szekeres models with the gamma-ray bursts

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Contents

1. Motivation and background

In a Robertson – Walker spacetime, every light ray emitted at the Big Bang (BB) reaches all later observers with ***infinite redshift*** ($z_{obs} \to \infty$, frequency $\nu_{obs} \to 0$).

In ***Lemaître – Tolman (L-T)*** and ***Szekeres*** spacetimes (see further), ***some rays*** emitted at the BB reach all observers with ***infinite blueshift*** ($z_{obs} \to -1$, $\nu_{obs} \to \infty$).

→ Rays emitted *close* to the BB can display strong (finite) blueshifts ($\nu_{obs} >> \nu_{em}$).

A necessary condition for $\nu_{obs} >> \nu_{em}$ is that the BB time at the emission point is not constant and not extremum ($dt_B/dr \neq 0$) in comoving coordinates. [1]

In L-T, the other necessary condition is that the ray is ***radial*** [2].

The L-T and Szekeres families of spacetimes contain Friedmann as a subcase, and constant BB is an exception even in this small set.

When these models are applied to cosmology, blueshifts must be accommodated.

[1] P. Szekeres, Naked singularities. In: *Gravitational Radiation, Collapsed Objects and Exact Solutions*. Edited by C. Edwards. Springer (Lecture Notes in Physics, vol. 124), New York, pp. 477 -- 487 (1980).
[2] C. Hellaby and K. Lake, The redshift structure of the Big Bang in inhomogeneous cosmological models. I. Spherical dust solutions. *Astrophys. J.* **282**, 1 (1984) + erratum *Astrophys. J.* **294**, 702 (1985).

On the other hand, astronomers do observe impulses of high-frequency electromagnetic radiation (for example, X rays and gamma-ray bursts, GRBs).

It is known nearly for sure that GRB sources are a few billion light-years away.

What if?

the GRBs were emitted simultaneously with the radiation now seen as the CMB, but were blueshifted by the L-T/Szekeres mechanism?

The CMB rays were emitted $\tau \approx 380\,000$ years after the BB [3].

Can any rays emitted then reach us now with a blueshift instead of redshift?

Can the blueshift account for the frequencies of the GRBs?

Yes to both questions! – see Refs. [4 – 6] reported here.

[3] http://astronomy.swin.edu.au/cosmos/e/epoch+of+recombination
[4] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[5] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).
[6] A. Krasiński, Modeling sources of the gamma-ray bursts using quasi-spherical Szekeres metrics. ArXiv 1704.08145, submitted for publication.

2. Basic properties of gamma-ray bursts (GRBs) [7] [(H) = hypotheses].

(1) The GRB frequencies are contained in the range [8]

$\nu_{\gamma\ min} \approx \mathbf{0.24 \times 10^{19}\ Hz} < \boldsymbol{\nu} < \mathbf{1.25 \times 10^{23}\ Hz} \approx \nu_{\gamma\ max}$. (Converted from keV to Hz by $\nu = E/h$)

(2) GRBs typically last from < 1 second to a few minutes (but a few lasted between 2 and 30 hours) [9].

(3) Most GRBs are followed by longer-lived and fainter **afterglows** at longer waves.

(H) It is believed that **all GRBs have afterglows**, but some of them were missed by observers [10].
Nearly all knowledge about GRBs comes from observations of the afterglows [10].

(4) **(H)** GRBs are probably **focussed into narrow jets**.

(5) **(H)** Nearly all GRBs come from distances 10^8 ly < d < several billion ly.

Why **(H)**? The distances are calculated from redshifts measured for the afterglows using the Friedmann relations, so they may be grossly underestimated [4] – see Appendix.

(6) About one GRB per day is observed [7], so the sources must be many.

[4] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[7] D. Perley, http://w.astro.berkeley.edu/~dperley/pub/grbinfo.html
[8] A. Goldstein *et al*., The *Fermi* GBM gamma-ray burst spectral catalog: the first two years. *Astrophys. J. Suppl.* **199**, 19 (2012).
[9] S. J. Smartt, A twist in the tale of the γ-ray bursts. *Nature* **523**, 164 (2015).
[10] Gamma Ray Burst Afterglow, http://astronomy.swin.edu.au/cosmos/G/gamma+ray+burst+afterglow

(1) Frequencies ≈ 0.24×10^{19} Hz < ν < ≈ 1.25×10^{23} Hz.
(2) Lasting from <1s to a few minutes, exceptionally to 30 hours.
(3) Afterglows.
(4) Probably focussed into narrow jets.
(5) Come from distances 10^8 < d < several billion ly.
(6) The sources must be many.

No explanation of origins of the GRBs is universally accepted.

Different explanations apply to different classes of GRBs: gravitational collapse to a black hole, supernova explosions or collisions of ultra-dense neutron stars.

The models presented further on account satisfactorily for properties **(1)**, **(4)**, **(5)** and **(6)** and qualitatively for **(2)** and **(3)**.

Qualitatively = the effect is there, but the implied numbers do not agree with observations, so the models need improvement.

3. The Lemaître - Tolman (L-T) models

The metric of these models is

$$ds^2 = dt^2 - \frac{{R,_r}^2}{1+2E(r)}dr^2 - R^2(t,r)\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \qquad (3.1)$$

where E(r) is arbitrary and R(t,r) is determined by

$$R,_t{}^2 = 2E(r) + \frac{2M(r)}{R} - \frac{1}{3}\Lambda R^2, \qquad (3.2)$$

M(r) is one more arbitrary function and Λ is the cosmological constant.

This is a dust solution of Einstein's equations (p = 0), with the mass density

$$\frac{8\pi G}{c^2}\rho = \frac{2M,_r}{R^2 R,_r}, \qquad (3.3)$$

This solution was first found from the Einstein equations by Lemaître [11] in 1933, then investigated by Tolman [12] in 1934 and Bondi [13] in 1947.

And by > 100 other authors in later years. The number is still growing.

[11] G. Lemaître, L'Univers en expansion [The expanding Universe], *Ann. Soc. Sci. Bruxelles* **A53**, 51 (1933); *Gen. Rel. Grav.* **29**, 641 (1997).
[12] R. C. Tolman, Effect of inhomogeneity on cosmological models, *Proc. Nat. Acad. Sci. USA* **20**, 169 (1934); *Gen. Rel. Grav.* **29**, 935 (1997).
[13] H. Bondi, Spherically symmetrical models in general relativity. *Mon. Not. Roy. Astr. Soc.* **107**, 410 (1947; *Gen. Rel. Grav.* **31**, 1783 (1999).

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{{R,_r}^2}{1+2E(r)}\mathrm{d}r^2 - R^2(t,r)\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right) \qquad (3.1)$$

$${R,_t}^2 = 2E(r) + \frac{2M(r)}{R} - \frac{1}{3}\Lambda R^2, \qquad (3.2)$$

When E(r) > 0 and Λ = 0, the solution of (3.2) is:

$$\begin{aligned} R(t,r) &= \frac{M}{2E}(\cosh\eta - 1), \\ \sinh\eta - \eta &= \frac{(2E)^{3/2}}{M}\left[t - t_B(r)\right]. \end{aligned} \qquad (3.4)$$

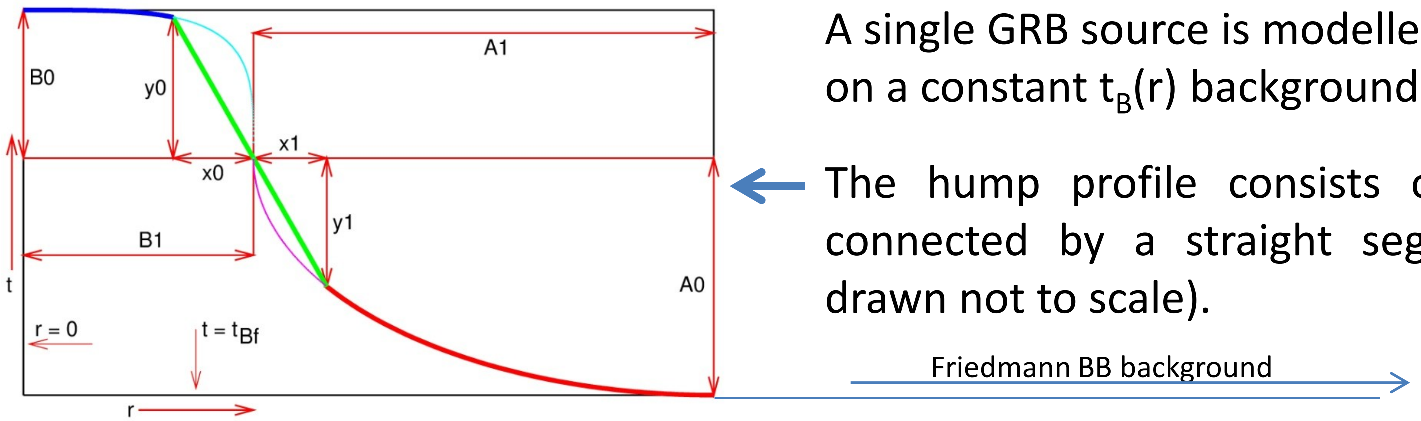
M(r), E(r) and $t_B(r)$ are "constants of integration" of the Einstein eqs.; t = $t_B(r)$ is the Big Bang.

The Friedmann limit follows from (3.1) when $M^{2/3}/E$ and t_B are constant; then R(t,r) = $M^{1/3}$ S(t), where S(t) is the Friedmann scale factor.

I will consider a composite model consisting of a Friedmann background, into which an L-T island is matched.

Recall: necessary conditions for z = -1 in L-T are: $dt_B/dr \neq 0$ at emission and the ray being radial.

4. An L-T model of a single GRB source



A single GRB source is modelled by a hump on a constant $t_B(r)$ background.

The hump profile consists of two arcs connected by a straight segment (here drawn not to scale).

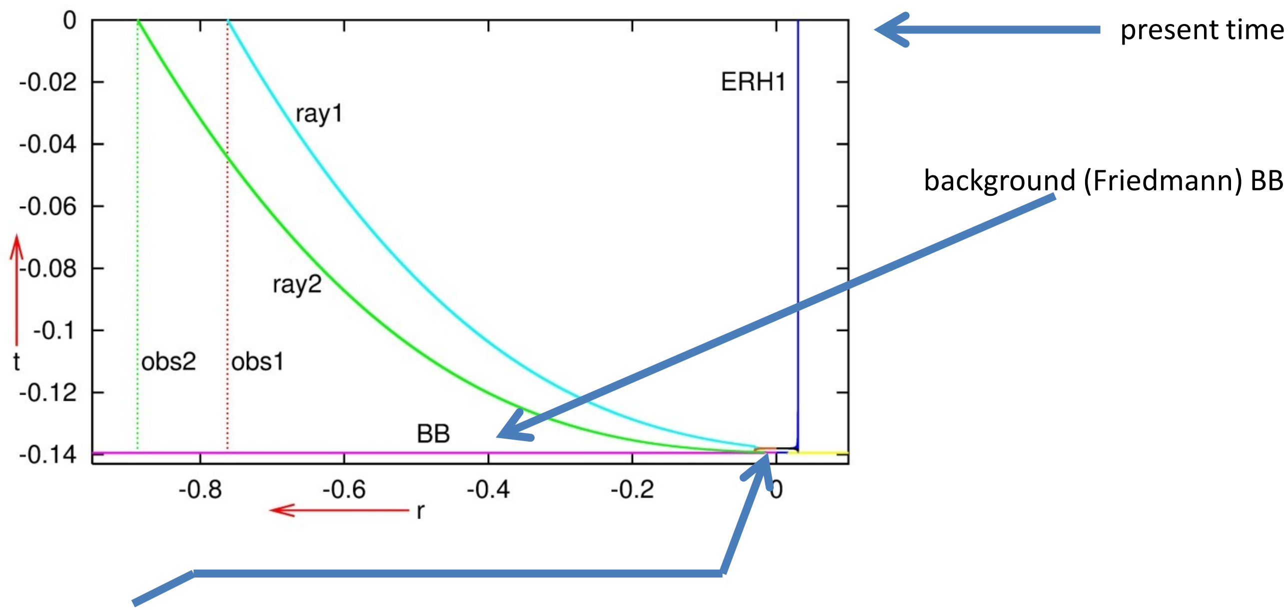
The upper-left arc is a segment of an ellipse-like curve:

$$\frac{r^4}{{B_1}^4} + \frac{(t - t_{\mathrm{Bf}} - A_0)^4}{{B_0}^4} = 1 \quad \text{or} \quad \frac{r^6}{{B_1}^6} + \frac{(t - t_{\mathrm{Bf}} - A_0)^6}{{B_0}^6} = 1. \tag{4.1}$$

The lower-right arc is a segment of an ellipse.

The straight segment prevents $dt_B/dr \to \infty$ at the junction of full arcs.

The free parameters are A_0, A_1, B_0, B_1 and x_0.



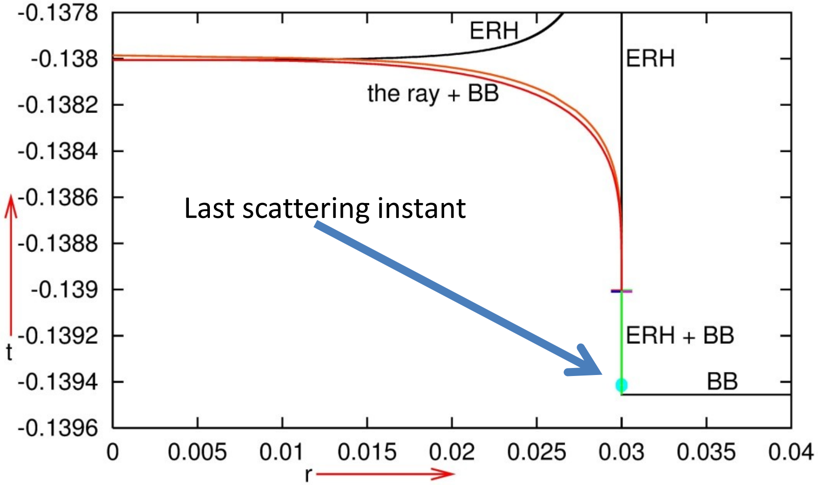
Here two humps are drawn in proportion to the age of the Universe

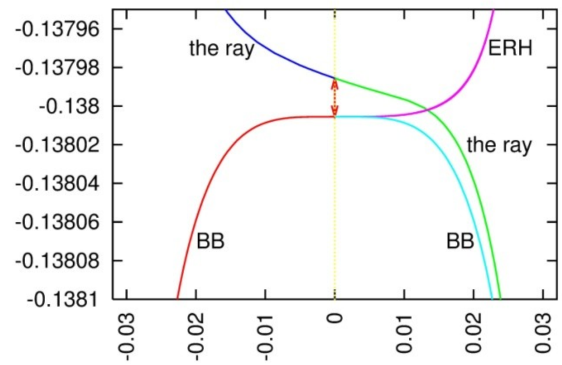
The lower hump (together with ray 2) constitutes a model of a GRB source of the lowest observed energy.

It has the height 8.9×10^{-4} × (the age of the Universe) ≈ 1.23×10^{7} years (the Universe age taken from the ΛCDM model),

encompasses the mass ≈ 3.1×10^{6} masses of our Galaxy,

and its upper arc is of 6-th degree.





Top of the hump, magnified

(imagine each figure being rotated around the r = 0 axis)

The real shape of the hump, and the blueshifted ray near the BB

Backward in time along the ray, z increases up to the first intersection with the ERH (***Extremum Redshift Hypersurface***).

Further into the past, z decreases until the next intersection of the ray with the ERH (or until the ray hits the BB).

The hump parameters are chosen such that

$$2.5 \times 10^{-8} < 1 + z_{\text{observed now}} < 1.7 \times 10^{-5}$$

which moves the frequencies from the hydrogen emission range to the GRB range:

$$0.24 \times 10^{19} < \nu_{GRB} < 1.25 \times 10^{23} \text{ Hz.}$$

The multitude of the GRBs is accounted for by putting many such BB humps into a Friedmann background.

Models of this type account for [4]:

(1) The observed frequency range of the GRBs $[0.24 \times 10^{19}\mathrm{Hz} \leq \nu \leq 1.25 \times 10^{23}\mathrm{Hz}]$;

(2) Their limited duration (observed: up to 30 hours);

(3) The afterglows (observed durations: up to several hundred days);

(5) The large distances to their sources ($n \times 10^9$ ly);

(6) The multitude of the observed GRBs (observed: about 1/day, the best model implies up to ≈11 000 potential sources in the whole sky at present).

Properties (2) and (3) are accounted for qualitatively (the effect is there, but the numbers do not agree with observations and the model needs improvements).

Property **(4)** (hypothetical collimation of the GRBs into narrow jets) is not accounted for in an L-T model because of its spherical symmetry (the blueshifted rays are emitted isotropically). But it is implied by the Szekeres models (see further).

[4] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).

5. The quasi-spherical Szekeres (QSS) models

The quasi-spherical Szekeres solutions [14] have the metric

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}t^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} \mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \\ \mathcal{E} &\overset{\mathrm{def}}{=} \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2}, \end{aligned} \tag{5.1}$$

where E(r), M(r), P(r), Q(r) and S(r) are arbitrary functions, and

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{5.2}$$

The mass density is

$$\kappa\rho = \frac{2\left(M/\mathcal{E}^3\right),_r}{(\Phi/\mathcal{E})^2 \left(\Phi/\mathcal{E}\right),_r}, \qquad \kappa = \frac{8\pi G}{c^2}. \tag{5.3}$$

Eq. (5.2) is the same as in the L--T model, and its solution is:

$$\int\limits_0^\Phi \frac{\mathrm{d}\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{5.4}$$

[14] P. Szekeres, A class of inhomogeneous cosmological models. *Commun. Math. Phys.* **41**, 55 (1975).

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} \mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{5.1}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{5.2}$$

$$\int_0^\Phi \frac{\mathrm{d}\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{5.4}$$

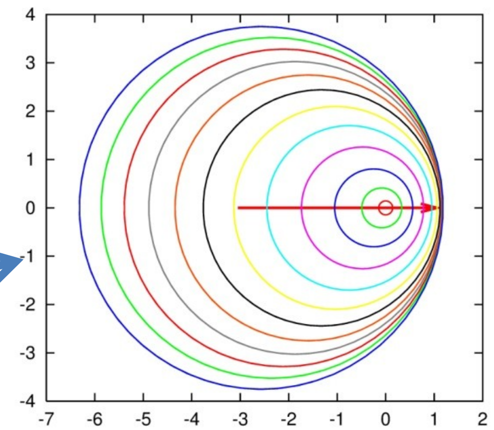
The source in the Einstein equations is dust.

A general QSS metric has no symmetry.

The surfaces of constant t and r

ds^2 = $\dfrac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right)$

are ***nonconcentric spheres***,
x and y are stereographic coordinates on them.



The L-T models are the limit of constant (P, Q, S) – then the spheres become concentric.

$$ds^2 = dt^2 - \frac{\mathcal{E}^2(\Phi/\mathcal{E}),_r{}^2}{1+2E(r)}dr^2 - \frac{\Phi^2}{\mathcal{E}^2}\left(dx^2+dy^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{5.1}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} \tag{5.2}$$

6. Blueshifts in axially symmetric QSS models

In L-T, z = -1 was possible only on radial rays. But a general Szekeres model has no symmetry, so no radial directions. Can large blueshifts exist in it at all?

In an *axially symmetric* QSS model, a *necessary condition* for infinite blueshift is that the ray is ***axial*** (intersects every space of constant t on the symmetry axis) [5].

That this condition is also ***sufficient*** was numerically verified for an exemplary QSS model, in which

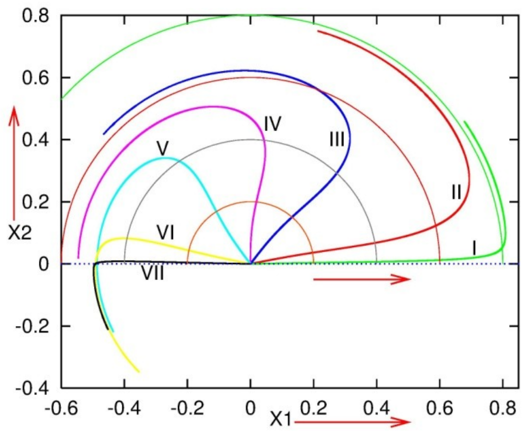
$2E(r) = -k r^2$ (the same as in Friedmann), with $k = -0.4$, (6.1)

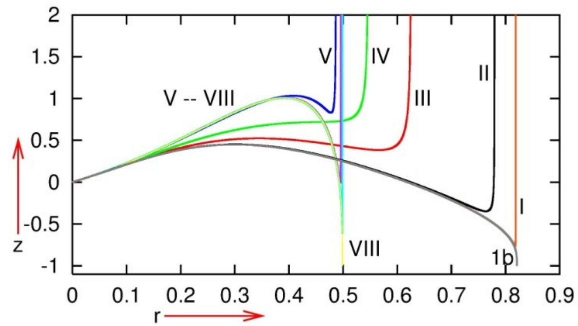
$P = Q = 0$ (for axial symmetry), $S^2(r) = a^2 + r^2$ (for simplicity), (6.2)

$$t_B(r) = \begin{cases} A\left(e^{-\alpha r^2} - e^{-\alpha {r_b}^2}\right) + t_{BB} & \text{for } r \le r_b, \\ t_{BB} & \text{for } r \ge r_b, \end{cases} \tag{6.3}$$

where A, α, r_b and t_{BB} are constants.

[5] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).





Rays 1b and VIII are axial.

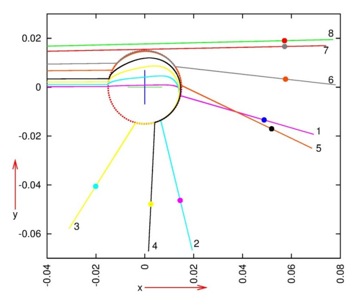
Rays projected on a surface of constant t and φ (left) and z-profiles along them (right)

The line $X_2 = 0$ is the projection of the symmetry axis.

$z_{min} \rightarrow -1$ when the ray approaches axial. On rays 1b and VIII, $1 + z_{min} < 10^{-5}$.

Non-axial rays hit the BB hump tangentially to r = constant surfaces, with $z_{obs} \rightarrow \infty$ (the same happens with nonradial rays in L-T).

Rays overshooting the hump would be strongly deflected and would hit the BB in the Friedmann region with $z_{obs} = \infty$.



$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} \mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{5.1}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} \tag{5.2}$$

7. Blueshifts in nonsymmetric QSS models

An analogous investigation was done in a QSS model without symmetry [5].

There was no hint whether blueshifted rays exist at all in this case; the search for them had to be done numerically all the way.

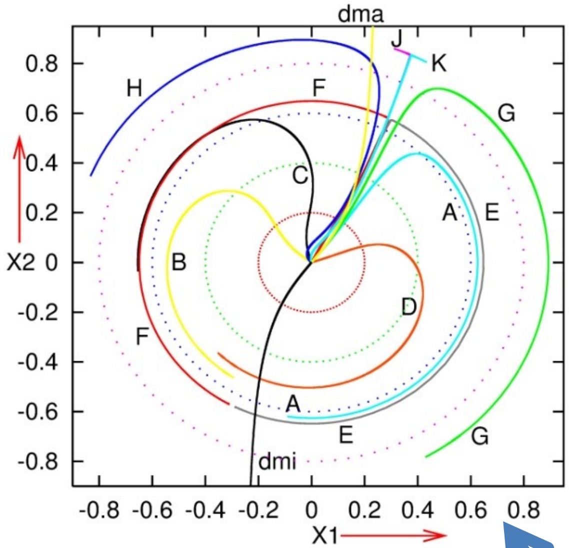
The E(r), S(r) and t_B(r) functions were the same as before,

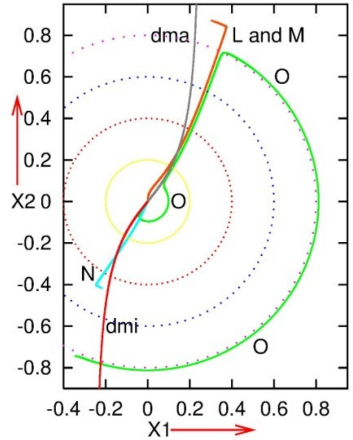
but P and Q had to be nonconstant to destroy the symmetry:

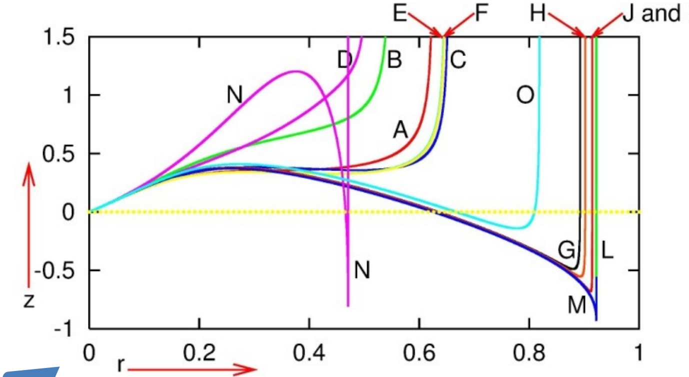
$$P(r) = \frac{pa}{2\left(a^2 + r^2\right)}, \qquad Q(r) = \frac{qa}{\sqrt{a^2 + r^2}}, \tag{7.1}$$

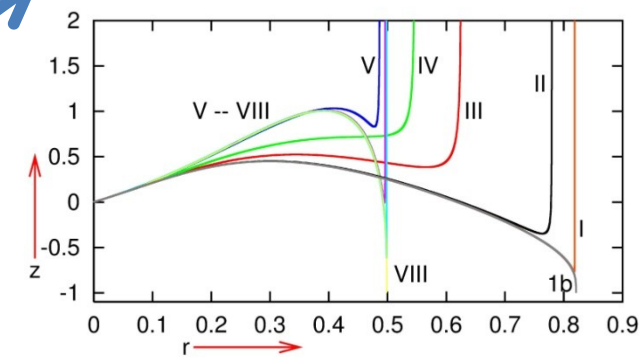
where *p* and *q* are constant parameters.

[5] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).









Projections of exemplary rays and redshift profiles along them in the nonsymmetric QSS model

The z(r) graphs are similar to those in the axially symmetric case.

→ Along two opposite null directions in a general Szekeres model (which is of Petrov type D) strong blueshifts exist.

But these directions ***do not*** coincide with the two principal null directions of the Weyl tensor, except in the axially symmetric case [5].

[5] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).

8. A realistic QSS model of a GRB

The exemplary QSS models were illustrative, but unrelated to cosmology.

The next thing to do was to superpose a QSS perturbation on the L-T model presented in the first part of this talk and see how much it will improve. This was done in Ref. [6].

The immediate improvement is the collimation of the GRBs: $z \to -1$ is possible only along two opposite directions.

With an axially symmetric QSS superposed on L–T, $1 + z$ along the axial direction becomes smaller.

→ The GRBs could be accounted for with a lower BB hump.

A lower hump reduces the angular size of the source, increases its distance from the observer → more potential sources fit into the observer's field of view (≈ 11 000). But the progress was not dramatic.

[6] A. Krasiński, Modeling sources of the gamma-ray bursts using quasi-spherical Szekeres metrics. ArXiv 1704.08145, submitted for publication.

In the current best Szekeres/Friedmann model, the angular radius of a GRB source is

$0.9681^\circ < \vartheta < 0.9783^\circ$,

depending on the direction of observation.
The current precision in determining the direction to an observed GRB source is a disk in the sky of radius ≈ 0.5°.
Thus, the whole sky could accommodate

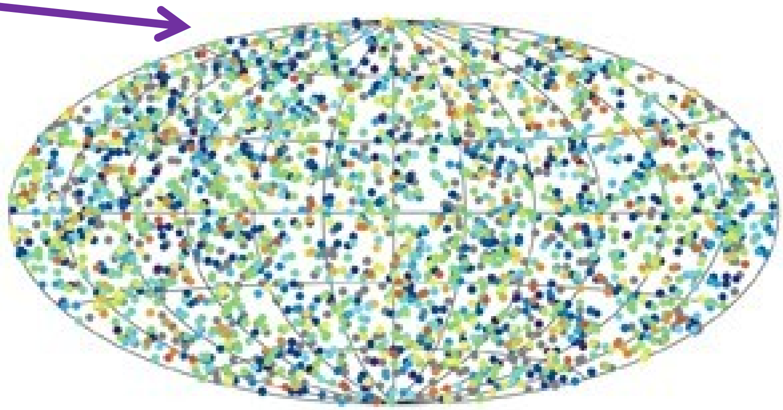
11 005 > *N* > 10 076

such objects.

The best now-existing detectors (BATSE = Burst And Transient Source Experiment) detected 2704 GRBs during their first 9 years (between 1991 and 1999) [15].

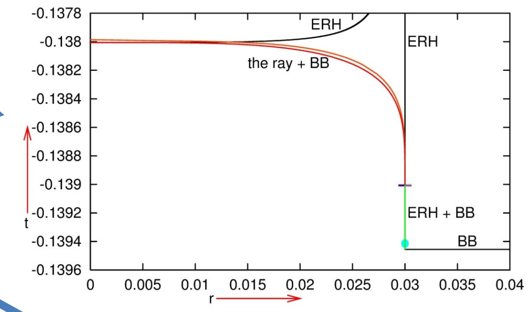
So, during the 27 years up to now the BATSE detectors should have discovered **8112** GRBs.

→ The **numbers** in the model and in the observations are consistent.



[15] Gamma-Ray Bursts, http://swift.sonoma.edu/about_swift/grbs.html

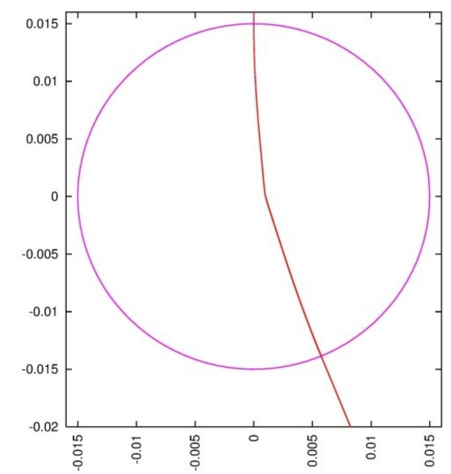
The Szekeres model should be capable of explaining the short durations of the GRBs themselves and of their afterglows by means of the ***cosmic parallax*** effect [16], also called ***non-repeatability of the light paths*** (non-RLP) [17].



A non-axial ray propagating above the BB hump is ***deflected***,
but the angle of deflection depends on time (on how high above the hump the ray is flying) - because of the non-RLP phenomenon [17].

→ With time, the maximally blueshifted ray changes direction, so after a while it will miss the observer.

A numerical illustration of this effect is being prepared.



[16] C. Quercellini, L. Amendola, A. Balbi, P. Cabella, M. Quartin, Real-time cosmology. *Phys. Rep.* **521**, 95 -- 134 (2012).
[17] A. Krasiński and K. Bolejko, Redshift propagation equations in the β' ≠ 0 Szekeres models. *Phys. Rev*. **D83**, 083503 (2011).

9. Expression of hope

Astronomers do not take inhomogeneous models seriously, and try to dismiss blueshifts.

In several papers blueshifts were argued to cause assorted disasters.

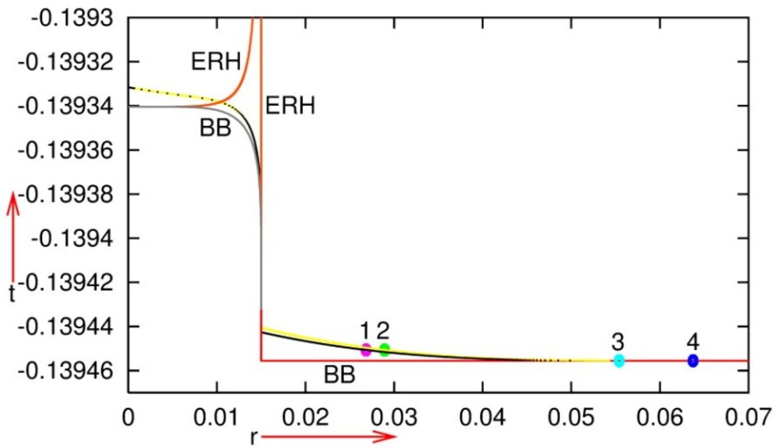
The disasters are spurious – some implications of the inhomogeneous models differ from expectations based on Friedmannian intuitions, which are heavily model-dependent.

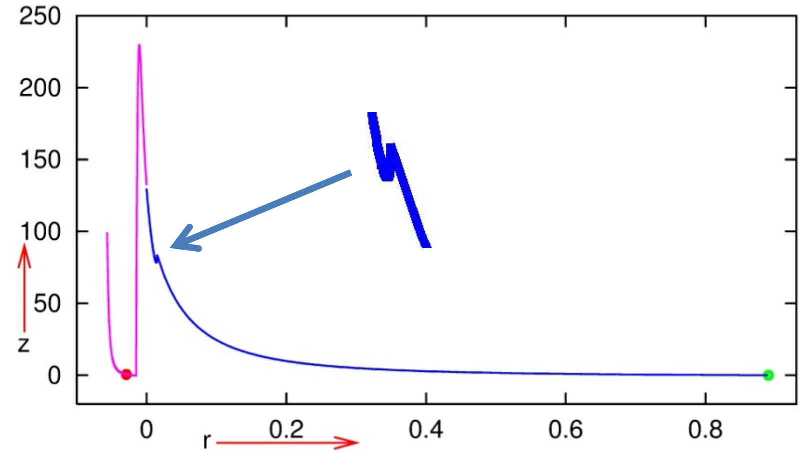
My aim was to show that L-T and Szekeres models have interesting geometries, in particular that blueshifts imply interesting bits of optics.

History of science teaches us that if a well-tested theory predicts a phenomenon, then the prediction has to be taken seriously and put to experimental tests.

Perhaps this will happen with the results reported here (but will it during our lifetime?).

10. Appendix: non-monotonicity of redshift along light rays





When local blueshifts are present, ***redshift fails*** to be a distance indicator.

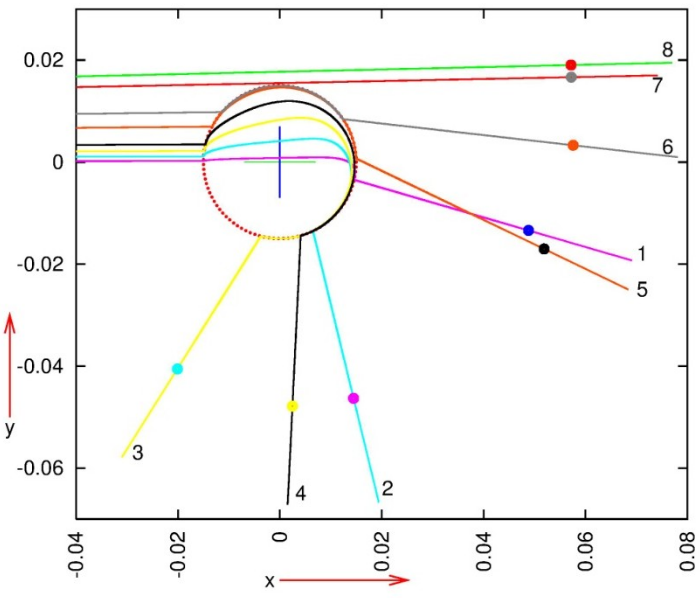
The right graph shows z(r) seen by the observer sitting at r ≈ 0.9 (**green** dot), calculated along the **yellow ray** of the left graph.

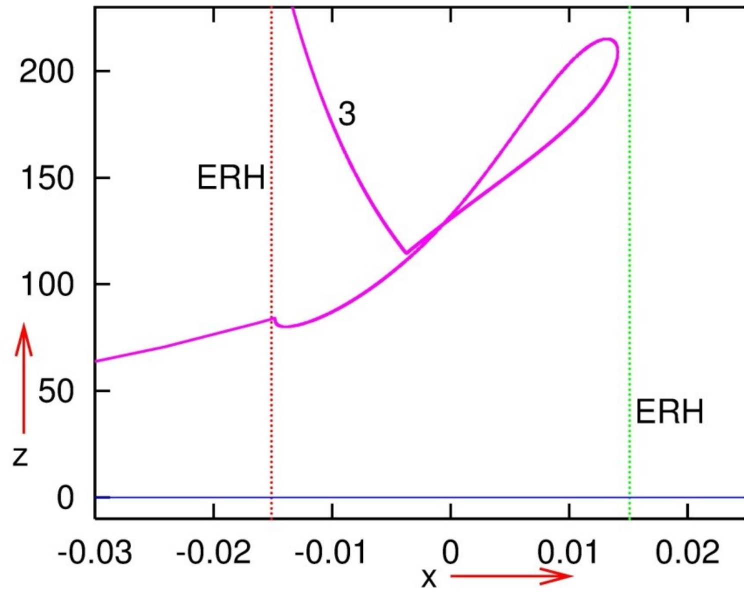
The redshift first increases toward the past, then decreases under the ERH.

At the **red dot** in the right graph z = 0.598. The standard formula would imply the source to lie 5.9×10^9 years to the past [18,19].

In this model, the source lies 1.37×10^{10} years to the past.

[18] E. L. Wright, A Cosmology Calculator for the World Wide Web. *Publ. Astr. Soc. Pac.* **118**, 1711 (2006).
[19] E. L. Wright, http://www.astro.ucla.edu/~wright/ACC.html





Nonradial rays propagating above the hump toward the same observer (left graph)

Along them, too, z is not monotonic (right graph is for ray 3).

The big dots are where the rays hit the last scattering hypersurface.

The present observer sees all these rays within a 2^o cone around the central ray (the uncertainty in determinig the direction to a GRB source is 1^o).

This cone may be made still narrower when the model is improved.

The presence of these rays makes the model falsifiable against observations.