BIG RIP IN AN INHOMOGENEOUS UNIVERSE

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BIG RIP

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Phantom Energy: Dark Energy with $w < -1$ Causes a Cosmic Doomsday

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We explore the consequences that follow if the dark energy is phantom energy, in which the sum of the pressure and energy density is negative. The positive phantom-energy density becomes infinite in finite time, overcoming all other forms of matter, such that the gravitational repulsion rapidly brings our brief epoch of cosmic structure to a close. The phantom energy rips apart the Milky Way, solar system, Earth, and ultimately the molecules, atoms, nuclei, and nucleons of which we are composed, before the death of the Universe in a "big rip."

FLRW model with "phantom energy" w<-1, and spatially flat.

Big Rip in ~22 Gyr from now! Friedmann equation yields:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\Omega_{\rm m} a^{-3} + \Omega_\Lambda a^{-3(1+w)}\right)$$

$$t_{\rm rip} \approx \frac{2}{3H_0}\frac{1}{-(1+w)}\frac{1}{\sqrt{1-\Omega_{\rm m}}}$$

TABLE I. The history and future of the Universe with $w = -3/2$ phantom energy.

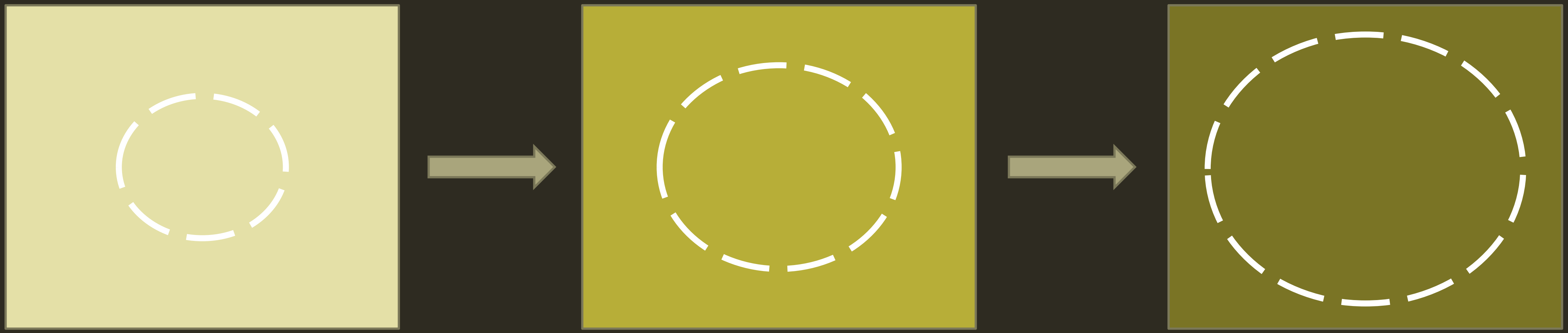
Time	Event
$\sim 10^{-43}$ s	Planck era
$\sim 10^{-36}$ s	Inflation
First three minutes	Light elements formed
$\sim 10^5$ yr	Atoms formed
~ 1 Gyr	First galaxies formed
~ 15 Gyr	*Today*
$t_{\rm rip} - 1$ Gyr	Erase galaxy clusters
$t_{\rm rip} - 60$ Myr	Destroy Milky Way
$t_{\rm rip} - 3$ months	Unbind solar system
$t_{\rm rip} - 30$ min	Earth explodes
$t_{\rm rip} - 10^{-19}$ s	Dissociate atoms
$t_{\rm rip} = 35$ Gyr	Big rip

EQUATION OF STATE

$$p = w\rho$$

Model the dark energy as a "fluid" (that is, it has pressure, unlike "dust"). Perfect fluid.

		w	-3(1+w)
Radiation / Ultra-relativistic matter		1/3	-4
Matter (non-relativistic)		0	-3
Curvature		-1/3	-2
Dark energy	Cosmological constant	-1	0
	Phantom energy	< -1	>0

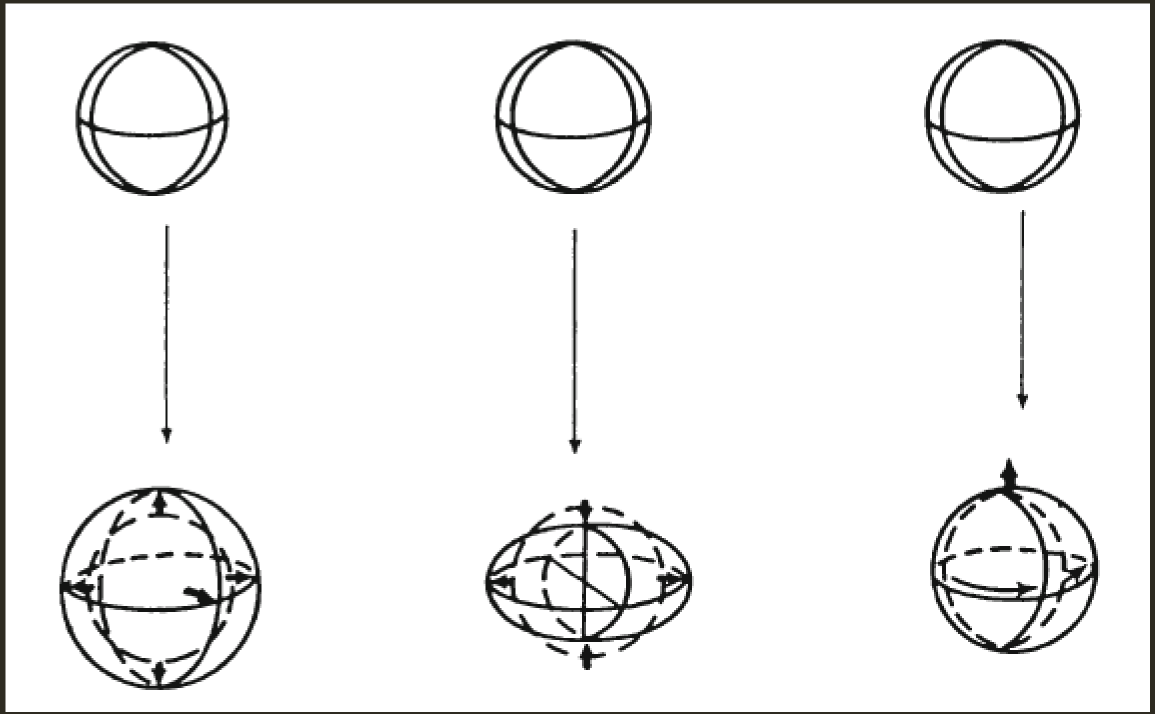


COSMIC FLUID

Einstein field equations and continuity equation

become fluid properties: expansion, shear, rotation, etc

See Ellis 1971 or Ehlers 1961 for review



	Properties
Fluid characteristics	Density
	Pressure
	Energy transfer
Velocity field	Expansion rate
	Shear
	Rotation
	Acceleration
Spacetime curvature	Electric part
	Magnetic part
etc. …	…

SILENT UNIVERSE

Existing studies assume the homogeneous case.

We generalise to the "silent universe" case.

Definition: Each worldline evolves independently of the other worldlines, with no communication between them.
This means no pressure gradients, no energy flux, no gravitational radiation, etc.

We also assume no rotation, for now.

Local Hubble parameter and 3-volume:

$$H \equiv \frac{\Theta}{3} \qquad \dot{\mathcal{V}} = \Theta \mathcal{V}$$

$$\dot{\rho} = -(\rho + p)\Theta$$

$$\dot{\Theta} = -\frac{\Theta^2}{3} - 4\pi(\rho + 3p) - 6\Sigma^2$$

$$\dot{\Sigma} = -\frac{2}{3}\Theta\Sigma + \Sigma^2 - \mathcal{W}$$

$$\dot{\mathcal{W}} = -\Theta\mathcal{W} - 4\pi\rho\Sigma - 3\Sigma\mathcal{W}$$

SHEAR = ZERO

$$\Sigma \equiv 0$$

Motivation: original Big Rip proposal.

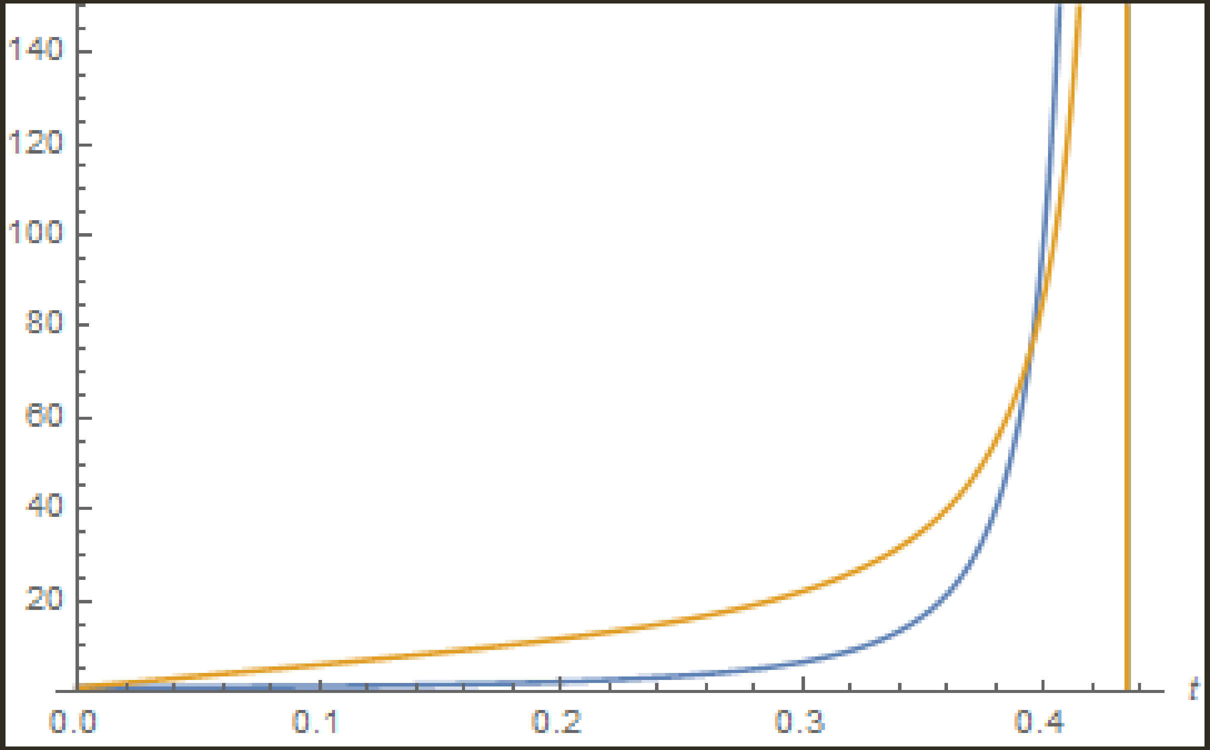
For a homogeneous and isotropic fluid, shear and Weyl scalar are zero.

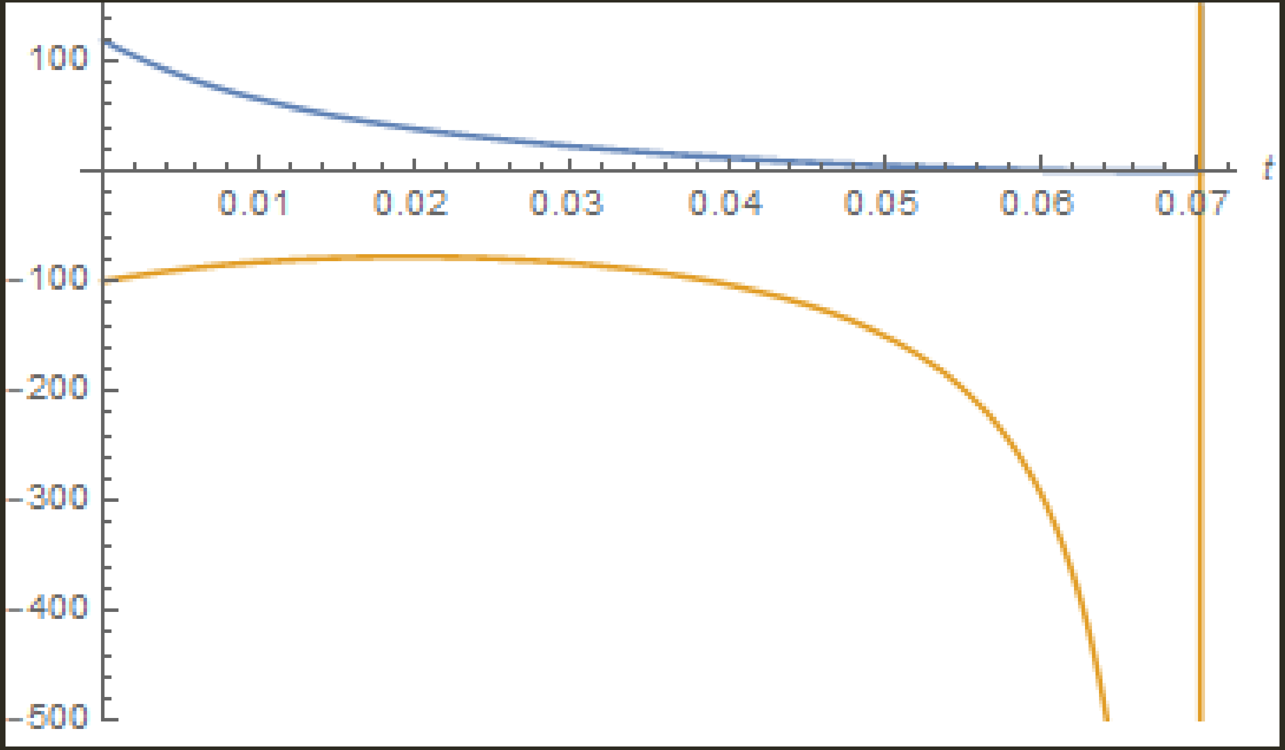
$$\dot{\rho} = -(1+w)\rho\Theta$$

$$\dot{\Theta} = -\frac{\Theta^2}{3} - (1+3w)4\pi\rho$$

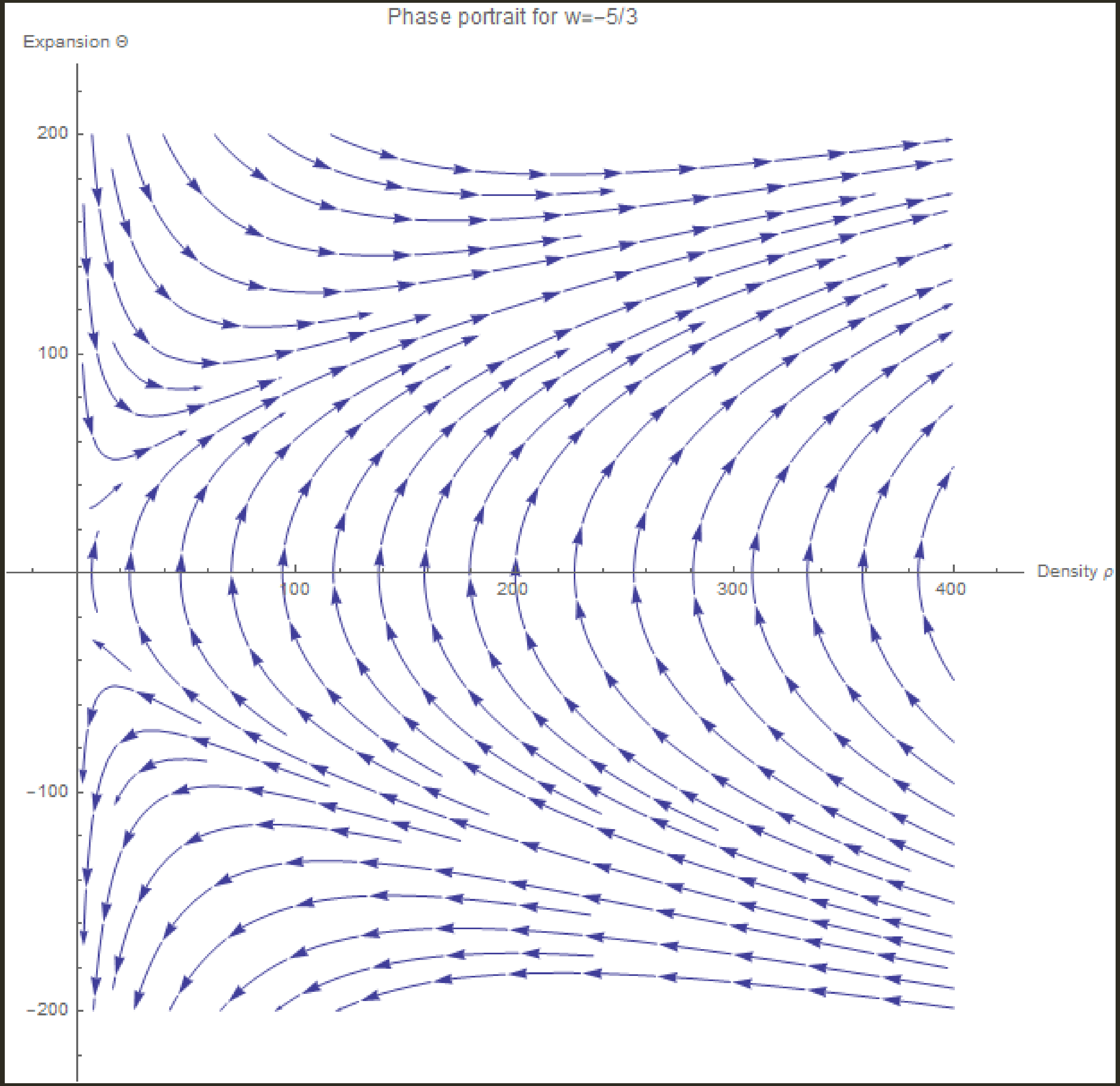
These are scalar fields over all spacetime. We pick initial conditions and integrate. This corresponds to following a single “location” / fluid element over time.

NUMERICAL SOLUTIONS



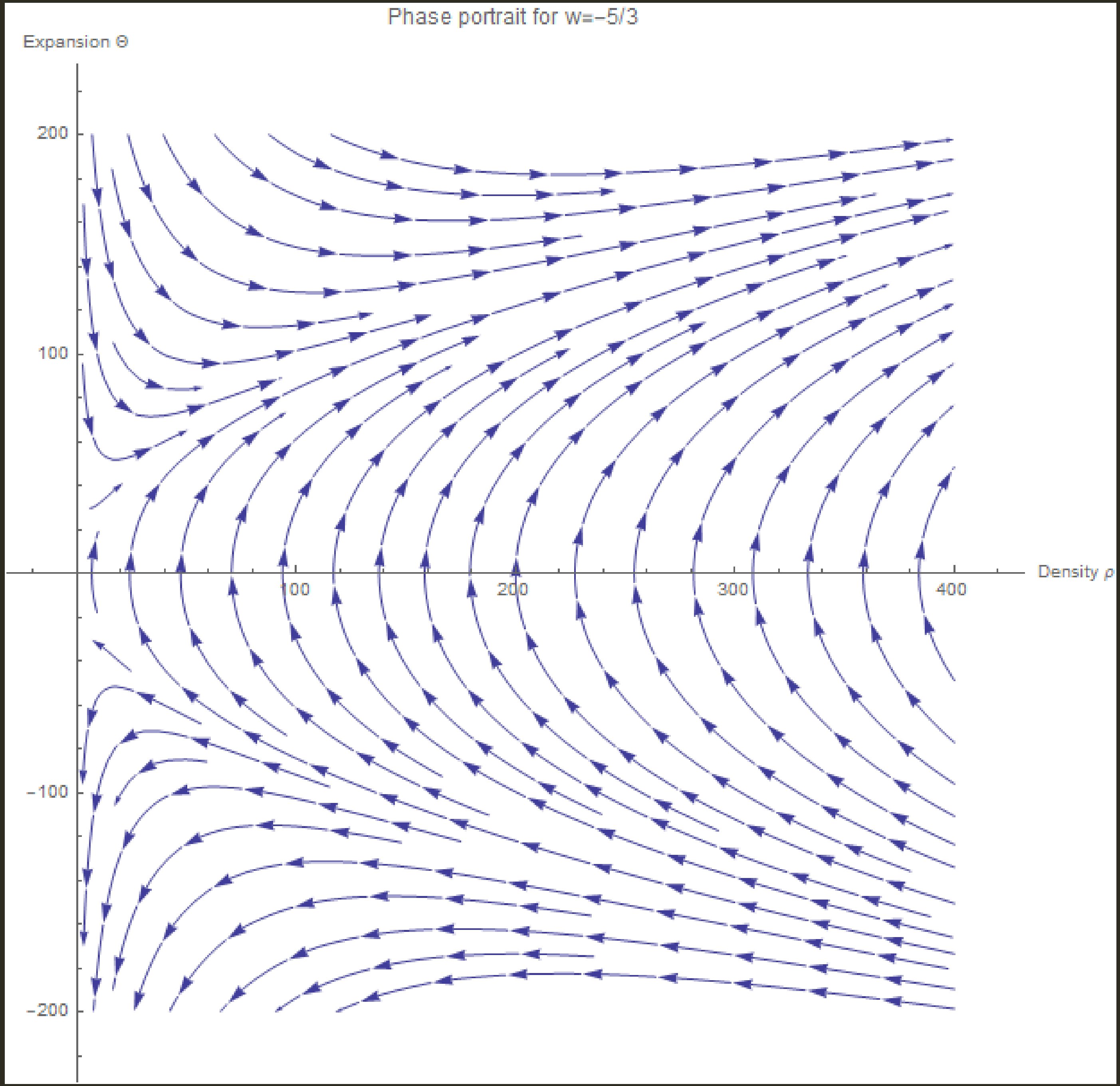


Blue = density, orange = expansion



Phase portrait, shows “all” parameter space

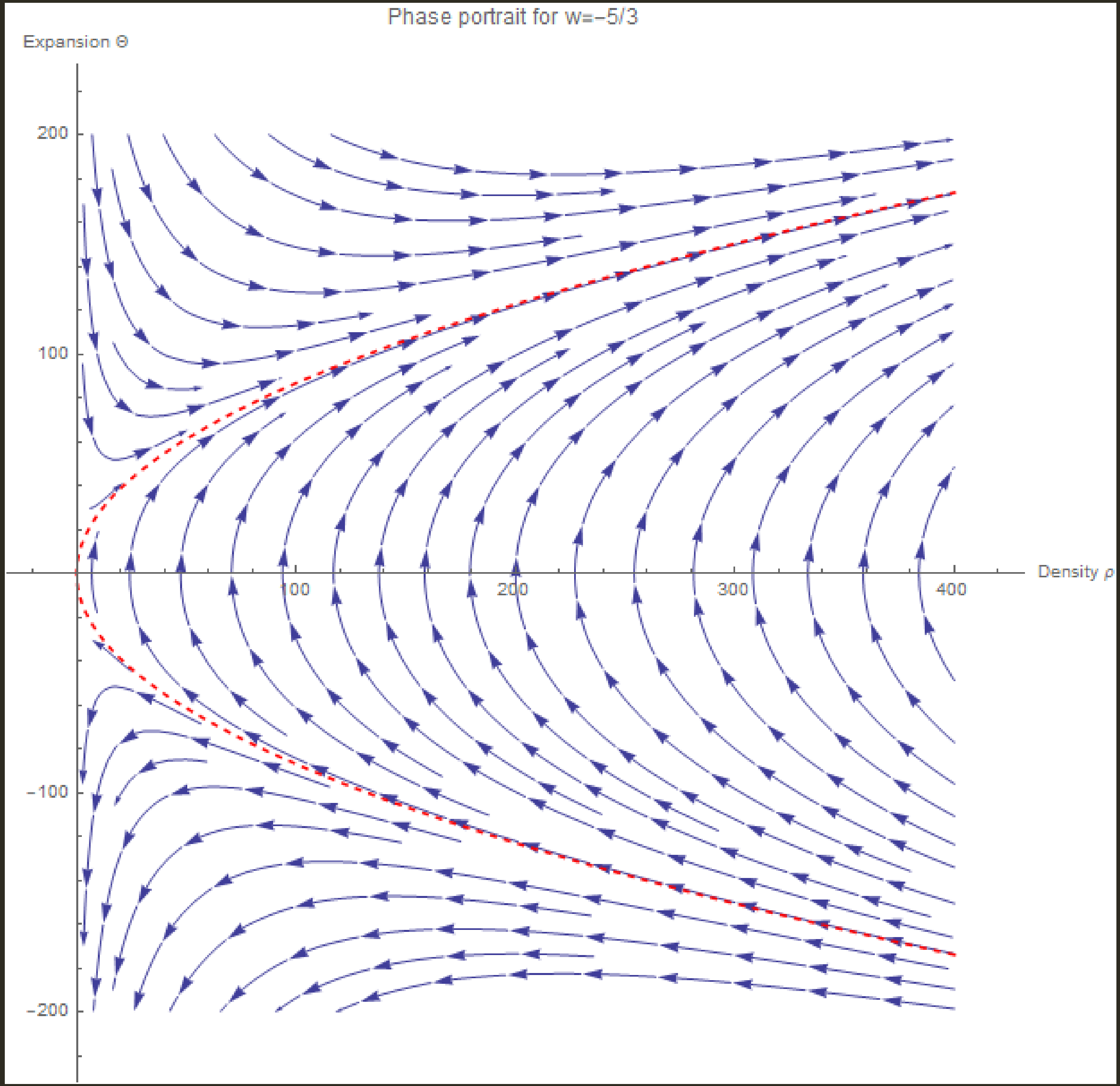
Collapse is good!



Try parabola trajectory:

$$\rho = k\Theta^2$$

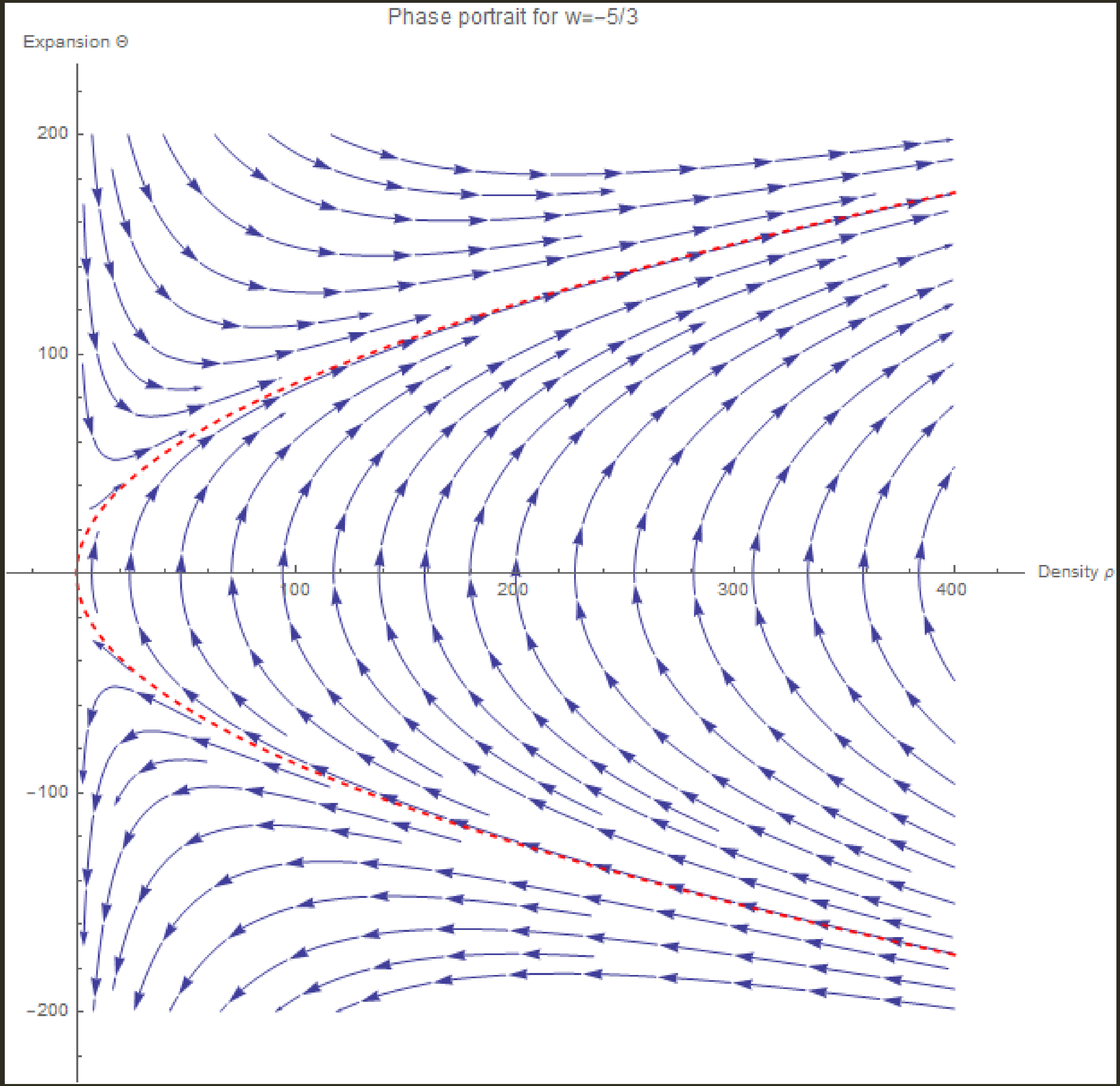
$$\dot{\rho} = 2k\Theta\dot{\Theta}$$



Critical trajectory (in phase space):

$$k = \frac{1}{24\pi}$$

$$\rho = \frac{1}{24\pi}\Theta^2$$



Critical regions:

- Rho=Theta=0
- Rho=0, Theta>0
- Middle and upper
- Lower
- Parabola

SUMMARY FOR SHEAR=0

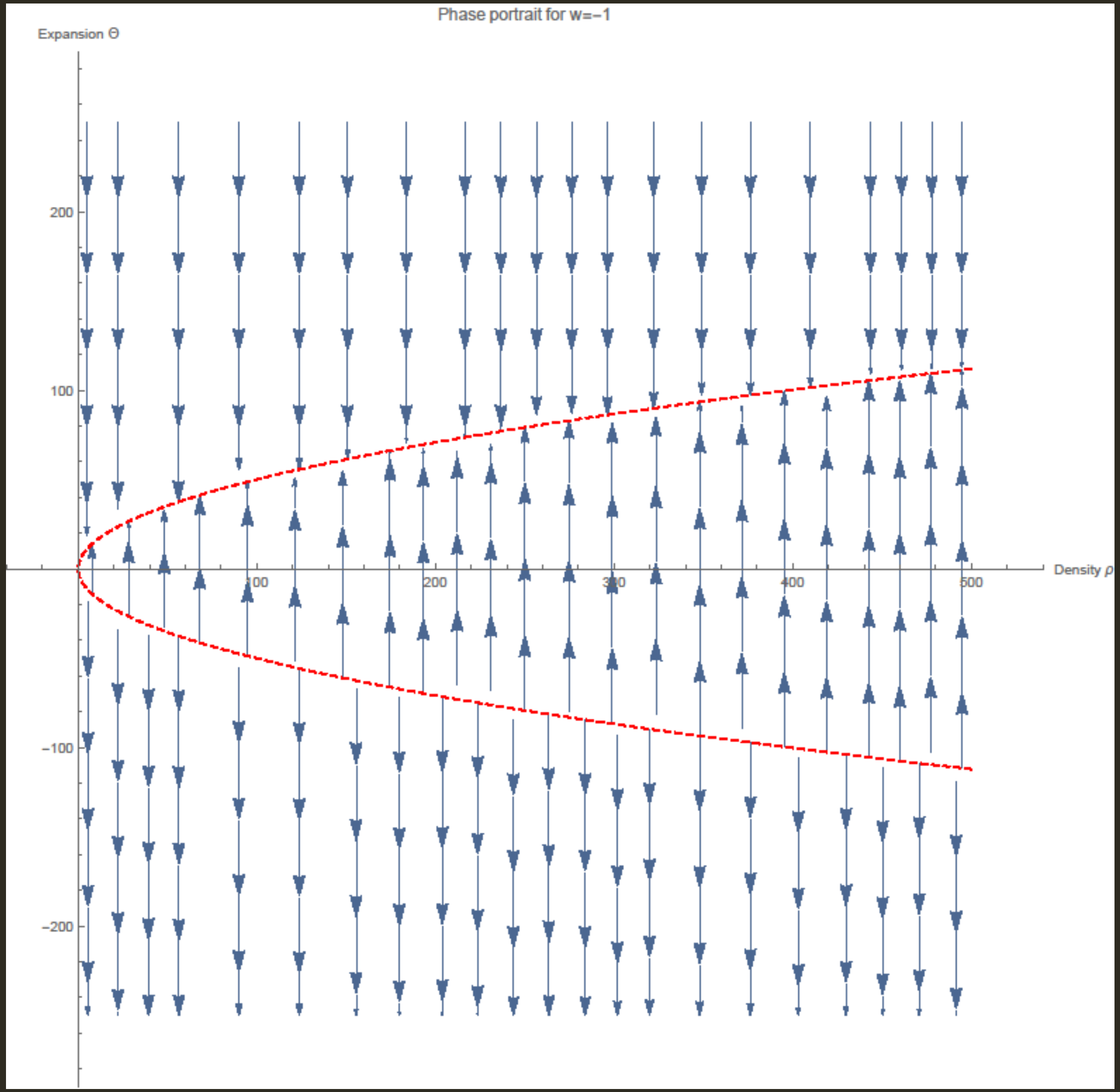
Phantom energy, w<-1

Initial conditions	Result
$\rho = \Theta = 0$	static empty universe
$\rho = 0,\ \Theta > 0$	slowing expansion
$\Theta > -\sqrt{\rho/24\pi}$	Big Rip
$\Theta = -\sqrt{\rho/24\pi}$	asymptotic collapse
$\Theta < -\sqrt{\rho/24\pi}$	Big Crunch ($\Theta < \rho = 0$ is included)

BONUS: COSMOLOGICAL CONSTANT W=-1

Cosmic "no-hair" theorem

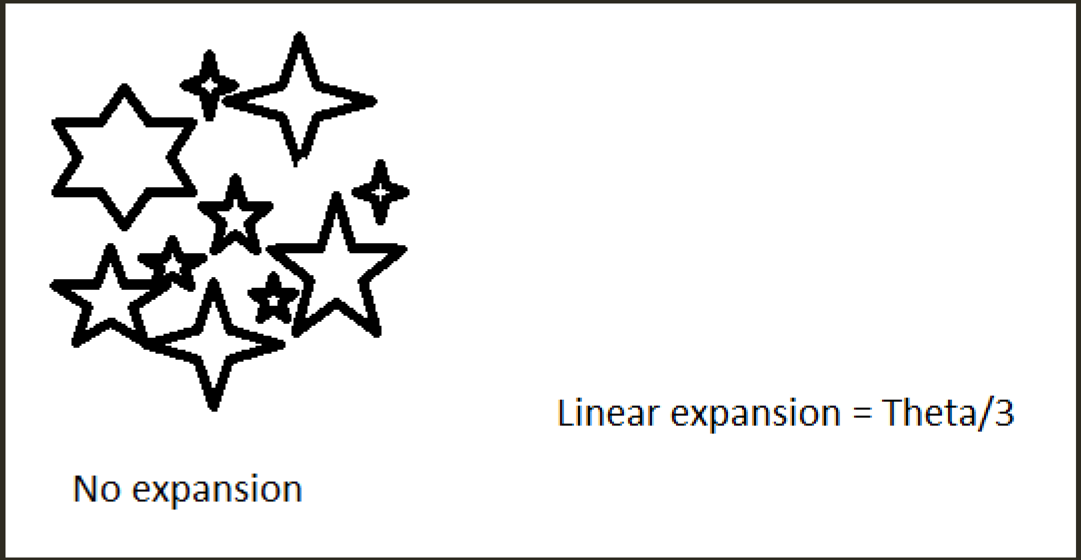
Hawking, Wald etc. (1980s)



SHEAR = EXPANSION/3

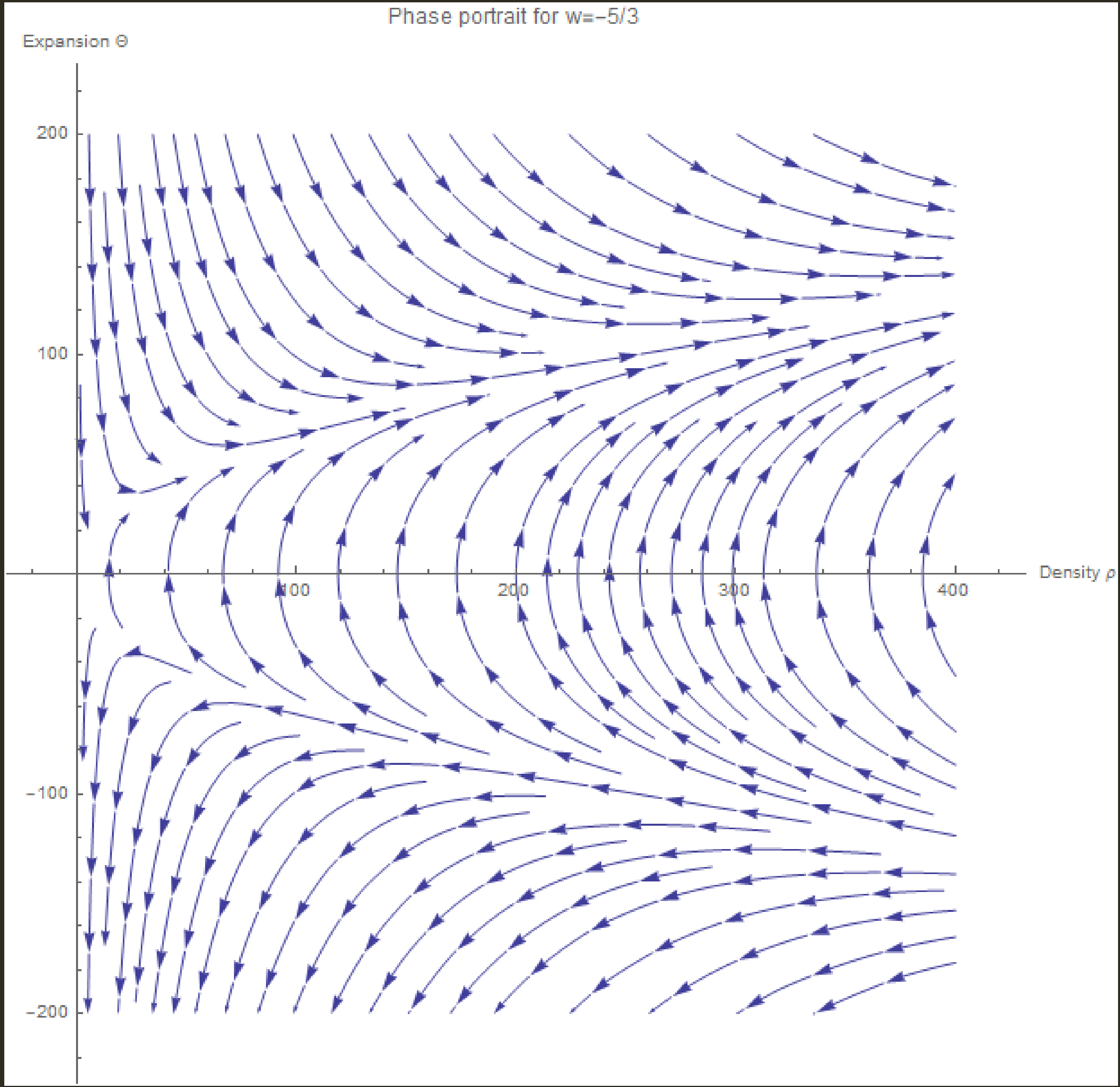
$$\Sigma \equiv \frac{\Theta}{3}$$

Physical motivation: this is a crude approximation to a non-expanding galaxy within an expanding universe. We simply replace the gradient with the value.



$$\dot{\rho} = -(1+w)\rho\Theta$$

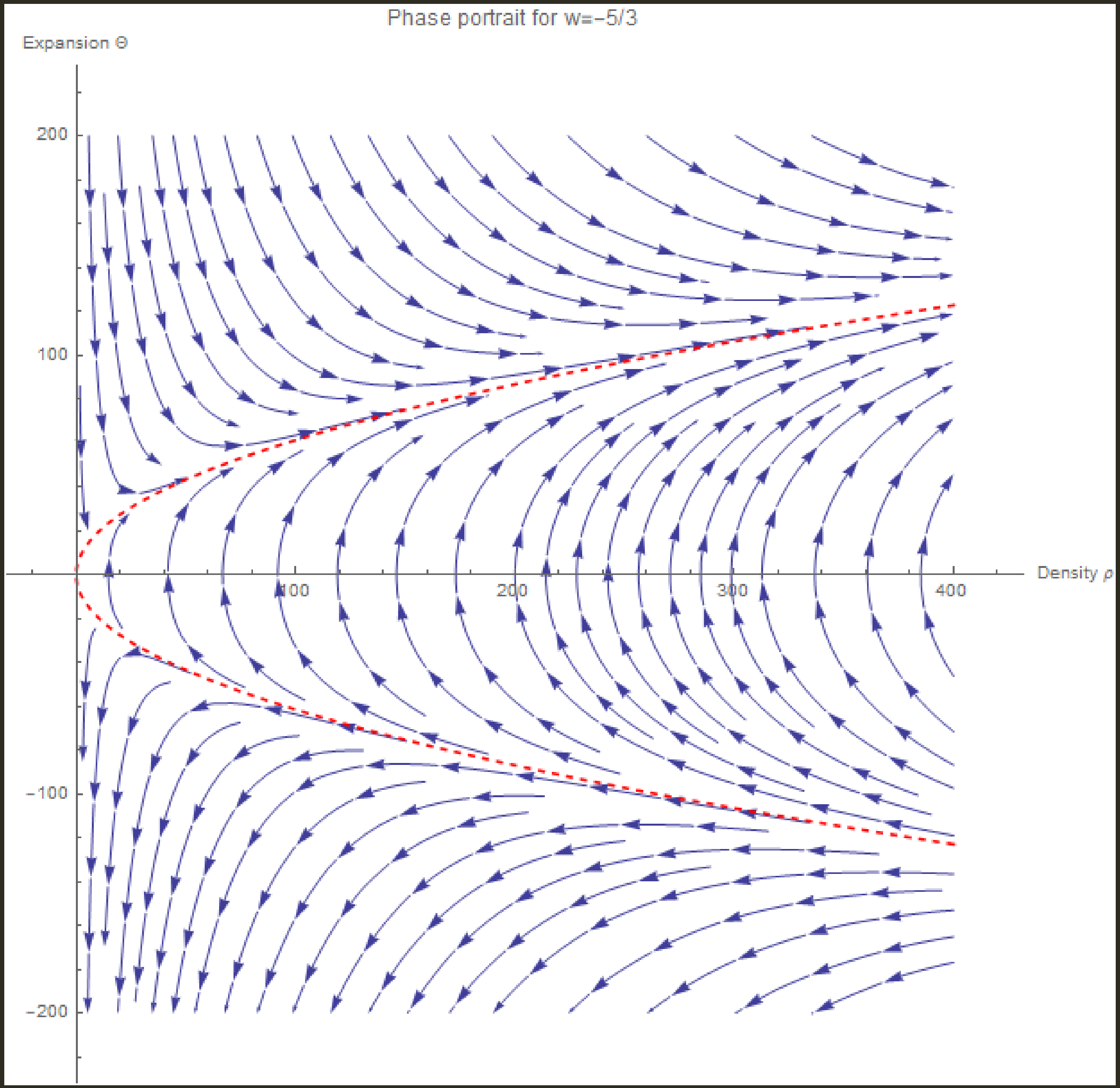
$$\dot{\Theta} = -\Theta^2 - (1+3w)4\pi\rho$$



Again, guess a parabola trajectory:

$$\rho = k\Theta^2$$

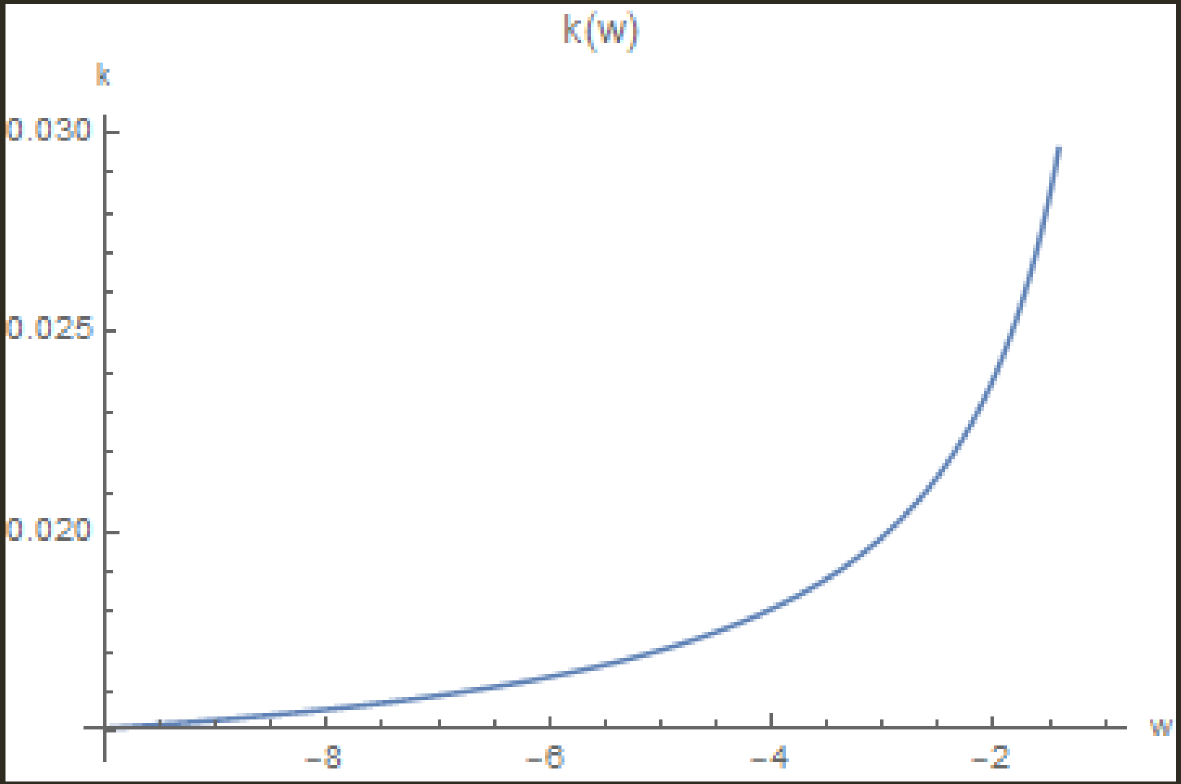
$$\dot{\rho} = 2k\Theta\dot{\Theta}$$



$$\rho = k\Theta^2$$

$$k = \frac{w-1}{8\pi(1+3w)}$$

BONUS: K(W)



SUMMARY FOR: SHEAR = EXPANSION/3

Qualitatively the same as zero shear case. Just *k* is different

Initial conditions	Result
$\rho = \Theta = 0$	static empty universe
$\rho = 0,\ \Theta > 0$	slowing expansion
$\Theta > -\sqrt{\rho/k}$	Big Rip
$\Theta = -\sqrt{\rho/k}$	asymptotic collapse
$\Theta < -\sqrt{\rho/k}$	Big Crunch

SHEAR = DENSITY

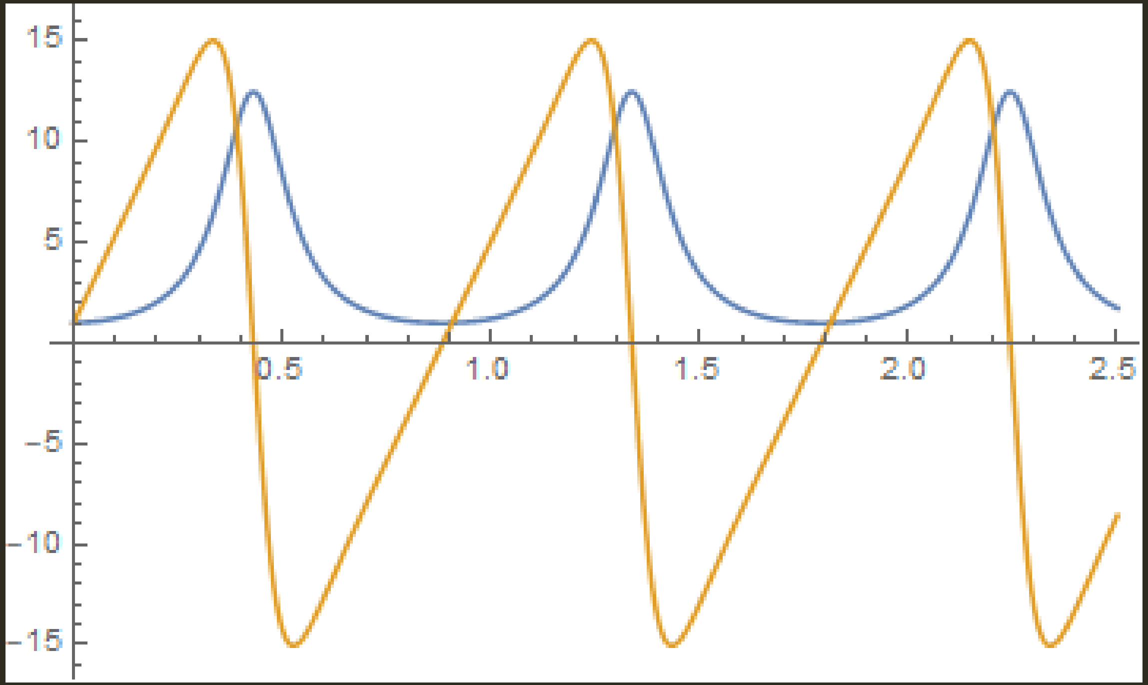
$$\Sigma \equiv \rho$$

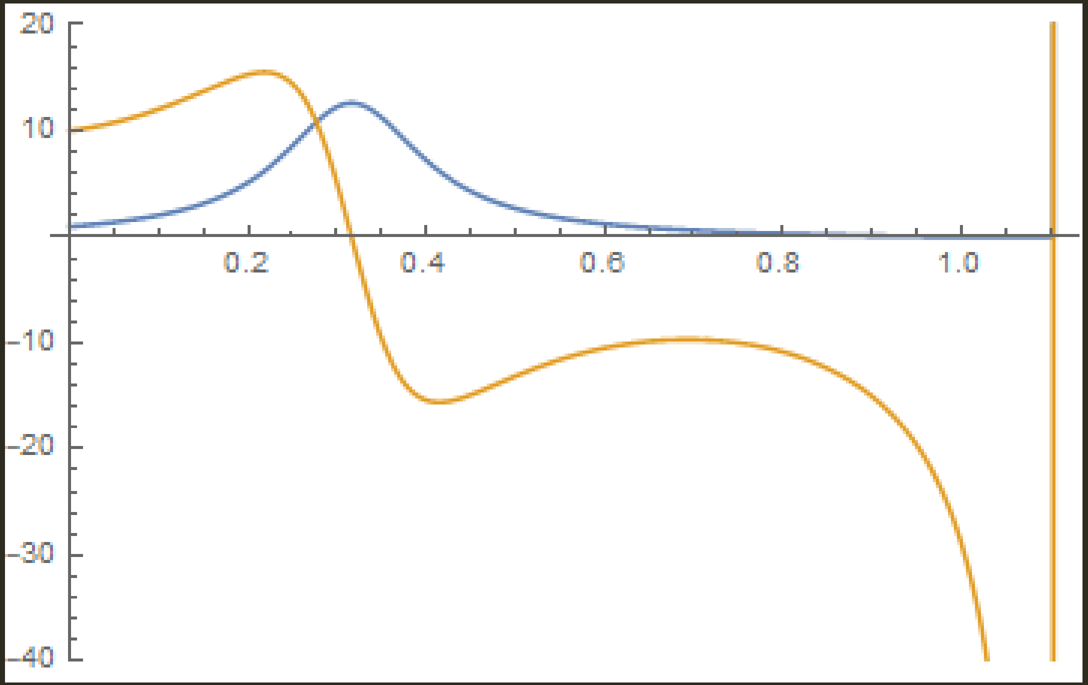
Motivation: arbitrary choice!

$$\dot{\rho} = -(1+w)\rho\Theta$$

$$\dot{\Theta} = -\frac{\Theta^2}{3} - (1+3w)4\pi\rho - 6\rho^2$$

NUMERICAL INTEGRATIONS





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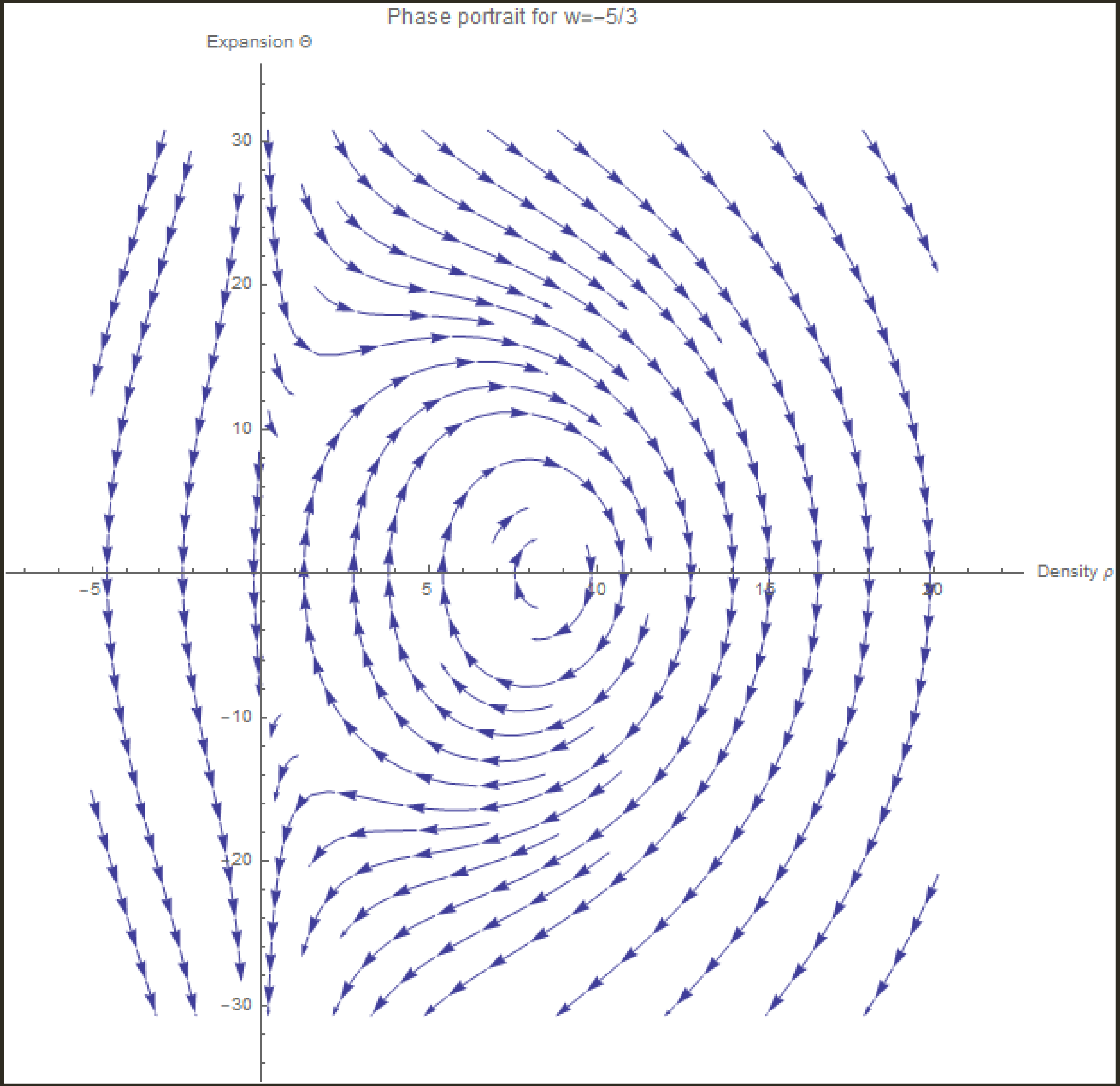
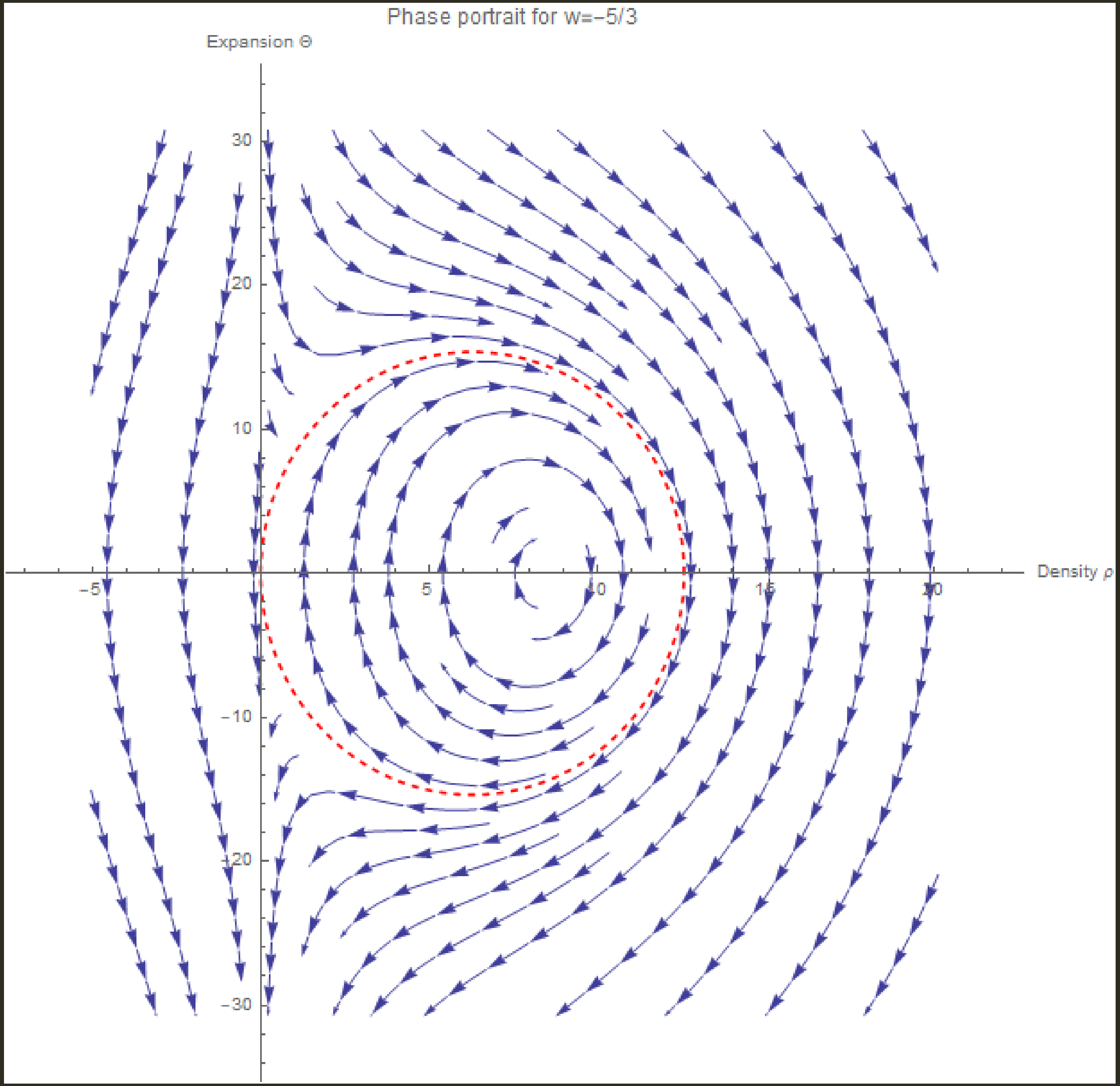


Guess ellipse trajectories:

$$\frac{(\rho - c)^2}{a^2} + \frac{\Theta^2}{b^2} = 1$$

$$\frac{2(\rho - c)\dot{\rho}}{a^2} + \frac{2\Theta\dot{\Theta}}{b^2} = 0$$



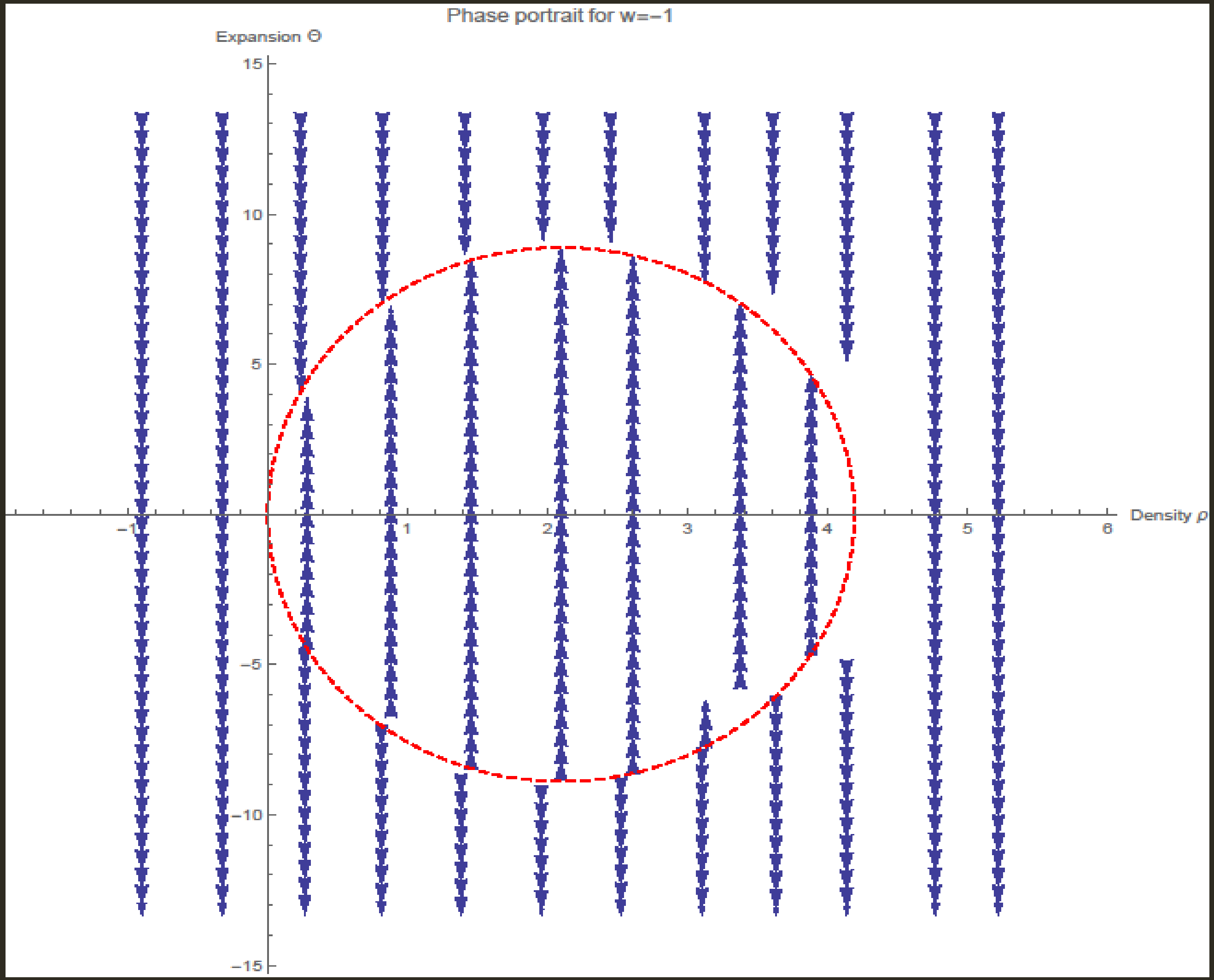
There is only one elliptical trajectory:

$$\Theta^2 = 6\rho\left(4\pi + \frac{3\rho}{2+3w}\right)$$

SUMMARY FOR SHEAR = DENSITY

Initial conditions	Result
$\rho = \Theta = 0$	static empty universe
$\rho = 0,\ \Theta > 0$	slowing expansion
$\rho = -(1+3w)\frac{2\pi}{3},\ \Theta = 0$	static
$\Theta^2 < 6\rho\left(4\pi + \frac{3\rho}{2+3w}\right)$	cyclic
$\Theta^2 = 6\rho\left(4\pi + \frac{3\rho}{2+3w}\right)$	(expansion, then) asymptotic collapse
$\Theta^2 > 6\rho\left(4\pi + \frac{3\rho}{2+3w}\right)$	Big Crunch

BONUS: COSMOLOGICAL CONSTANT W=-1



COMPARISON

This shows the effect of shear.

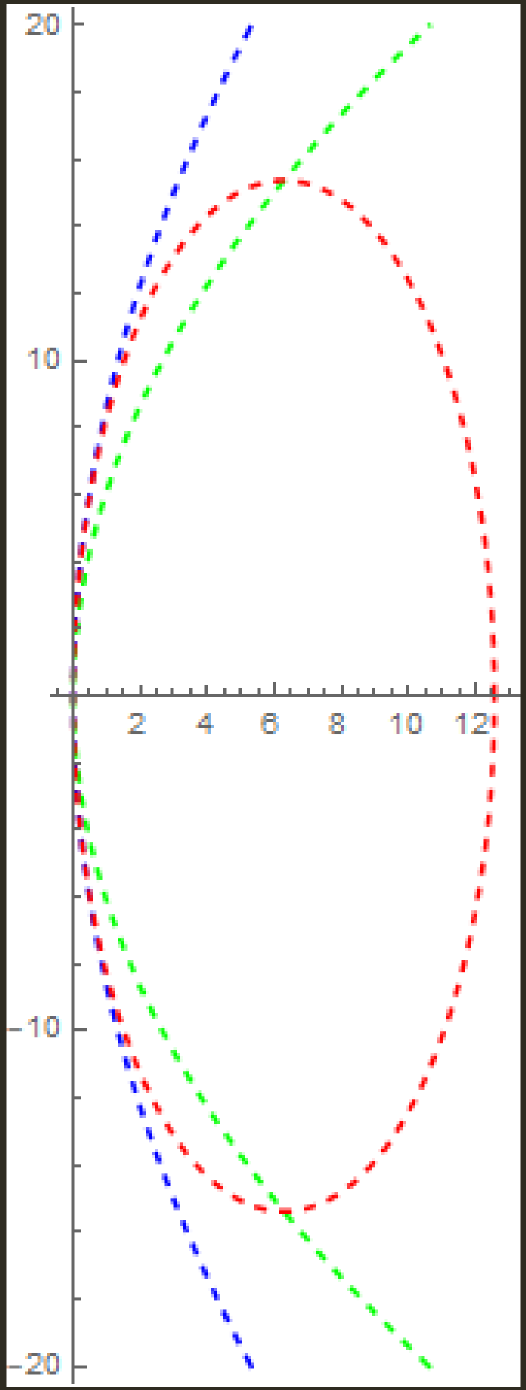
Blue: Shear = 0

Green: Shear = Expansion/3

Red: Shear = Density

Less initial conditions result in a Big Rip.

That is, for some initial conditions, the Big Rip is prevented!



CONCLUSIONS AND IMPLICATIONS

Shear can stop the Big Rip!

Backreaction effects from inhomogeneous cosmology can be strong (of order infinity… ☺)

Paper in preparation: MacLaurin & Bolejko.