Evolution of linear perturbations in (Λ)LTB models and light propagation

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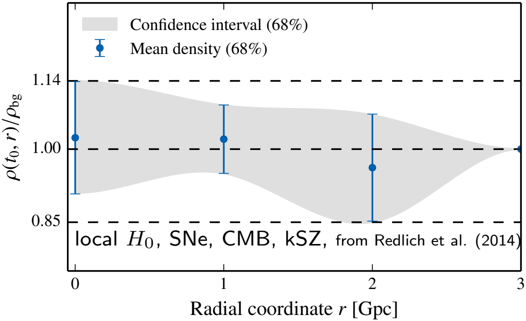
Outline

- ΛLTB: exact radially inhomogeneous solution to GR
- Λ + radial inhom. $\rightarrow$ ΛLTB
- constrain deviations from homogeneity $\Rightarrow$ test the Copernican Principle



investigate linear structure growth in (Λ)LTB models

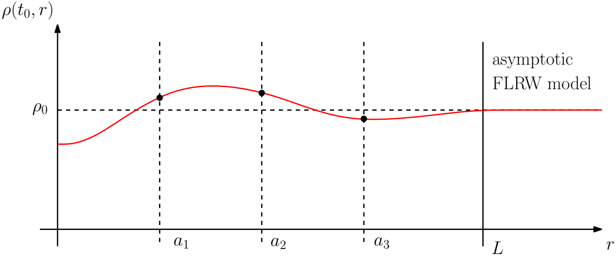
Provide **more observables** to

1. constrain ΛLTB models as best as possible
2. confirm and strengthen findings on spherical voids on a broad scientific basis

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{a_\parallel^2(t,r)}{1-\kappa(r)r^2}\mathrm{d}r^2 + r^2 a_\perp^2(t,r)\mathrm{d}\Omega^2$$

assumptions

- **homogeneous Big Bang** $t_B(r) = 0$, fixes global $t_0[h, \Omega_m, \Omega_\Lambda]$
- **radial gauge**: $a_\perp(t_0, r) = 1$, fixes $M(r)$ using $\rho(t_0, r)$



model parameters

density profile:
a_1, a_2, a_3, L
asymptotic FLRW:
h, Ω_m, Ω_Λ

2+2 split: $\mathcal{M}^4 = \mathcal{M}^2 \times \mathcal{S}^2$

expansion into **spherical harmonics** $Y^{(\ell m)}(\theta, \phi)$ and cov. derivatives *(Clarkson et al. (2010), Gundlach & Martín-García (2000), Gerlach & Sengupta (1978))*

$$\phi = \sum_{(\ell m)} \phi^{(\ell m)} Y^{(\ell m)}$$

$$\phi_a = \sum_{(\ell m)} \phi^{(\ell m)} Y_a^{(\ell m)} + \bar{\phi}^{(\ell m)} \bar{Y}_a^{(\ell m)}$$

$$\phi_{ab} = \sum_{(\ell m)} \phi^{(\ell m)} Y_{ab}^{(\ell m)} + \bar{\phi}^{(\ell m)} \bar{Y}_{ab}^{(\ell m)}$$

$\Rightarrow$ Scalar-Vector-Tensor expressions on $\mathcal{S}^2$

Sets of **gauge invariant quantities** can be found

different from FLRW gauge invariant quantities $\{\Psi, \Phi, V_i, h_{ij}\}$!

metric perturbations - Clarkson et al. (2010)

Abstract set of metric perturbations: $\left\{\chi^{(\ell m)}, \varphi^{(\ell m)}, \varsigma^{(\ell m)}, \eta^{(\ell m)}\right\}$

$$
\begin{aligned}
\ddot{\chi} - \frac{\chi'' + C\chi'}{Z^2} + 3H_{\|}\dot{\chi} - \left[A - \frac{(\ell-1)(\ell+2)}{r^2 a_\perp^2}\right]\chi &= \mathcal{S}_\chi(\varsigma, \varsigma', \varphi, \dot{\varphi}) \\
\ddot{\varphi} + 4H_\perp\dot{\varphi} - \left(\frac{2\kappa}{a_\perp^2} - \Lambda\right)\varphi &= \mathcal{S}_\varphi(\varsigma, \chi, \dot{\chi}, \chi', \ell) \\
\dot{\varsigma} + 2H_{\|}\varsigma &= \mathcal{S}_\varsigma(\chi') \\
\eta &= 0
\end{aligned}
$$

Similar for matter perturbations $\left\{\Delta^{(\ell m)}, w^{(\ell m)}, v^{(\ell m)}\right\}$

Dynamical coupling of gauge invariants !

main differences compared to FLRW:

- **dynamical coupling**
- ΛLTB gauge invariants $\neq$ FLRW gauge invariants

Numerical solution

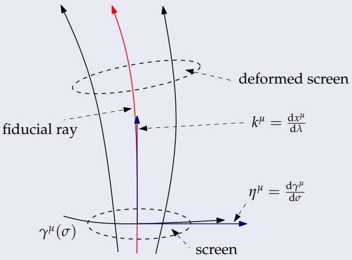
- initial scalar potential
  $P_{\mathcal{R}}(k) \rightarrow \Psi^{(\ell m)} = -\varphi^{(\ell m)}/2$
- discretization:

  1d finite elements in space
  method of lines

Dune
Distributed and Unified Numerics Environment

- estimate coupling strength

extract observables



$$\frac{\mathrm{d}^2 D_{ab}}{\mathrm{d}\lambda^2} = \mathcal{T}_{ac}[\eta, \varphi, \varsigma, \chi] D_{cb}$$

Numerical Solution - Overview

background model

- $\rho(t_0, r)$: L, a_1, a_2, ...
- asymptotic FLRW model: h, Ω_m, Ω_Λ

evolve backwards to initial time/redshift ($z_{\text{ini}} \sim 100$)

- **initial perturbations** (FLRW limit)
- $\varphi^{(\ell m)} = -2\Psi^{(\ell m)}$
- $\chi^{(\ell m)} = 0 = \varsigma^{(\ell m)}$

- Evolve linear PDEs for each (ℓ, m)-mode forward in time

$$C^\ell(z) = \sum_{m=-\ell}^{\ell} \frac{\left|a^{(\ell m)}(t(z), r(z))\right|^2}{2\ell + 1}$$

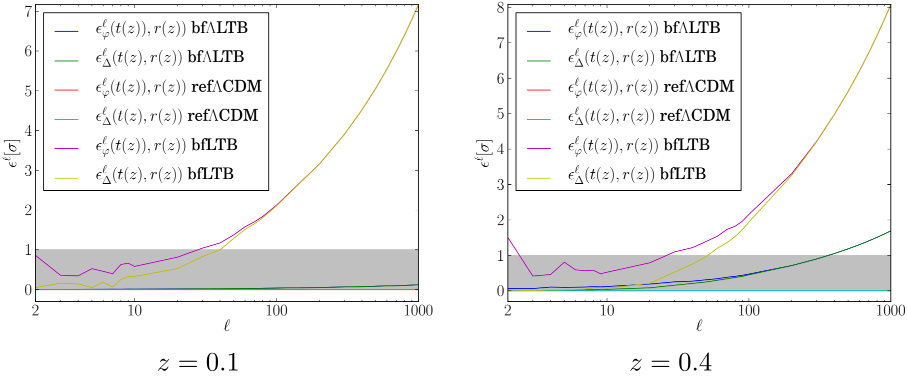
models from Redlich et al. (2014):

- best fit ΛLTB (**bf$\boldsymbol{\Lambda}$LTB**): local H_0, SNae, CMB, kSZ
- best fit LTB (**bfLTB**): local H_0, SNae, asymptotically EdS
- reference ΛCDM (**ref$\boldsymbol{\Lambda}$CDM**)

model	h	Ω_m	Ω_Λ	a_1	a_2	a_3
bfΛLTB	0.73	0.245	0.745	1.02	1.02	0.96
bfLTB	0.557	1.0	0.0	0.23	0.44	0.59
refΛCDM	0.73	0.245	0.745	1.0	1.0	1.0

coupling strength

$$\epsilon_X^\ell(z) = \frac{2}{2\ell+1}\frac{\left|C_X^\ell(z) - C_{X,\mathrm{uc}}^\ell(z)\right|}{C_{X,\mathrm{uc}}^\ell(z)}, \quad X = \varphi,\, \Delta$$



coupling negligibly small in "realistic" ΛLTB models!

Jacobi equation: $$\frac{\mathrm{d}^2 D_{ab}}{\mathrm{d}\lambda^2} = \mathcal{T}_{ac}[\eta, \varphi, \varsigma, \chi] D_{cb}$$

assumptions:

- **central observer** in the background ΛLTB spacetime
- **Born's approximation**: influences of perturbations integrated along the unperturbed lightpath k^μ

$$D_{ab} = (r a_\perp)(\lambda)\gamma_{ab} + D^{(1)}_{ab}(\lambda, \theta, \phi), \qquad \mathcal{T}_{ab} = \mathcal{T}^{(0)}_{ab}(\lambda) + \mathcal{T}^{(1)}_{ab}(\lambda, \theta, \phi)$$

Spherical harmonic decomposition:

$$\frac{\mathrm{d}^2 D^{X(\ell m)}}{\mathrm{d}\lambda^2} = -4\pi G\rho(1+z)^2 D^{X(\ell m)} + \mathcal{T}^{X(\ell m)}[\chi, \varphi, \varsigma, \eta]\; r a_\perp$$

$X =$ trace, trace-free

$$D^{X(\ell m)}(\lambda) = \int_0^\lambda \mathrm{d}\lambda' \, G(\lambda, \lambda') \, \mathcal{T}^{X(\ell m)}[\chi, \varphi, \varsigma, \eta](\lambda') \, ra_\perp(\lambda'), \quad X = T, TF$$

Greens function:

$$G(\lambda, \lambda') = ra_\perp(\lambda) ra_\perp(\lambda') \int_{\lambda'}^{\lambda} \frac{\mathrm{d}\lambda''}{(ra_\perp)^2(\lambda'')}$$

Weak lensing observables:

$$\kappa^{(\ell m)}(\lambda) = D^{T(\ell m)}(\lambda)/ra_\perp(\lambda), \quad \gamma^{(\ell m)}(\lambda) = D^{TF(\ell m)}(\lambda)/ra_\perp(\lambda)$$

ΛLTB: coupling small $\varphi^{(\ell m)}(t,r) = D_\varphi(t,r)\varphi^{(\ell m)}_{\rm ini}(r)$

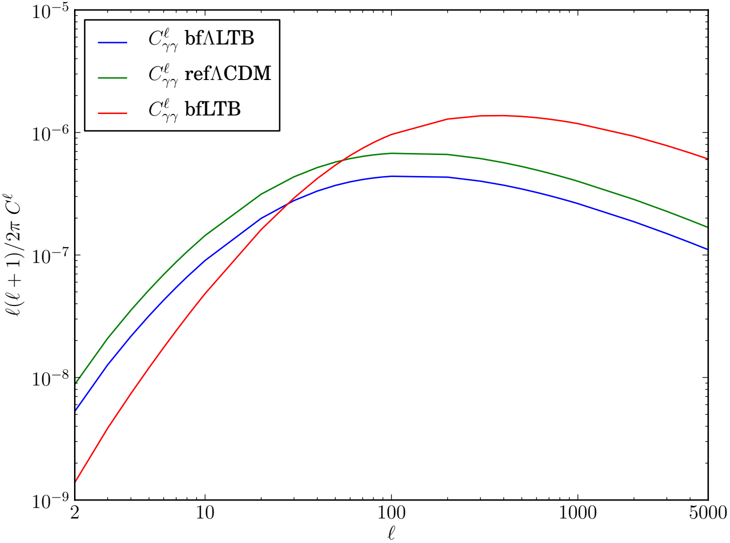
$$C^\ell_{\tilde X \tilde Z}\delta_{\ell\ell'}\delta_{mm'} = \int_0^\lambda \mathrm{d}x\ (ra_\perp)(x)\int_0^{\lambda'}\mathrm{d}x'\ (ra_\perp)(x')\int_x^\lambda \frac{\mathrm{d}y}{(ra_\perp)^2(y)} \int_{x'}^{\lambda'}\frac{\mathrm{d}y'}{(ra_\perp)^2(y')}\langle F^{X(\ell m)}(x)F^{Z(\ell' m')}(x')^*\rangle$$

with $\tilde X, \tilde Z = \kappa, \gamma$ and $X, Z = T,\ TF$.

Separable time evolution using field equations:

$$\langle F^{X(\ell m)}(\lambda)F^{Z(\ell' m')}(\lambda')^*\rangle = \sum_{p,q}\mathcal{L}^p_X(\lambda)\mathcal{L}^q_Y(\lambda')\langle\varphi^{(p)}_{\rm ini}(r(\lambda))\varphi^{(q)}_{\rm ini}(r(\lambda'))^*\rangle$$

where $p, q = 0, 1, 2$ and $\varphi^{(p)} = \partial^p_r\varphi$


$C^{\ell}_{\gamma\gamma}$ bfΛLTB
$C^{\ell}_{\gamma\gamma}$ refΛCDM
$C^{\ell}_{\gamma\gamma}$ bfLTB
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$\ell(\ell+1)/2\pi\ C^{\ell}$
2
10
100
1000
5000
ℓ

$z = 0.3$

Conclusion

- dynamical coupling is **negligible** for relevant ΛLTB models
- **Observables**: ΛLTB weak lensing analogous to FLRW in case of central observer
- κ, γ as line of sight integrals over gauge invariant perturbations

work in progress/future

- theoretical powerspectra for negligible/small couplings
- extend to ISW effect
- comparison to observationally inferred spectra
- full MCMC analysis for ΛLTB models including consistent linear perturbation theory