Scalar-averaged N-body simulations

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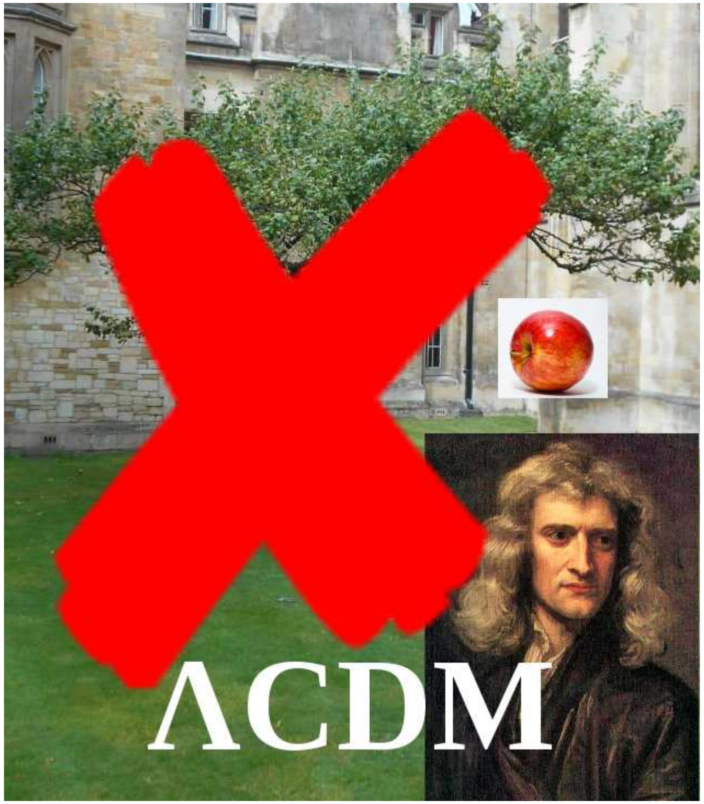
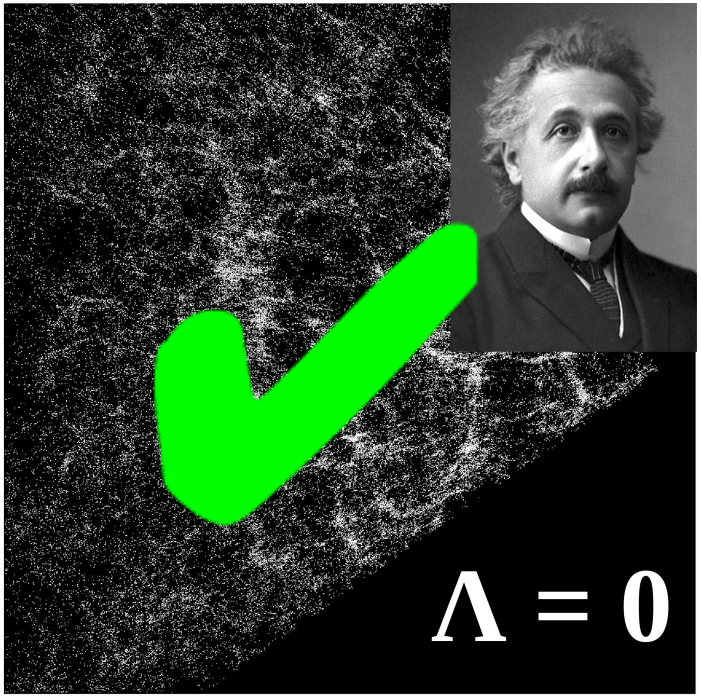
Newton vs Einstein

Universe = space–time





?

FLRW ($\ni \Lambda$CDM) models

- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T} + \Lambda\mathbf{g}$
- stress–energy tensor:

  "dust:" $p := 0$, $\mathbf{T} = \rho\,\mathbf{u} \otimes \mathbf{u}$
- homogeneity assumption of FLRW

  $\Rightarrow$ non-linear structure grav. collapse+virialisation process is relativistically ignored

scalar averaging: Raychaudhuri eq

averaged Raychaudhuri eq [RZA2, PRD arXiv:1303.6193, (9)]:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3}\frac{M_{\mathcal{D}_{\mathbf{i}}}\, a^3_{\mathcal{D}_{\mathbf{i}}}}{V_{\mathcal{D}_{\mathbf{i}}}\, a^3_{\mathcal{D}}} + \frac{\mathcal{Q}_{\mathcal{D}}}{3} + \Lambda\,,$$

remove a free parameter by setting $\Lambda := 0$

scalar averaging: Raychaudhuri eq

averaged Raychaudhuri eq [RZA2, PRD arXiv:1303.6193, (9)]:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3} \frac{M_{\mathcal{D}_{\mathbf{i}}}\, a^3_{\mathcal{D}_{\mathbf{i}}}}{V_{\mathcal{D}_{\mathbf{i}}}\, a^3_{\mathcal{D}}} + \frac{\mathcal{Q}_{\mathcal{D}}}{3} \,,$$

where kinematical backreaction [Newtonian case: BKS, arXiv:astro-ph/9912347, II.B., (5)]

$$\mathcal{Q}_{\mathcal{D}} := 2\, \langle \mathrm{II} \rangle_{\mathcal{D}} - \frac{2}{3} {\langle \mathrm{I} \rangle_{\mathcal{D}}}^2 \,,$$

scalar averaging: Raychaudhuri eq

with invariants of the peculiar expansion tensor [Newtonian case: Buchert 94, MNRAS arXiv:astro-ph/9309055]:

$$
\begin{aligned}
\mathrm{I}(v^i_{,j}) &:= \mathrm{tr}(v^i_{,j}) = v^i_{,i} = \nabla\cdot\mathbf{v} \\
\mathrm{II}(v^i_{,j}) &:= \tfrac{1}{2}\left\{\left[\mathrm{tr}\left(v^i_{,j}\right)\right]^2 - \mathrm{tr}\left[\left(v^i_{,j}\right)^2\right]\right\} \\
&= \tfrac{1}{2}\left((v^i_{,i})^2 - v^i_{,j}\, v^j_{,i}\right) \\
&= \tfrac{1}{2}\nabla\cdot\Big(\mathbf{v}(\nabla\cdot\mathbf{v}) - (\mathbf{v}\cdot\nabla)\mathbf{v}\Big) \\
\mathrm{III}(v^i_{,j}) &:= \det(v^i_{,j}).
\end{aligned}
$$

scalar averaging: QZA

$$\mathcal{Q}_{\mathcal{D}} = \frac{\dot{\xi}^2\left(\gamma_1 + \xi\gamma_2 + \xi^2\gamma_3\right)}{\left(1 + \xi\left\langle \mathrm{I}_{\mathbf{i}}\right\rangle_{\mathcal{I}} + \xi^2\left\langle \mathrm{II}_{\mathbf{i}}\right\rangle_{\mathcal{I}} + \xi^3\left\langle \mathrm{III}_{\mathbf{i}}\right\rangle_{\mathcal{I}}\right)^2} \,,$$

where

$$\begin{cases} \gamma_1 := 2\left\langle \mathrm{II}_{\mathbf{i}}\right\rangle_{\mathcal{I}} - \frac{2}{3}\left\langle \mathrm{I}_{\mathbf{i}}\right\rangle_{\mathcal{I}}^2 \\ \gamma_2 := 6\left\langle \mathrm{III}_{\mathbf{i}}\right\rangle_{\mathcal{I}} - \frac{2}{3}\left\langle \mathrm{II}_{\mathbf{i}}\right\rangle_{\mathcal{I}}\left\langle \mathrm{I}_{\mathbf{i}}\right\rangle_{\mathcal{I}} \\ \gamma_3 := 2\left\langle \mathrm{I}_{\mathbf{i}}\right\rangle_{\mathcal{I}}\left\langle \mathrm{III}_{\mathbf{i}}\right\rangle_{\mathcal{I}} - \frac{2}{3}\left\langle \mathrm{II}_{\mathbf{i}}\right\rangle_{\mathcal{I}}^2 \,. \end{cases}$$

QZA $=$ $\mathcal{Q}_{\mathcal{D}}$ Zel'dovich approximation:

- algebraic structure same in Newtonian and GR cases
- initial invariants conceptually differ
- initial invariants numerically approximated for zero curvature

Volume averaging

instead of verbal averaging (FLRW $\ni$ ΛCDM): mathematical averaging

$$a_{\rm eff}(t) := \left(\frac{\sum_{\mathcal{D}} a_{\mathcal{D}}^3(t)}{\sum_{\mathcal{D}} 1} \right)^{1/3} = \left(\frac{\sum_{\mathcal{D}} a_{\mathcal{D}}^3(t)}{n_{\mathcal{D}}} \right)^{1/3}$$

QZA model

- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T} \quad (\Lambda := 0)$
- $a_{\mathcal{D}}$: domain-averaged scale factor and Raychaudhuri equation for $\ddot{a}_{\mathcal{D}}$ ("silent" universe) on a spatial domain $\mathcal{D}$
- non-linear scales but no shell-crossing (gravitational collapse)
- Raychaudhuri + $\mathcal{Q}_{\mathcal{D}}$ analytical approximation (QZA)
- calculate global average scale factor: $a_{\rm eff}(t)$
- *expected result: generate* $a_{\rm eff}(t) \approx a_{\rm EdS}(t)$ *instead of assuming it*

V = virialisation model

Next step: how can we model virialisation?

- Newtonian: virialisation backreaction term: $-(\Pi^{ik}_{,k}/\rho)_{,i}$?

  [Al Roumi 2011; (2.20), (2.22), (2.23)]

- GR: $\mathcal{P}^{\rm T}_{\mathcal{D}}$? [Mourier+2017]

  i.e. add local negative effective pressure to $\mathbf{T}$? **TODO!**

- GR initial approximation: assume standard EdS virialisation overdensity:

$$a_{\mathcal{D}}(t > t_{\rm coll}) := a_{\rm EdS}(t_{\rm coll}) \,/ (18\pi^2)^{1/3}$$

- *Is this GR?* It violates $\mathbf{T} = \rho\, \mathbf{u} \otimes \mathbf{u}$, but not $\mathbf{G} = 8\pi \mathbf{T}$.
- *Is this Newtonian?* $a_{\rm eff}(t)$ will not be $a_{\rm EdS}(t)$.

VQZA model

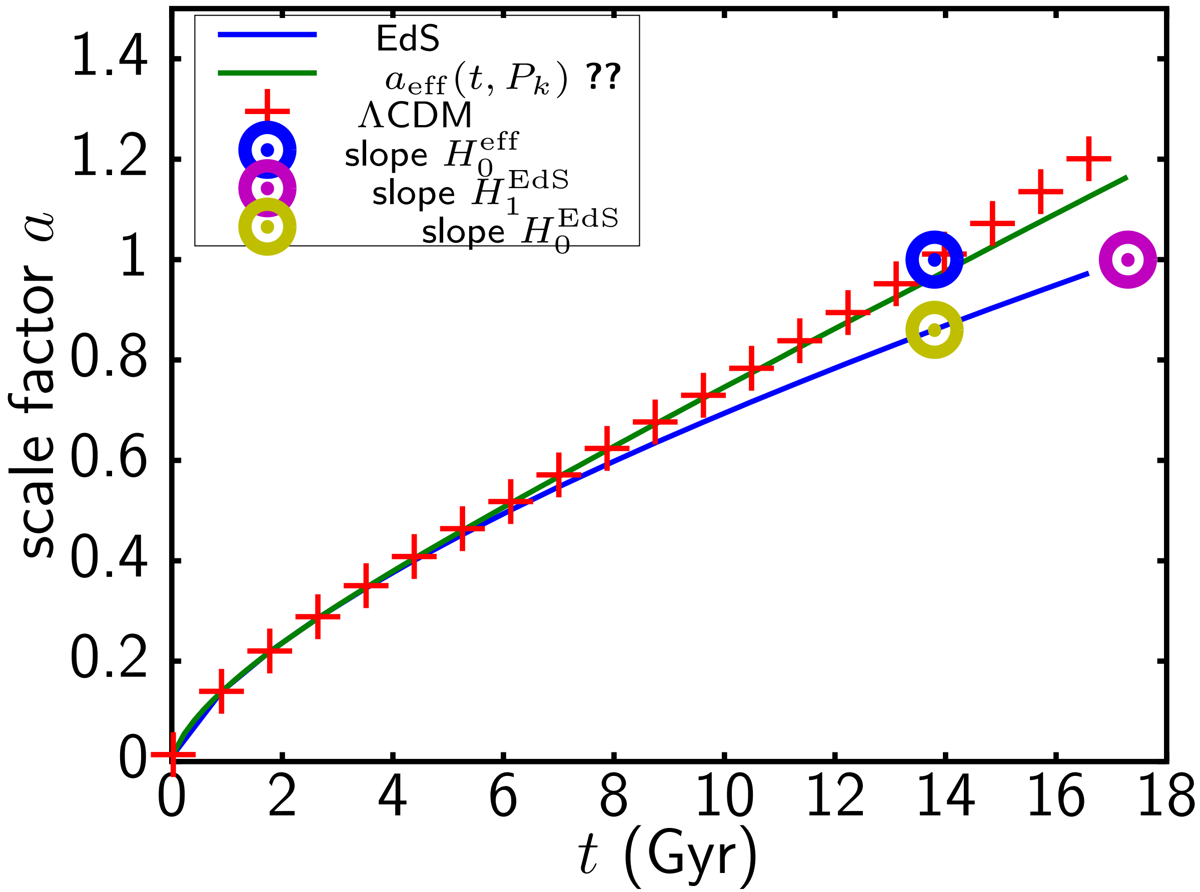
- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T} \quad (\Lambda := 0)$
- $a_{\mathcal{D}}$: domain-averaged scale factor and Raychaudhuri equation for $\ddot{a}_{\mathcal{D}}$ ("silent" universe) on a spatial domain $\mathcal{D}$
- non-collapsed domains: Raychaudhuri + $\mathcal{Q}_{\mathcal{D}}$ analytical approximation (QZA)
- grav. collapsed/virialised domains:
- assume stable clustering (V)

  [implicitly: introduce local negative effective pressure $p_{\mathcal{D}}^{\rm vir}$ into $\mathbf{T}$]

  virialisation statistically halts gravitational collapse
- V+QZA implicitly represents including $p_{\mathcal{D}}^{\rm vir}$ in Einstein eq RHS
- calculate global average scale factor: $a_{\rm eff}(t)$

initial conds: ΛCDM proxy

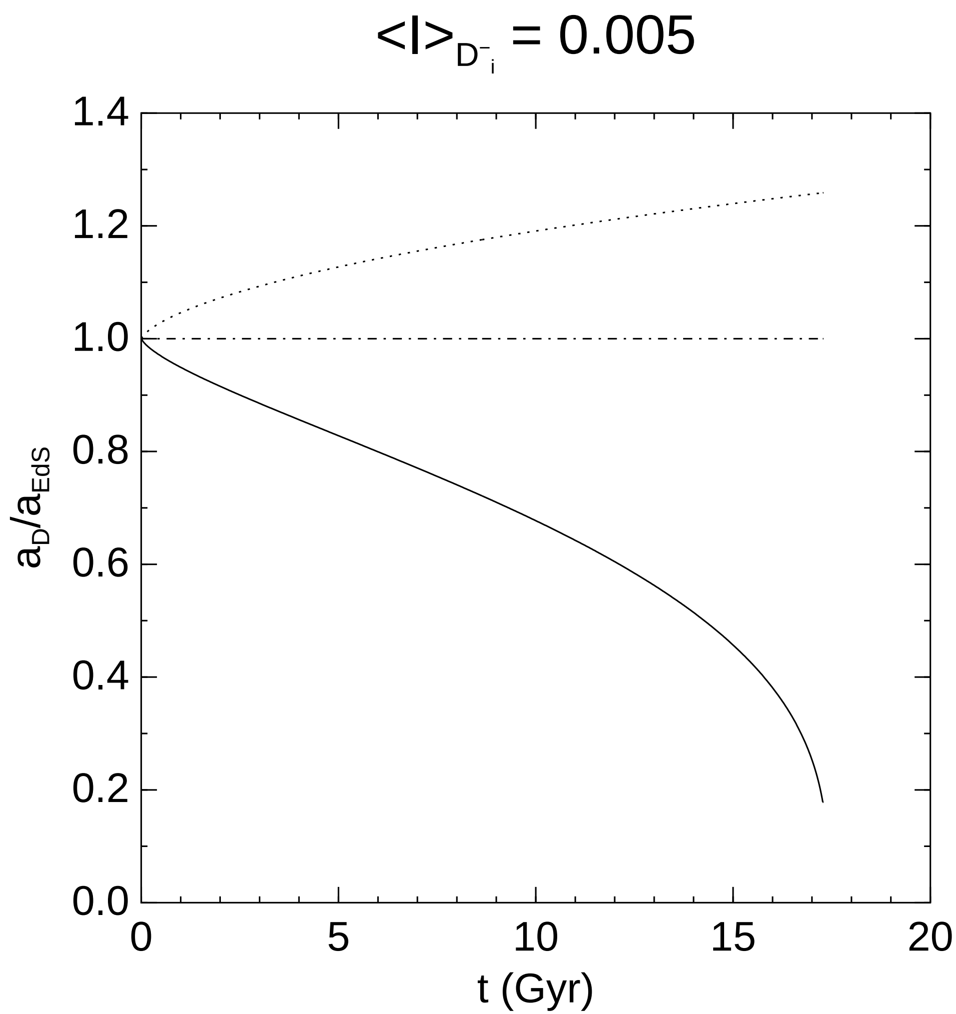
obsvns $\Rightarrow H_0^{\rm eff}, H_1^{\rm EdS}, H_0^{\rm EdS} = 67.74, 37.7, 47.24$ km/s/Mpc
(arXiv:1608.06004 Roukema+2016)



EdS + RZA(P_k) $\Rightarrow$ $\sim \Lambda$CDM ?

RZA = relativistic Zel'dovich approximation (PRD arXiv:1303.6193)
TCfA+CRAL sims N-body + RZA : *work in progress. . .*

Biscale example: QZA

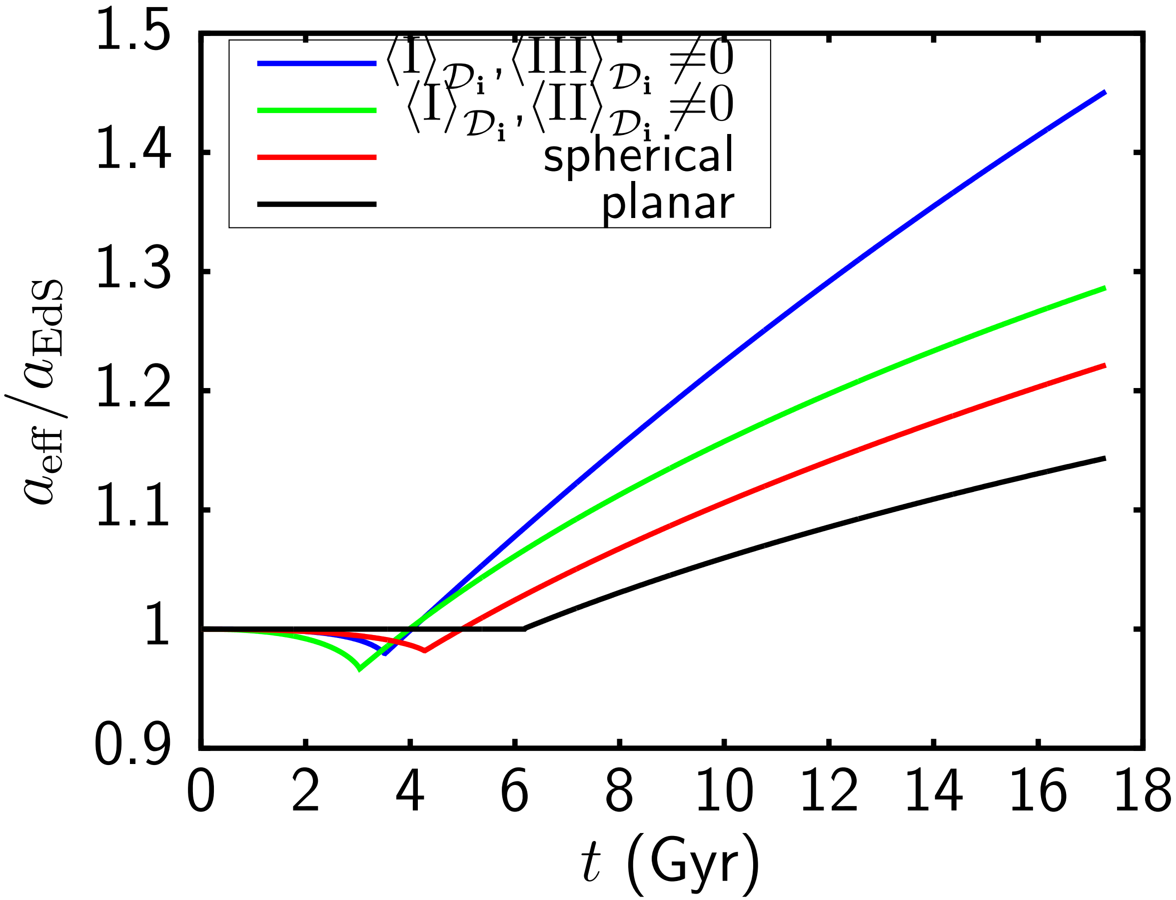


$a_{\mathrm{EdS}\,\mathbf{i}} = 0.005$, expanding domain D^-:

$(\langle \mathrm{I} \rangle_{\mathcal{D}^-_{\mathbf{i}}}, \langle \mathrm{II} \rangle_{\mathcal{D}^-_{\mathbf{i}}}, \langle \mathrm{III} \rangle_{\mathcal{D}^-_{\mathbf{i}}}) = (0.005, 0, 0)$ ("planar" case)

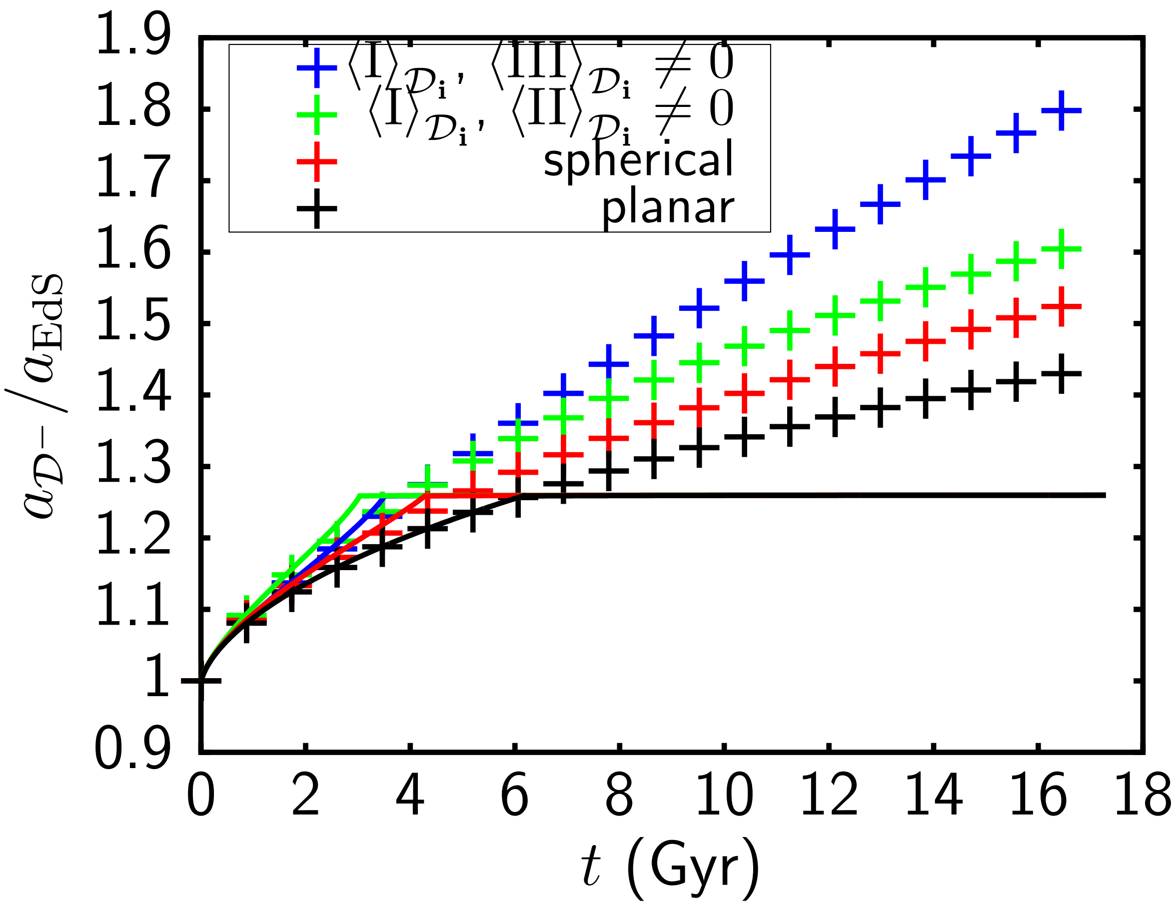
non-linear $a_{\mathcal{D}^-}$, $a_{\mathcal{D}^+}$ *cancel*

Biscale example: global evolution



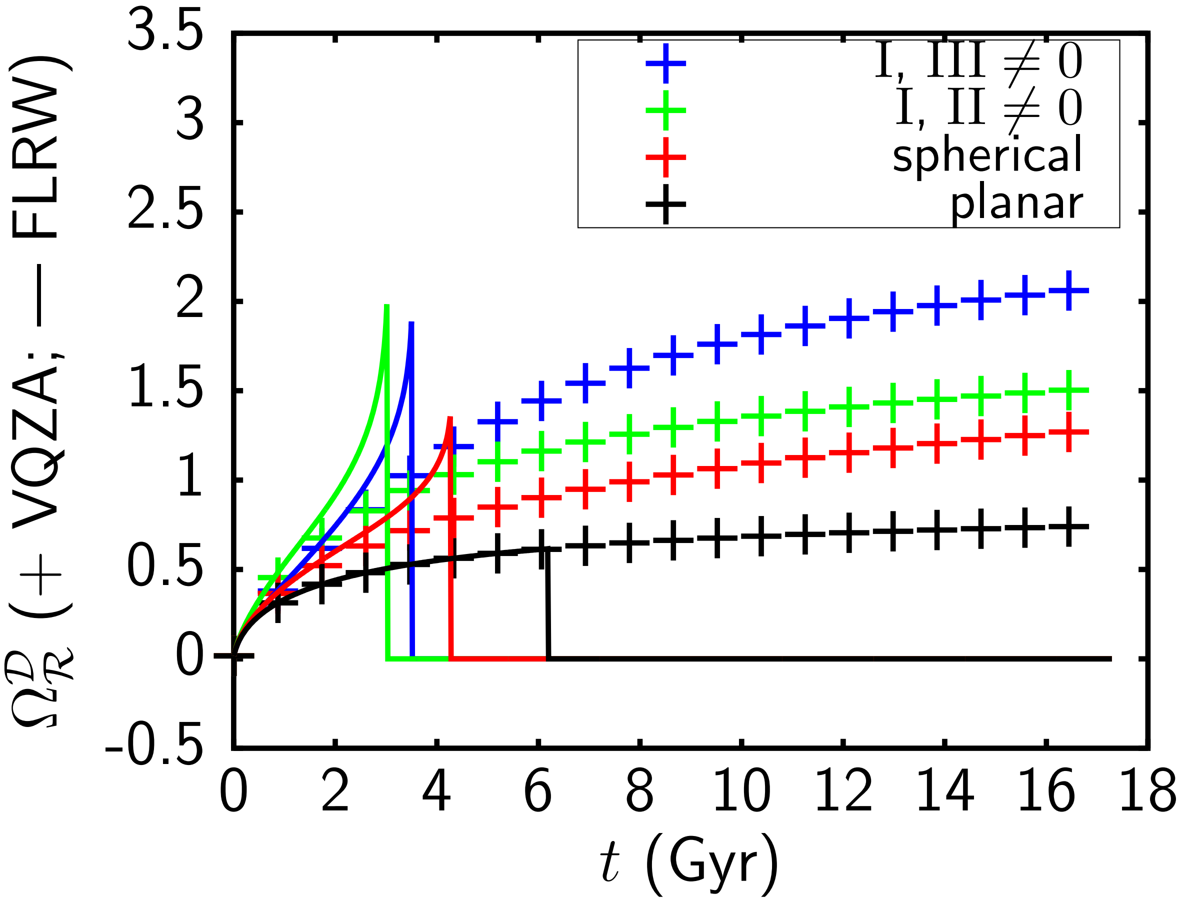
$a_{\mathrm{EdS}\mathbf{i}} = 0.005$, expanding domain D^{-}:
$(\langle \mathrm{I} \rangle_{\mathcal{D}^-_\mathbf{i}}, \langle \mathrm{II} \rangle_{\mathcal{D}^-_\mathbf{i}}, \langle \mathrm{III} \rangle_{\mathcal{D}^-_\mathbf{i}}) = (0.01, 0, 0)$ ("planar" case);
$(0.01, 10^{-4}/3, 10^{-6}/27)$ ("spherical" $\mathcal{D}^-$ case); $(0.01, 10^{-4}, 0)$;
$(0.01, 0, 10^{-6})$;

Biscale example: $a_{\mathcal{D}^-}$



— FLRW; + QZA

Biscale example: $\mathcal{D}^-$ curvature

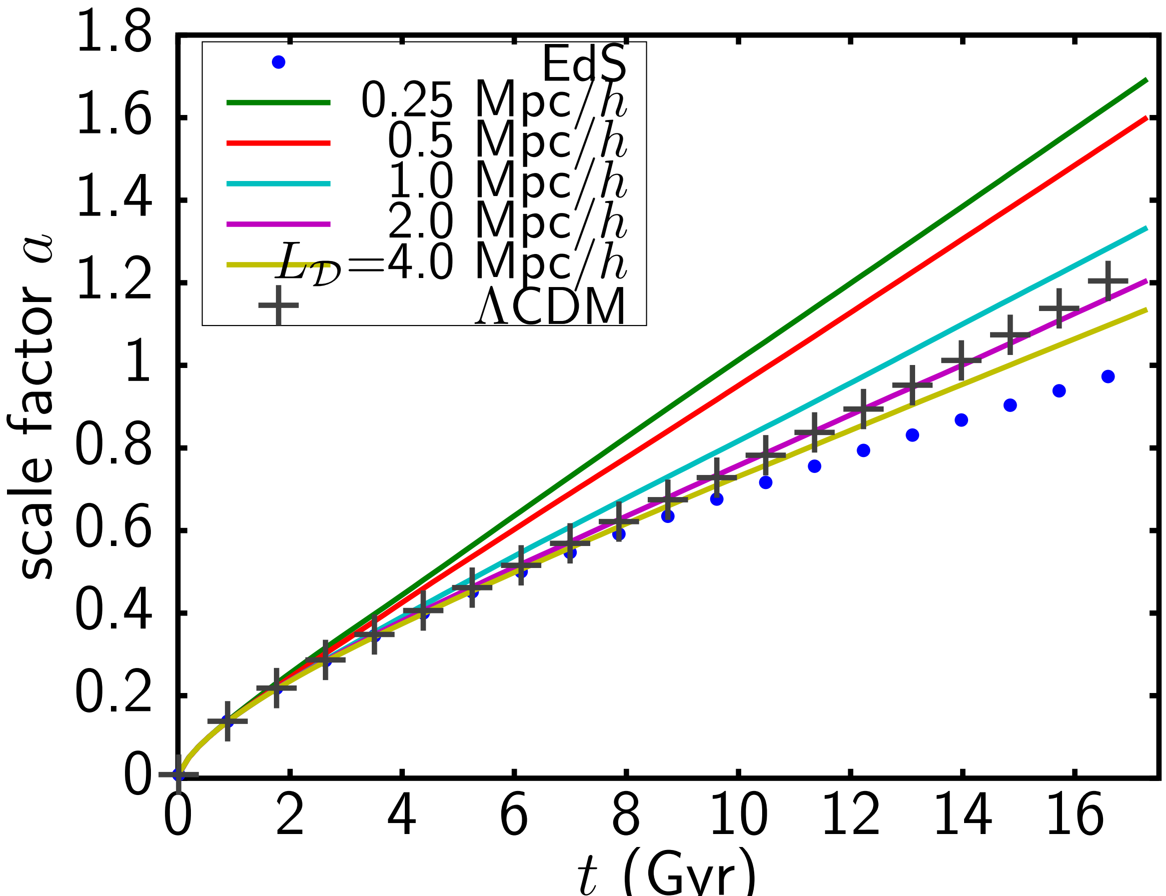


Hamiltonian constraint interpretation

software

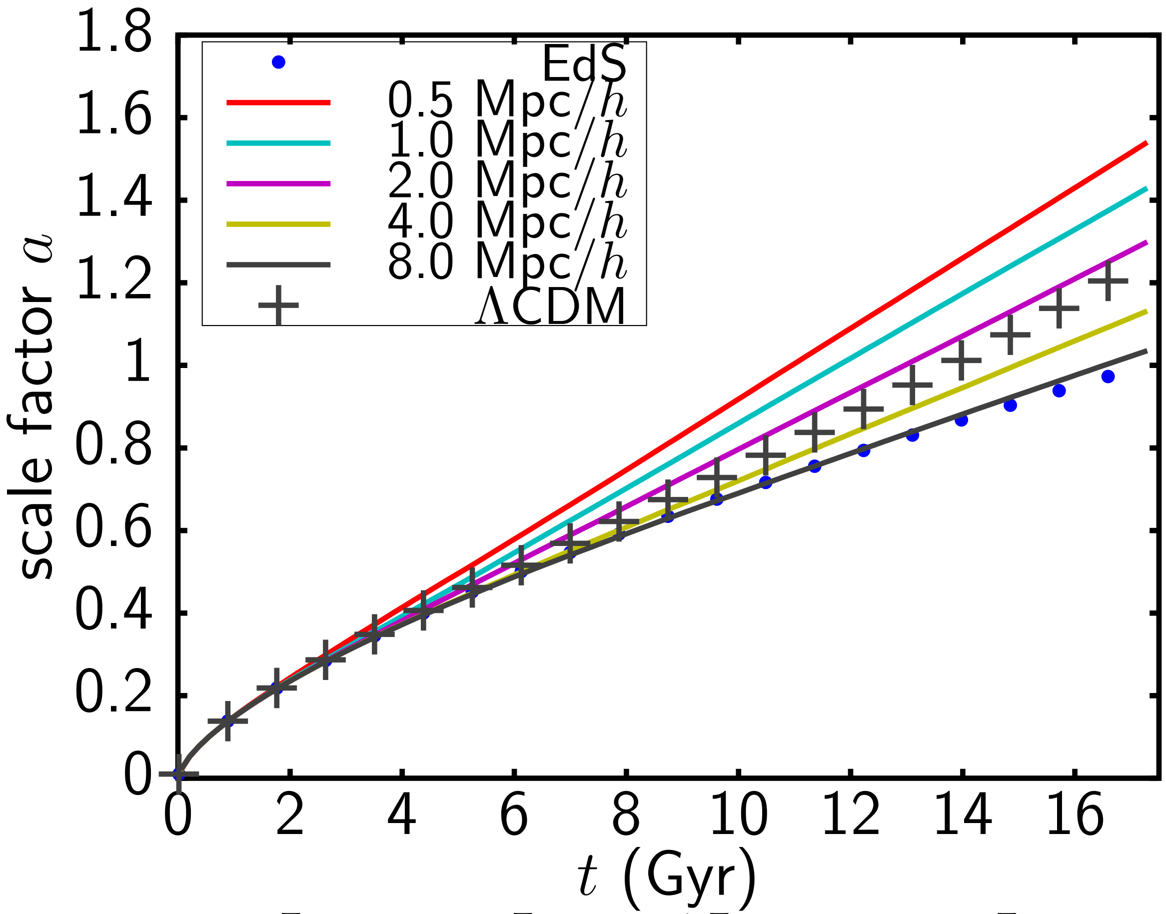
- MPGRAFIC — initial conditions (GPL, f90)
- DTFE — measure I, II, III (GPL, C++)
- INHOMOG — evolve QZA; stabilise virialised domains (GPL, C)
- RAMSES-SCALAV — extension of RAMSES as front end to the above (Cecill, f90)
- start at http://bitbucket.org/broukema/ramses-scalav

VQZA $a_{\mathrm{eff}}(t)$



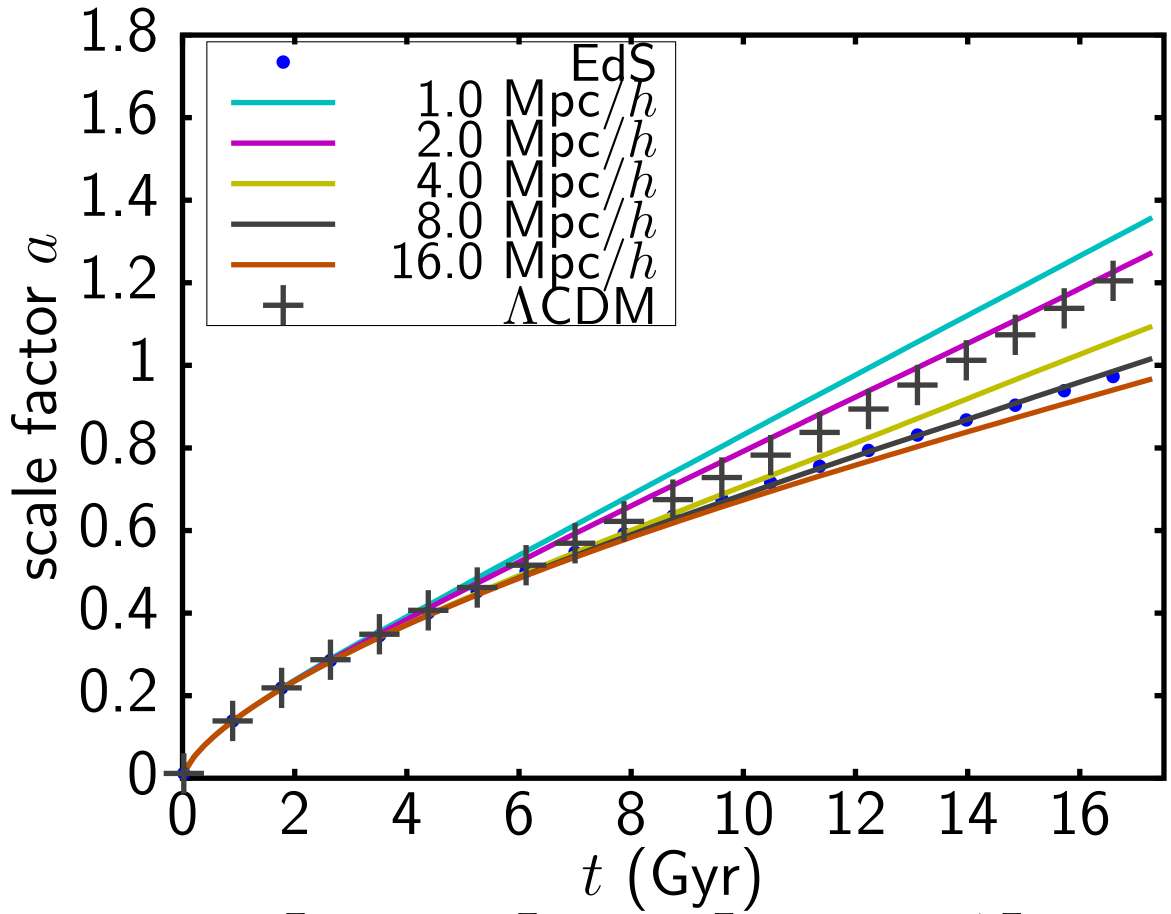
$$L_{\mathrm{box}} = 8L_{\mathcal{D}} = 2L_{\mathrm{DTFE}} = 16L_N$$

VQZA $a_{\mathrm{eff}}(t)$



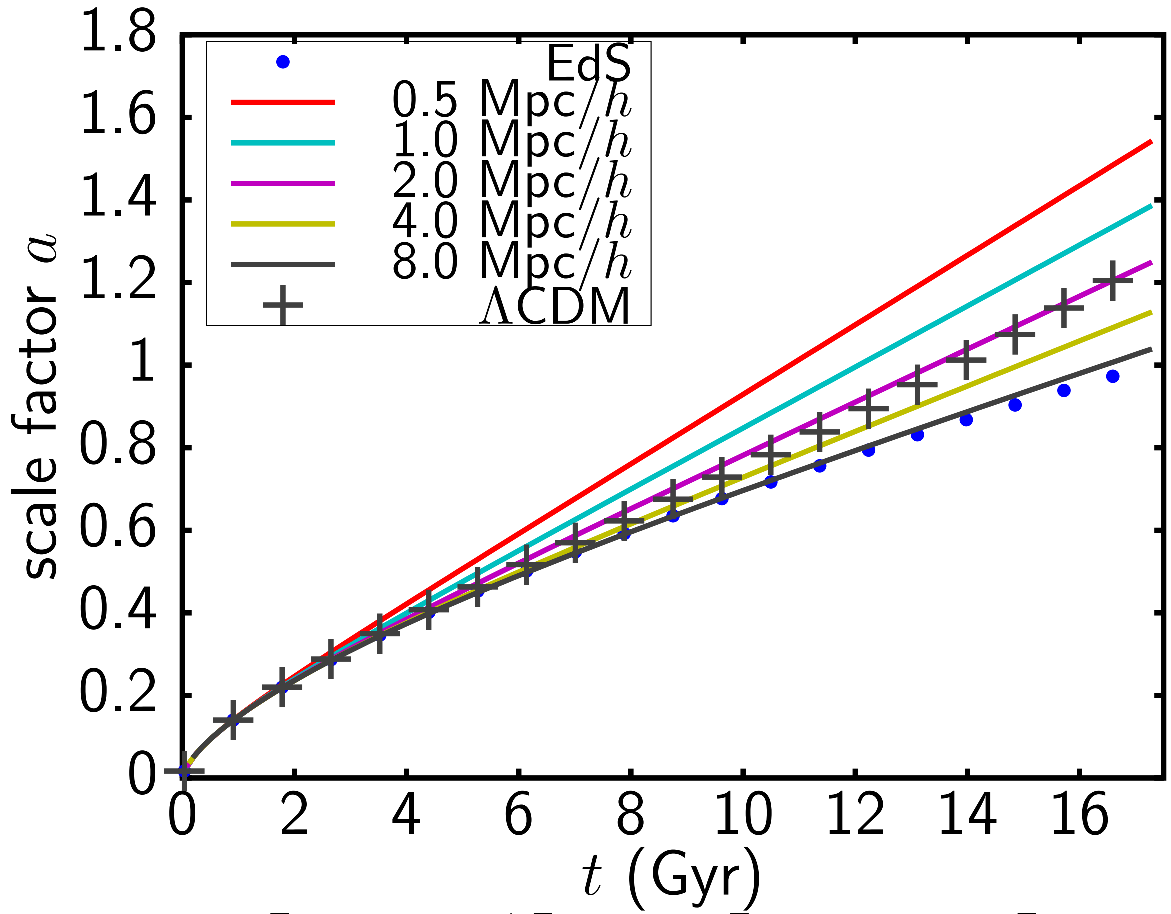
$$L_{\mathrm{box}} = 8L_{\mathcal{D}} = 4L_{\mathrm{DTFE}} = 8L_N$$

VQZA $a_{\mathrm{eff}}(t)$



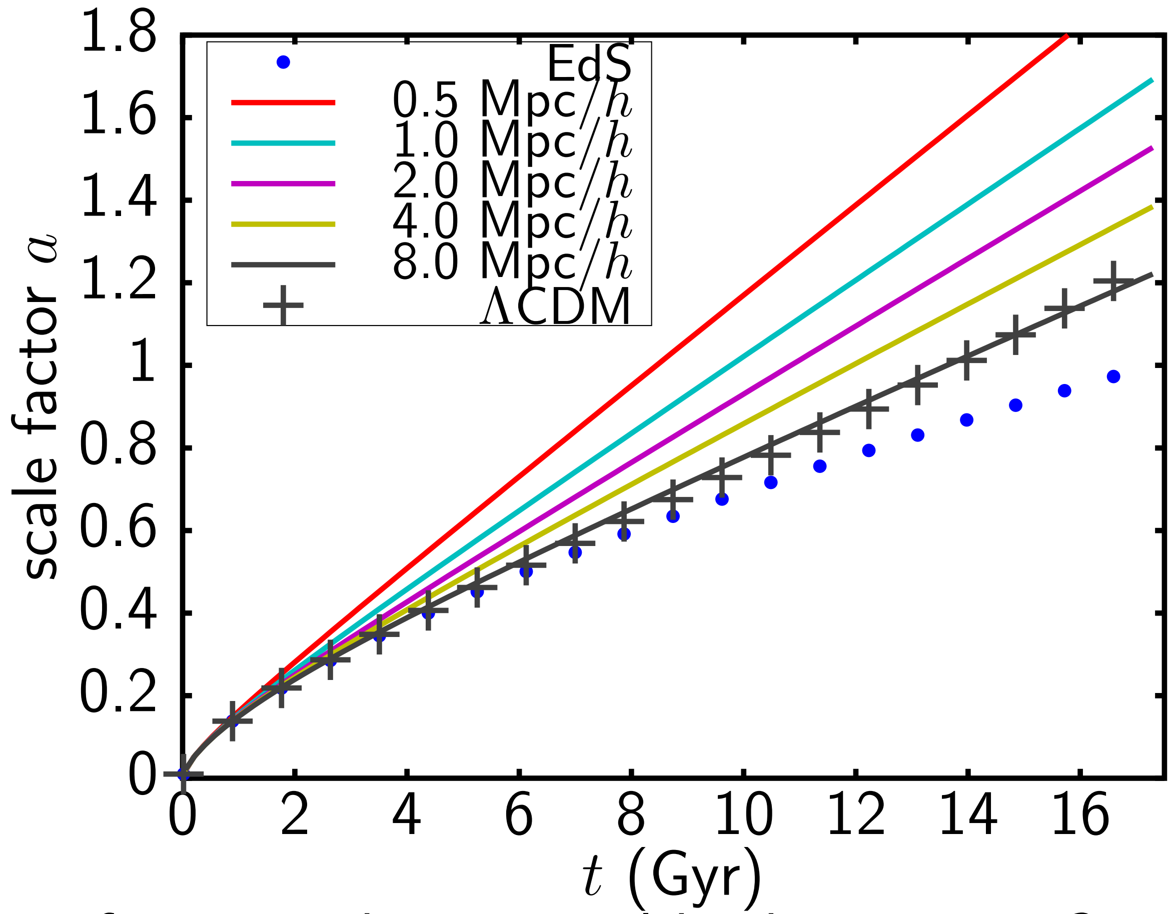
$$L_{\mathrm{box}} = 8L_{\mathcal{D}} = 8L_{\mathrm{DTFE}} = 4L_N$$

VQZA $a_{\text{eff}}(t)$



$$L_{\text{box}} = 64 L_{\mathcal{D}} = 2 L_{\text{DTFE}} = 2 L_N$$

$\mathcal{Q}_{\mathcal{D}} := 0$

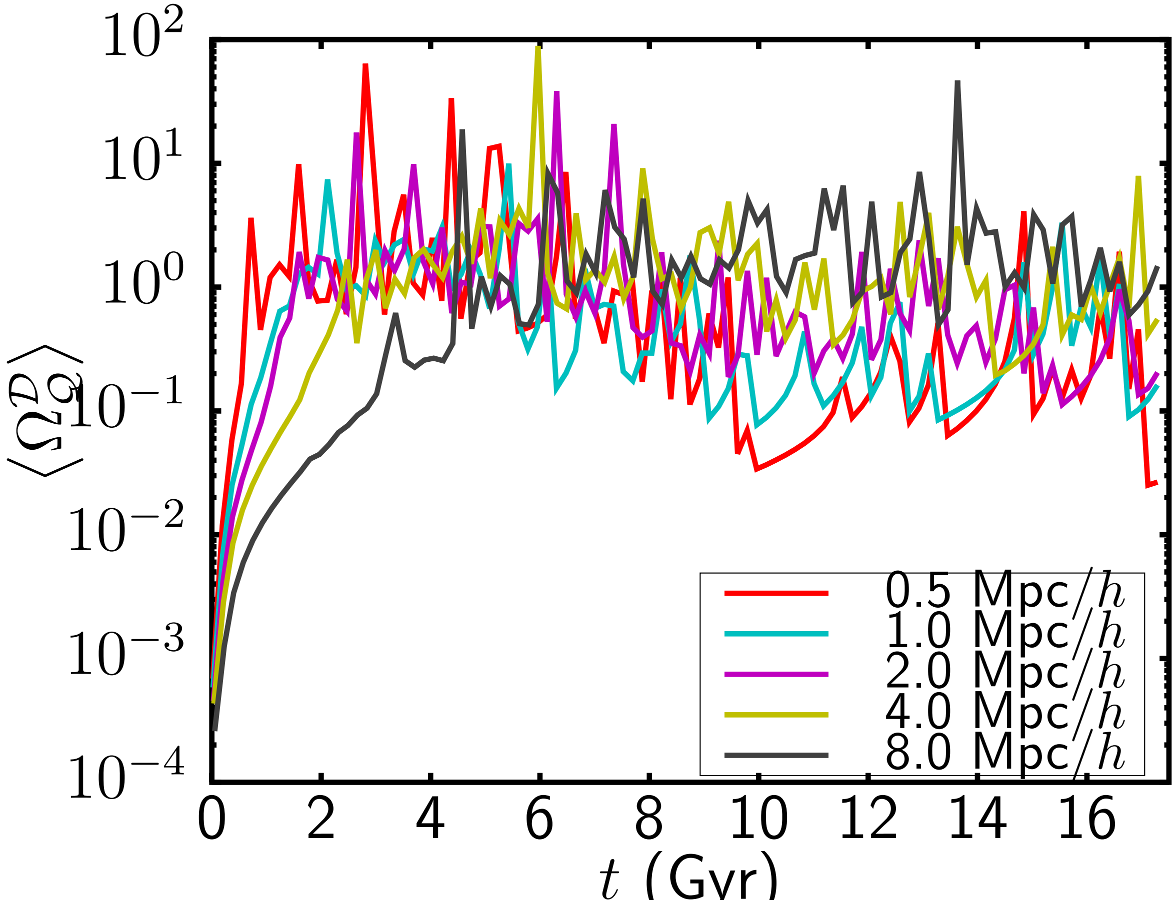


if ignoring kinematical backreaction: $\mathcal{Q}_{\mathcal{D}} := 0$

$\sim$ cf Rácz+2017

$$L_{\rm box} = 8L_{\mathcal{D}} = 4L_{\rm DTFE} = 8L_N$$

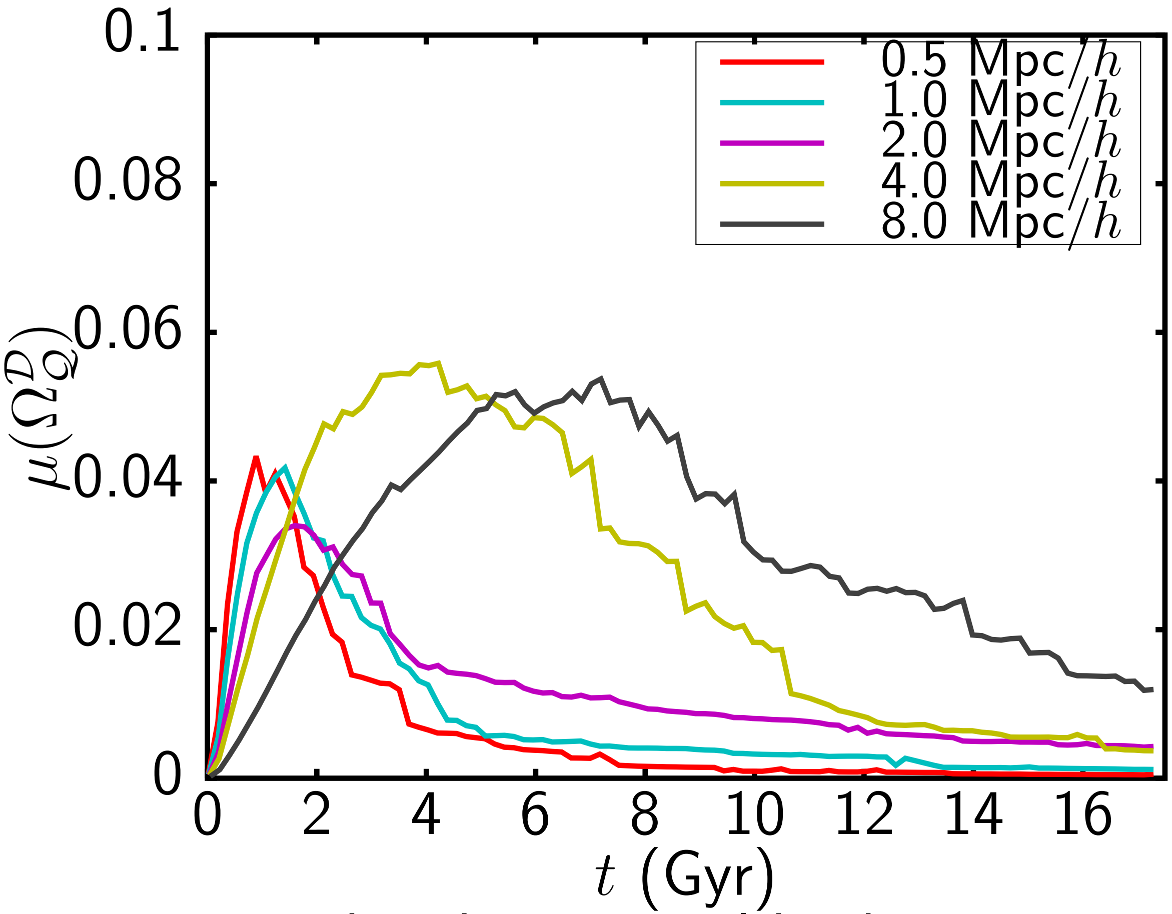
$\mathcal{Q}_{\mathcal{D}}$ evolution



mean kinematical backreaction

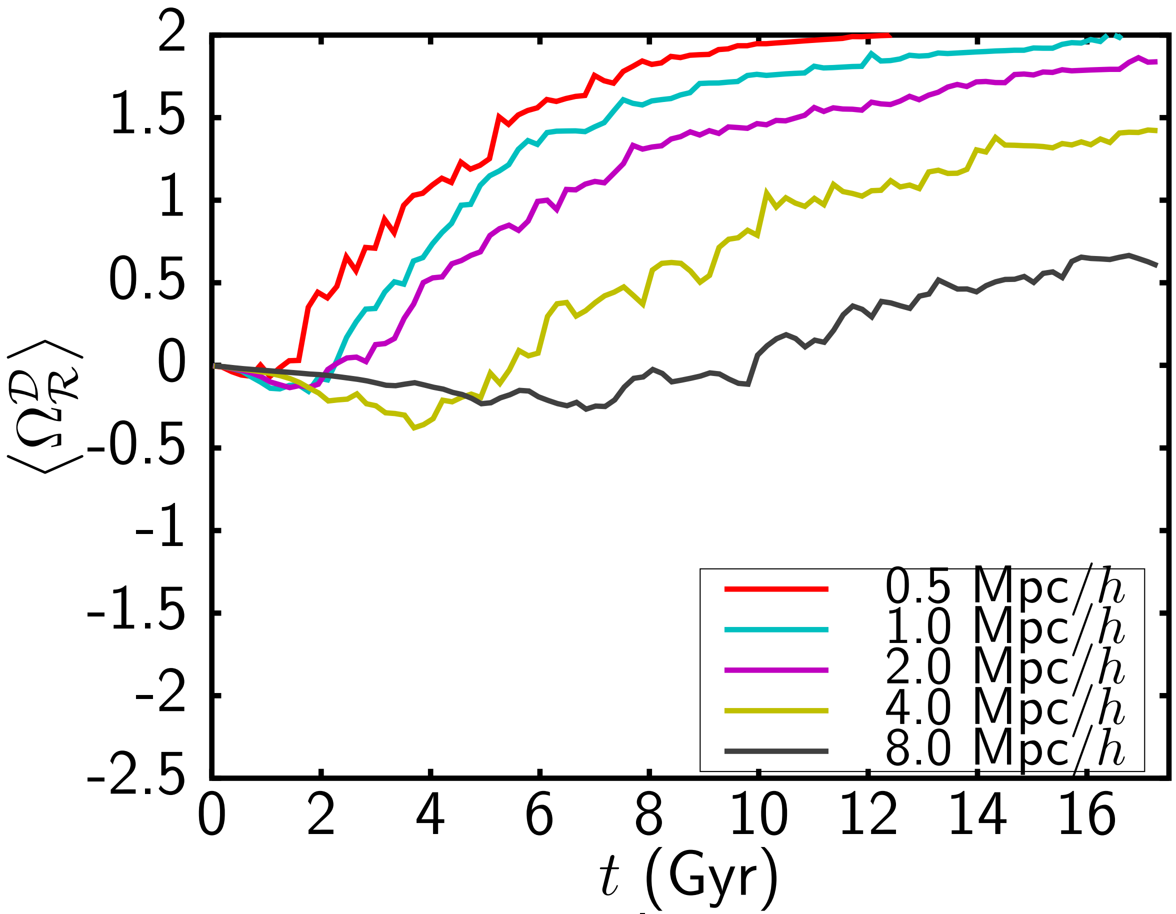
VQZA $L_{\rm box} = 8L_{\mathcal{D}} = 4L_{\rm DTFE} = 8L_N$

$\mathcal{Q}_\mathcal{D}$ evolution



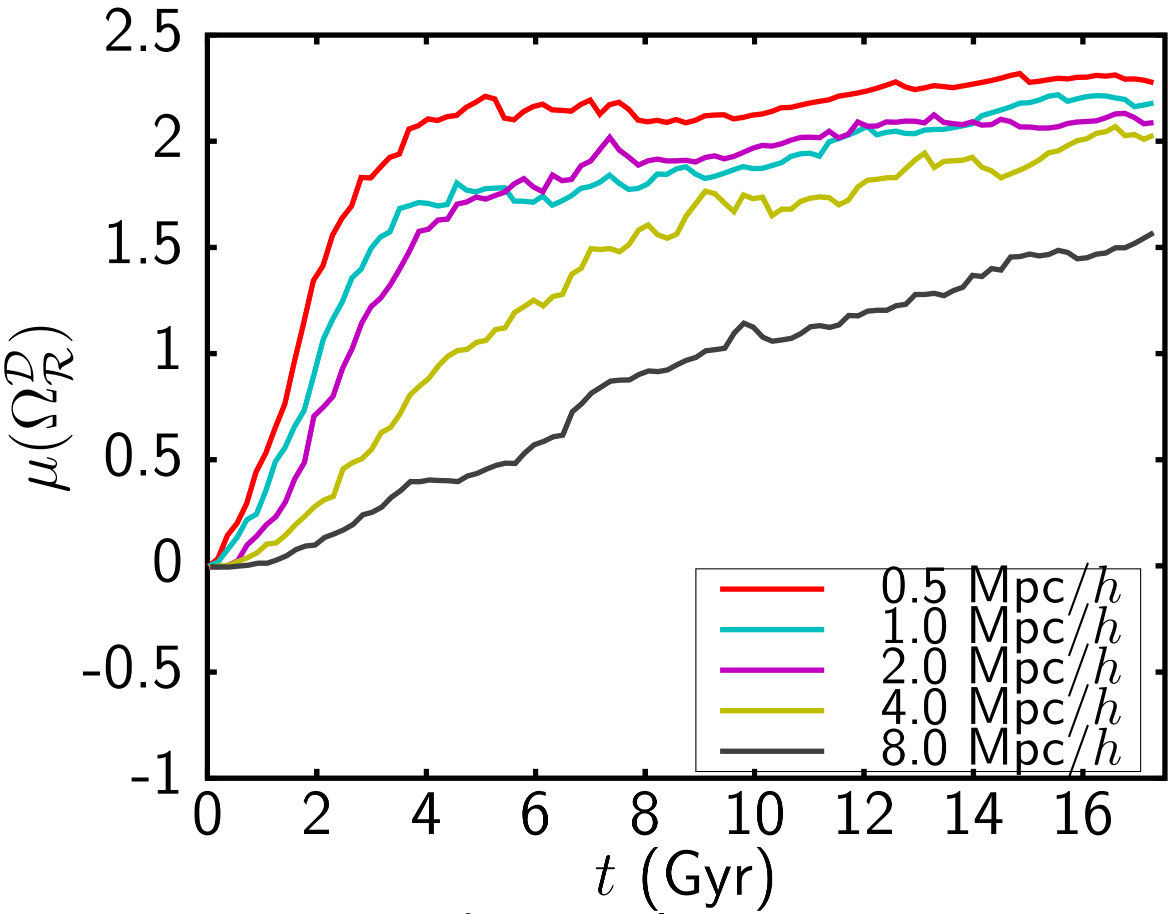
median kinematical backreaction

mean curvature evolution



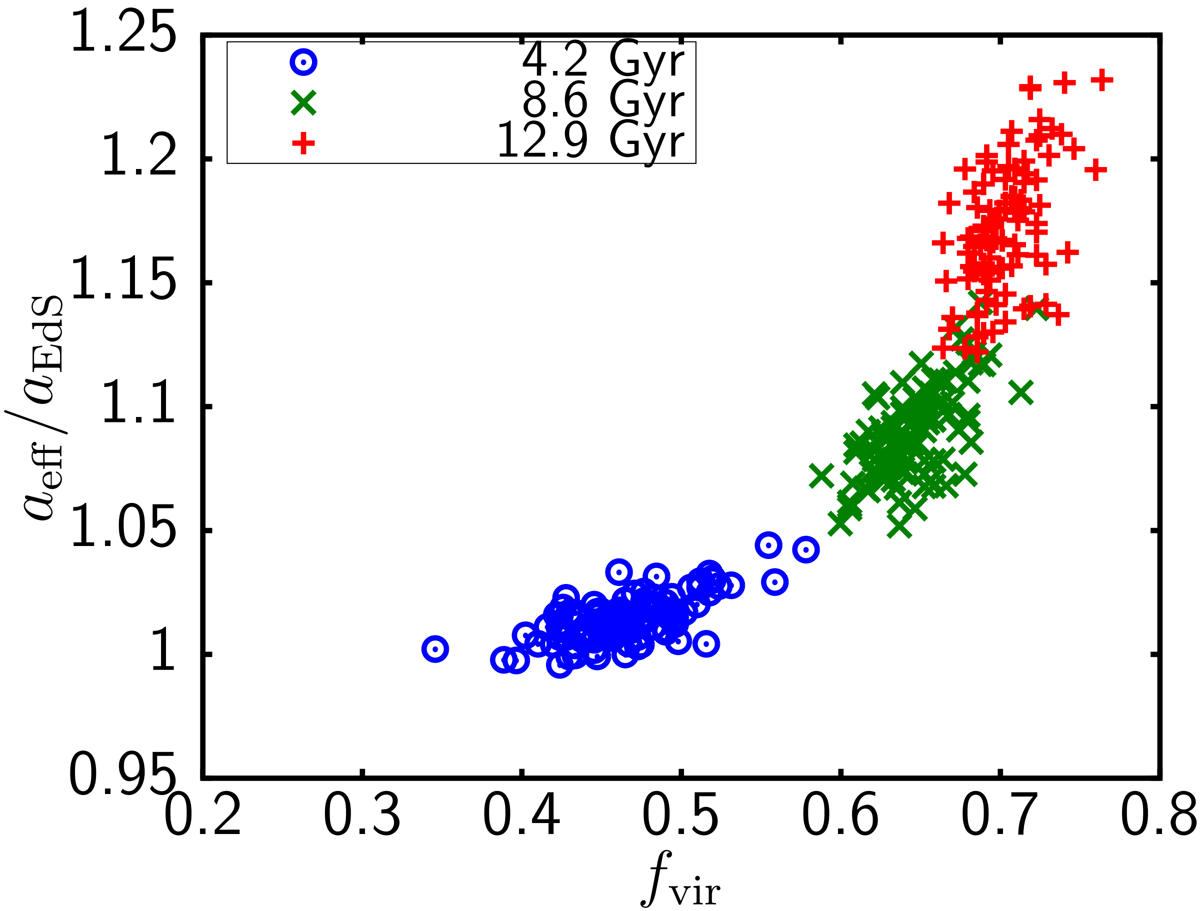
mean scalar curvature

mean curvature evolution



median scalar curvature

super-EdS growth vs $f_{\rm vir}$



100 VQZA simulations with $L_{\mathcal{D}} = 2$ Mpc/$h^{\rm eff}$ and
$L_{\rm box} = 8L_{\mathcal{D}} = 4L_{\rm DTFE} = 8L_N$

Conclusion

- assume EdS initial conditions, matched to ΛCDM obs proxy
- evolve Lagrangian domains (QZA) [silent universe approximation]
- biscale no-virialisation (QZA): $|a_{\mathrm{eff}}(t) - a_{\mathrm{EdS}}(t)| \ll 1$

Conclusion

- assume EdS initial conditions, matched to ΛCDM obs proxy
- evolve Lagrangian domains (QZA) [silent universe approximation]
- biscale no-virialisation (QZA): $|a_{\mathrm{eff}}(t) - a_{\mathrm{EdS}}(t)| \ll 1$
- assume stable clustering [implicitly: local statistical acceleration] (V)
- 2000 VQZA simulations at resolution 256^3:
  $L_{13.8} = 2.5^{+0.1}_{-0.4}$ Mpc/h^{eff} $\Rightarrow a_{\mathrm{eff}}(13.8\ \mathrm{Gyr}) \approx 1$ (16% above EdS)
- "DE" provided at order unity level from typical non-linear structure scale with this virialisation model
- Can better GR virialisation models avoid this?
- Roukema arXiv:1706.06179, submitted A&A