Special and General Relativity

Boud Roukema

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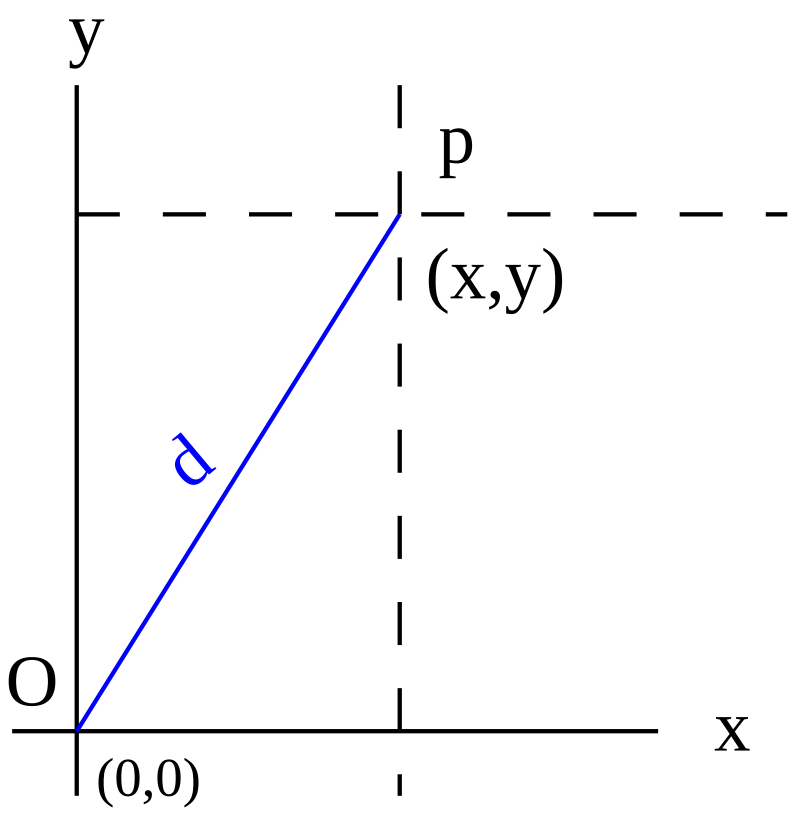
- GR: spacetime = a solution of the w:Einstein field equations

SR: Minkowski spacetime



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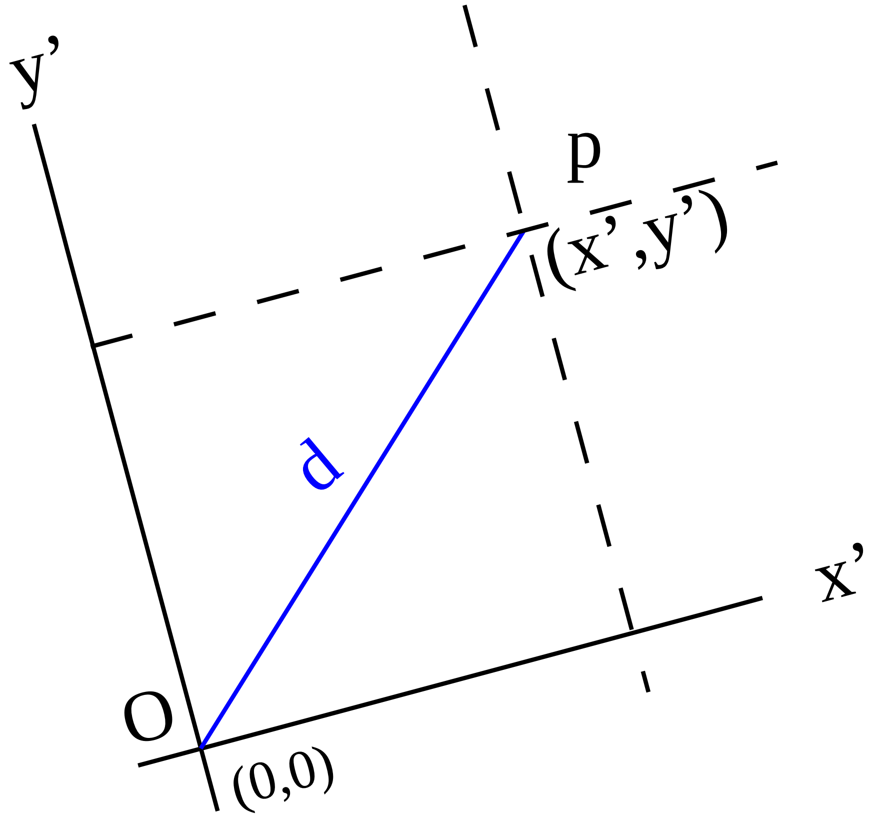

$$\begin{pmatrix} p_{\mathrm{x}'} \\ p_{\mathrm{y}'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \end{pmatrix}$$

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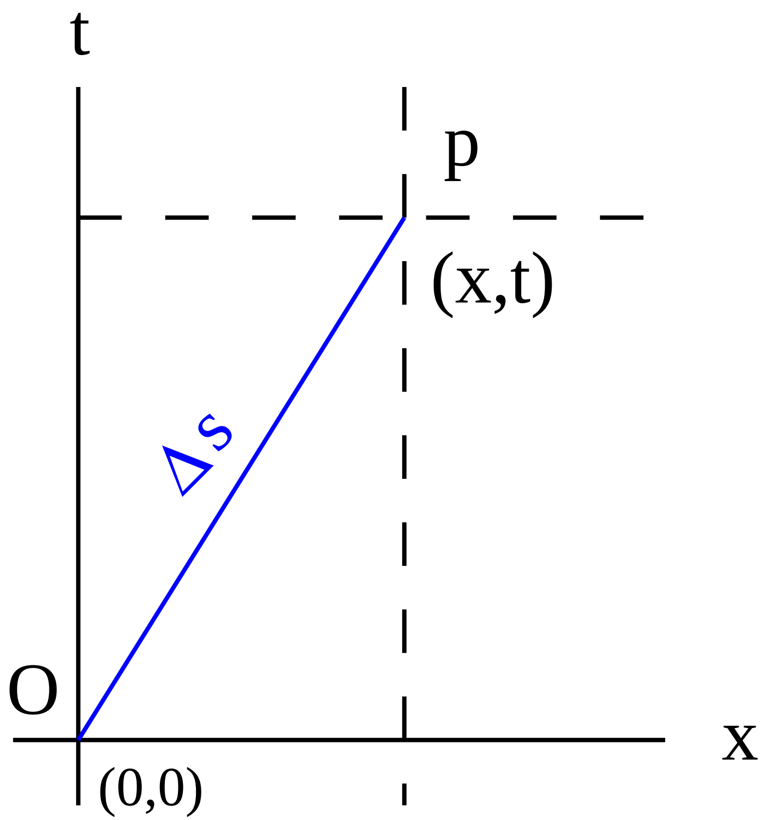

p at (x, t), w:invariant interval from observer at O is Δs
where $(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$

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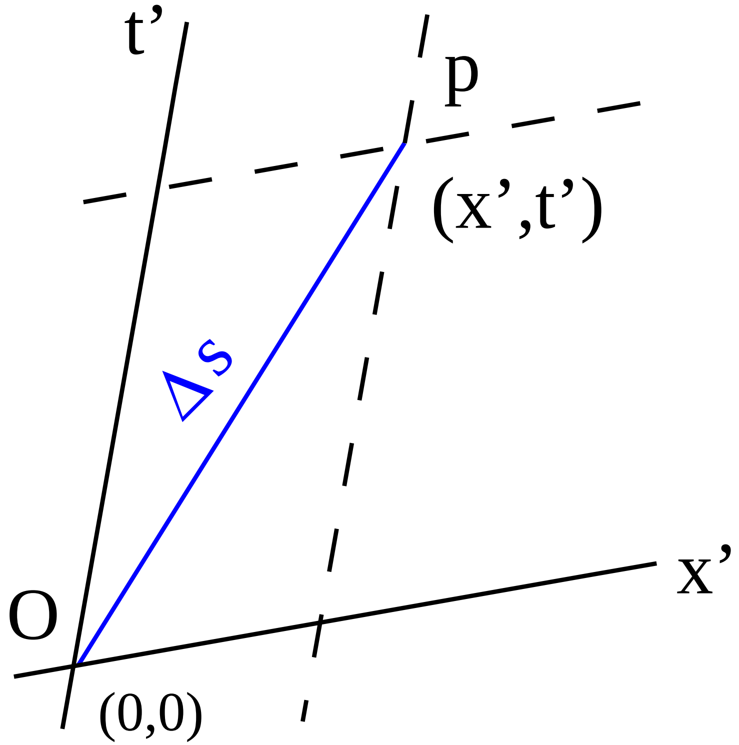

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

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$$= \frac{2.99792458\times10^8\times(1/2.99792458\times10^8) \text{ s}}{1 \text{ s}}$$

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$= \frac{1 \text{ s}}{1 \text{ s}} = 1$ (dimensionless)

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in A's coordinate system, B's worldline is:

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$$\Rightarrow x = t' \sinh \phi = t \tanh \phi = \beta t$$

where velocity $\beta := v/c \equiv v = \tanh \phi$

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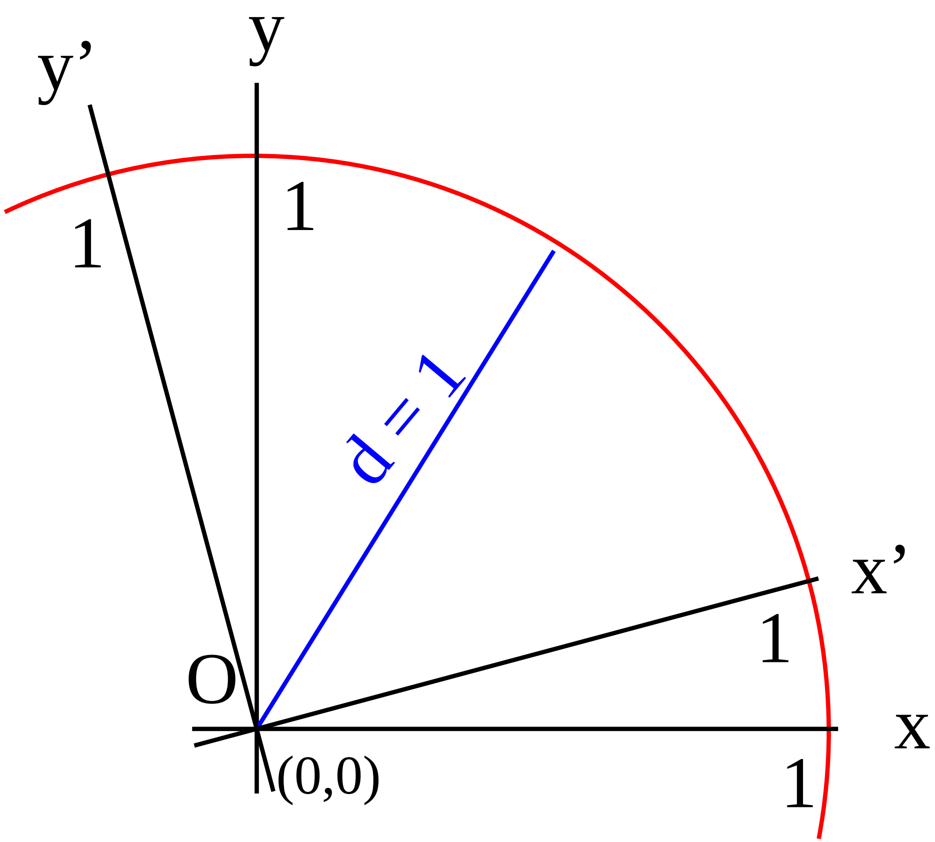

constant distance $d = 1$ from $(0, 0)$

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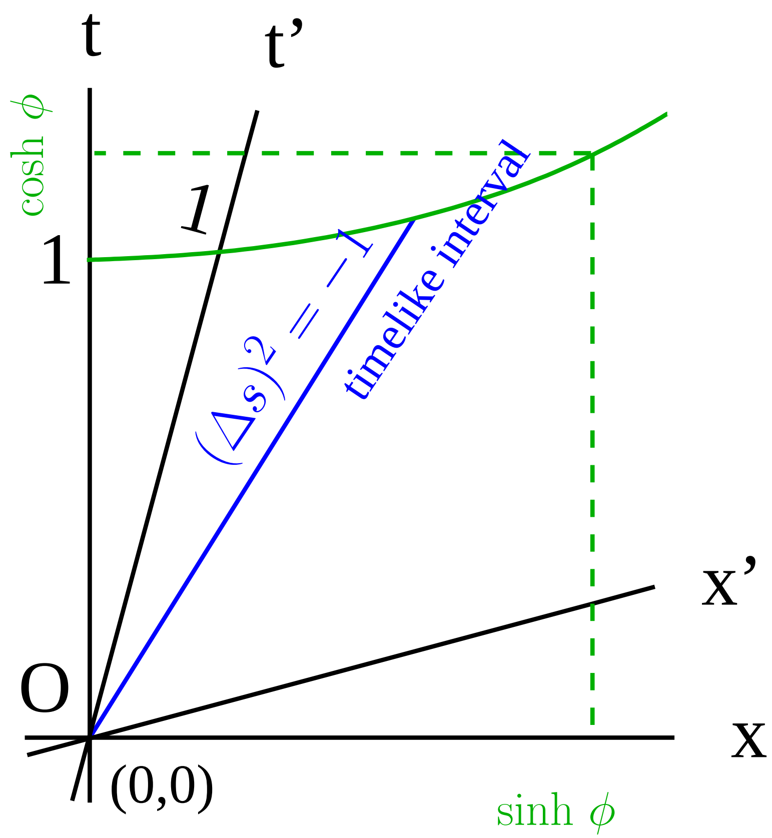

constant interval $(\Delta s)^2 = -1$ from $(0, 0)$

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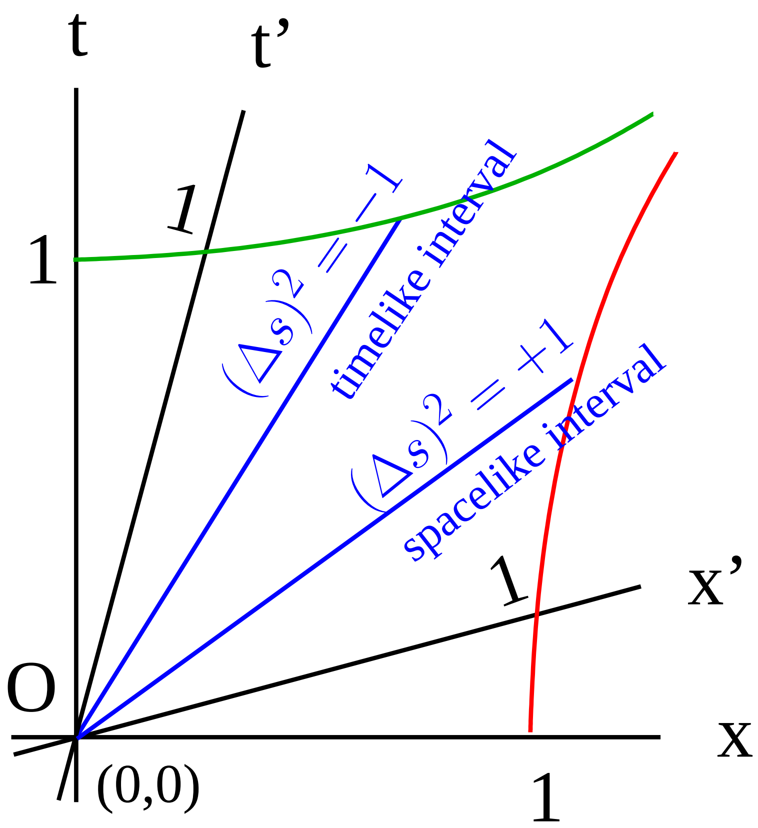

constant interval
$(\Delta s)^2 = +1$ from $(0,0)$

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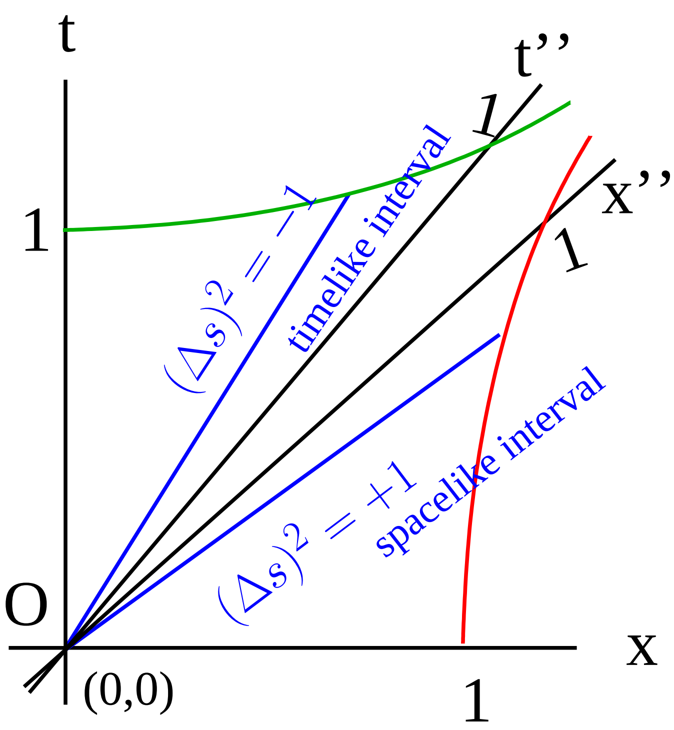

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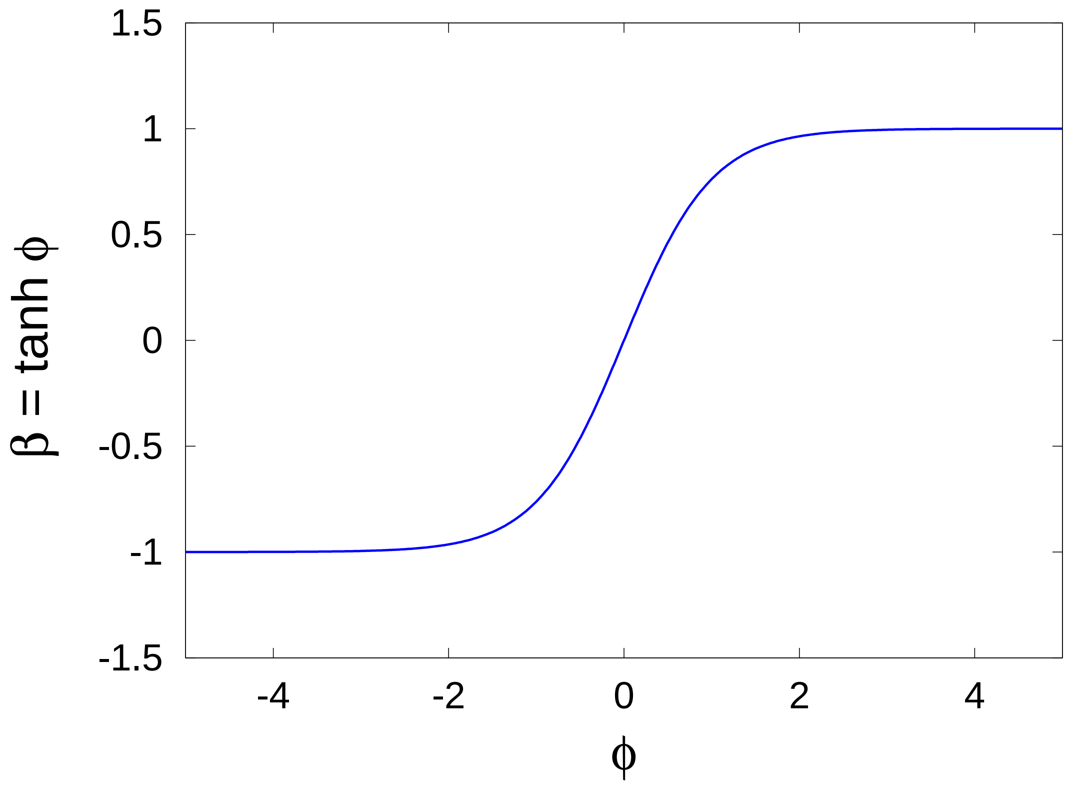

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- w:Michelson-Morley experiment (1887)

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interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?

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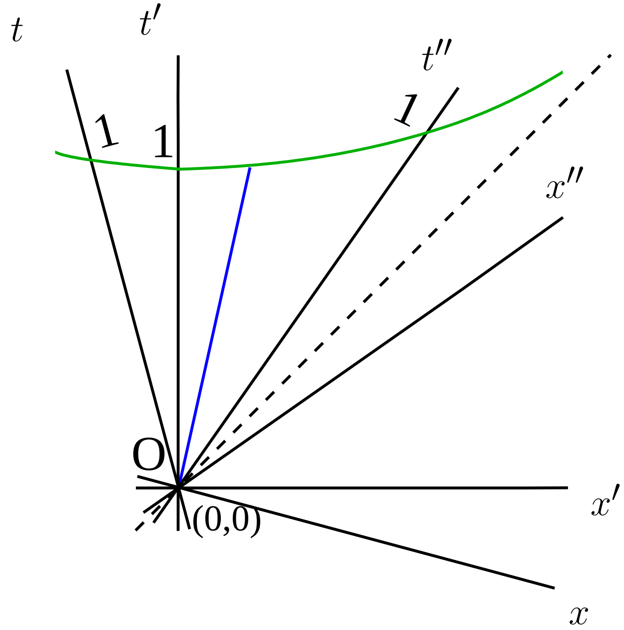

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where $\tanh\phi_1 = \beta_1 = 0.1$

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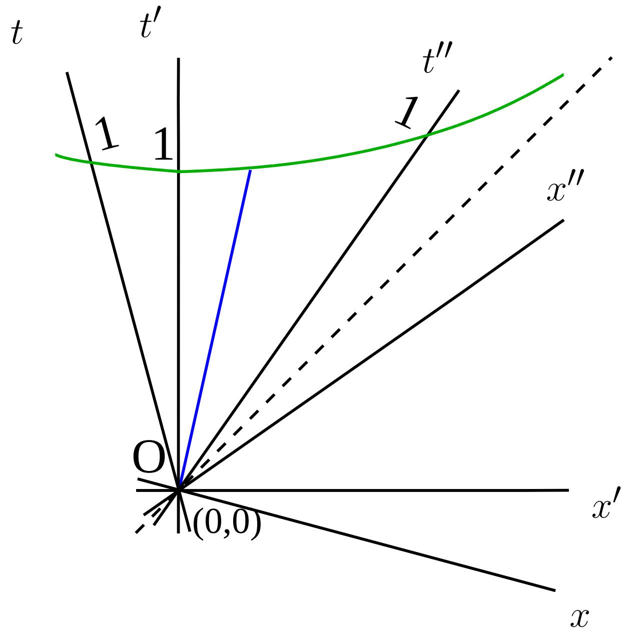

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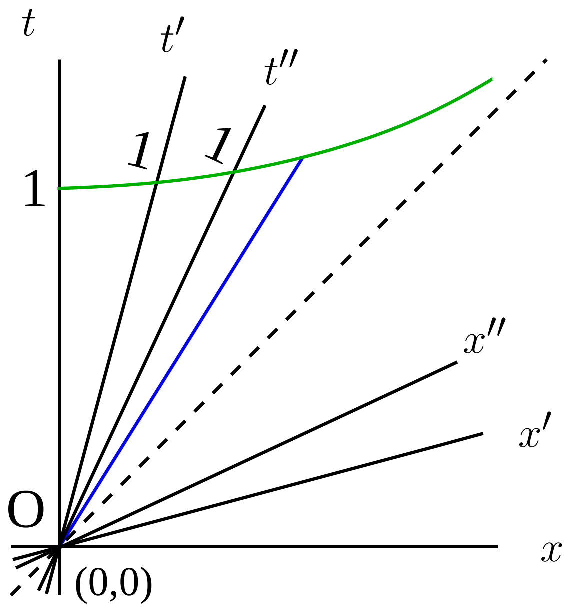

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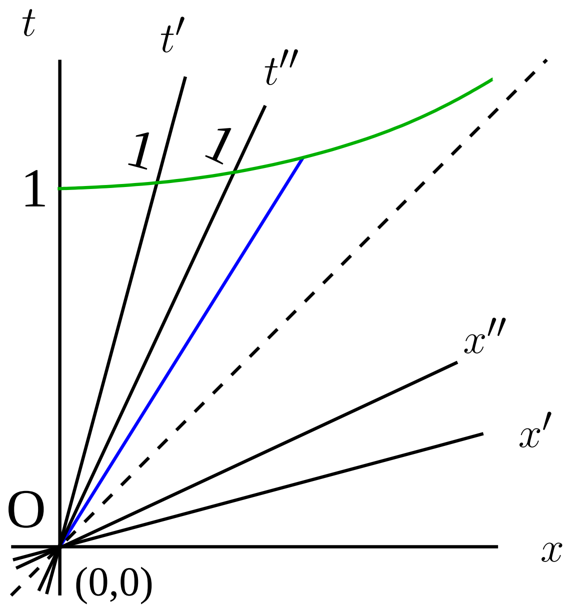

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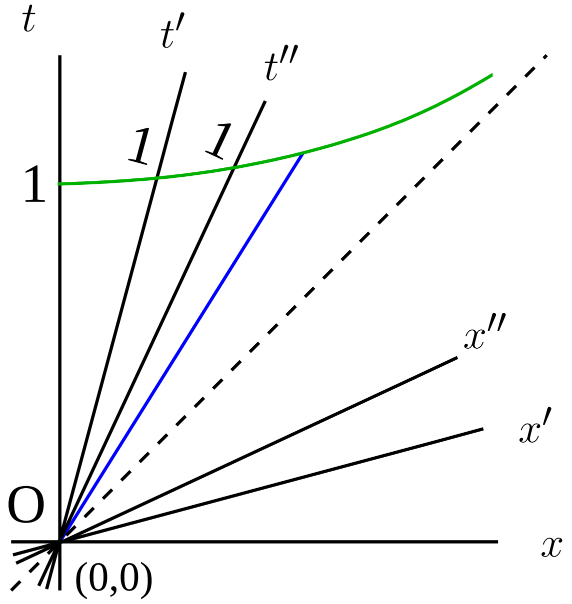

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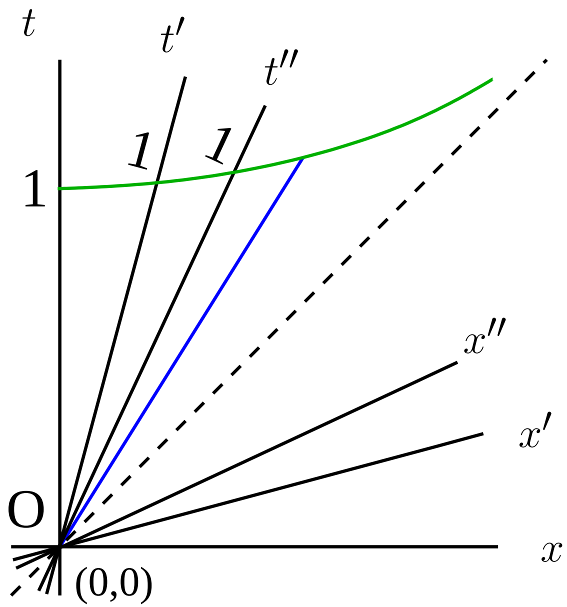

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cf. rotation θ_1 "plus" rotation θ_2 = rotation $\theta_1 + \theta_2$

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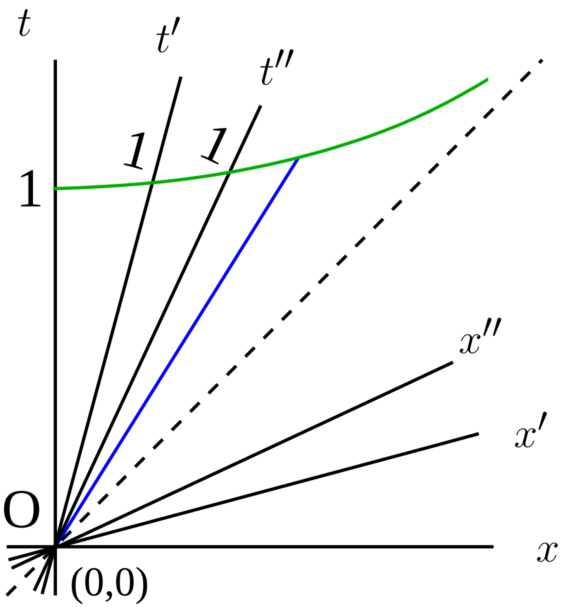

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so $\beta_3 = \tanh(\phi_1 + \phi_2)$

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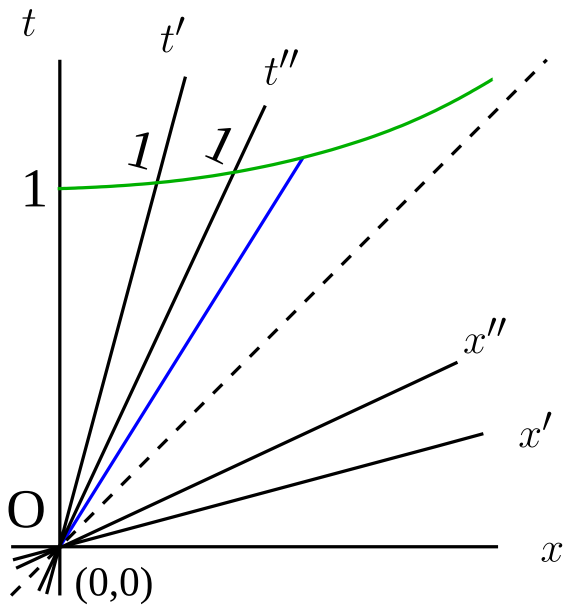

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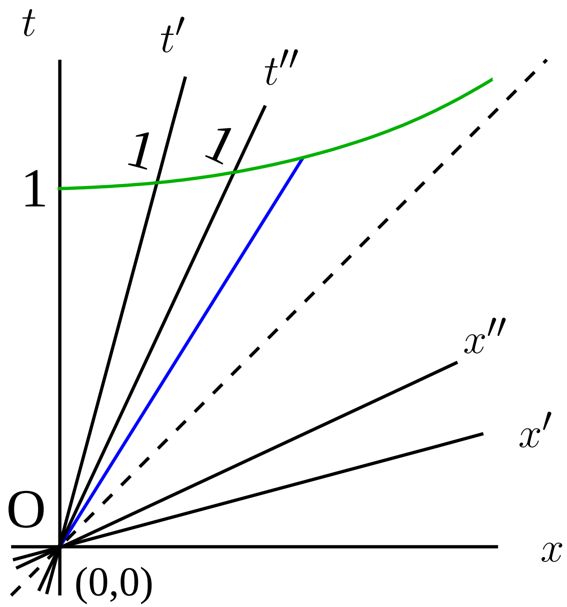
so $\beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\tanh\phi_1 + \tanh\phi_2}{1 + \tanh\phi_1 \tanh\phi_2}$

SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



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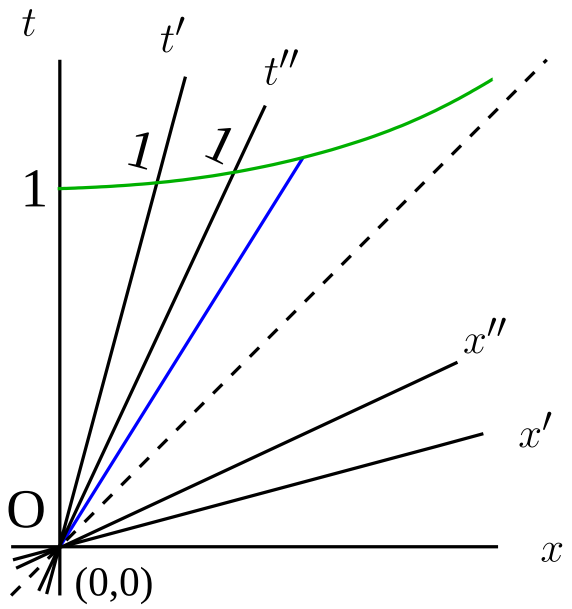

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SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

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$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

$$\cosh\phi := \frac{e^{\phi}+e^{-\phi}}{2}$$

$$\sinh\phi := \frac{e^{\phi}-e^{-\phi}}{2}$$

w:hyperbolic function

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

$$\Lambda(\beta) := \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}$$

$\beta = \tanh\phi$

$\gamma \ := \ (1 - \beta^2)^{-1/2}$ = Lorentz factor

$\gamma = \cosh\phi$

$\beta\gamma = \sinh\phi$

SR: model summary

Minkowski spacetime: draw a correct diagram

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Lorentz transformation (boost) $\Lambda(\phi)$ or $\Lambda(\beta)$

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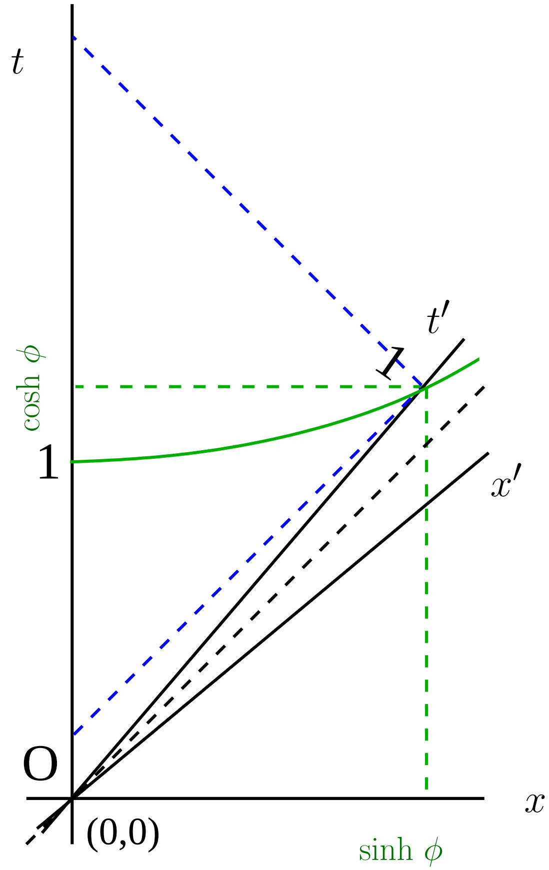
Lorentz transformation (boost) $\Lambda(\phi)$ or $\Lambda(\beta)$

refuse the assumption of absolute simultaneity (time)

SR: worldline time dilation



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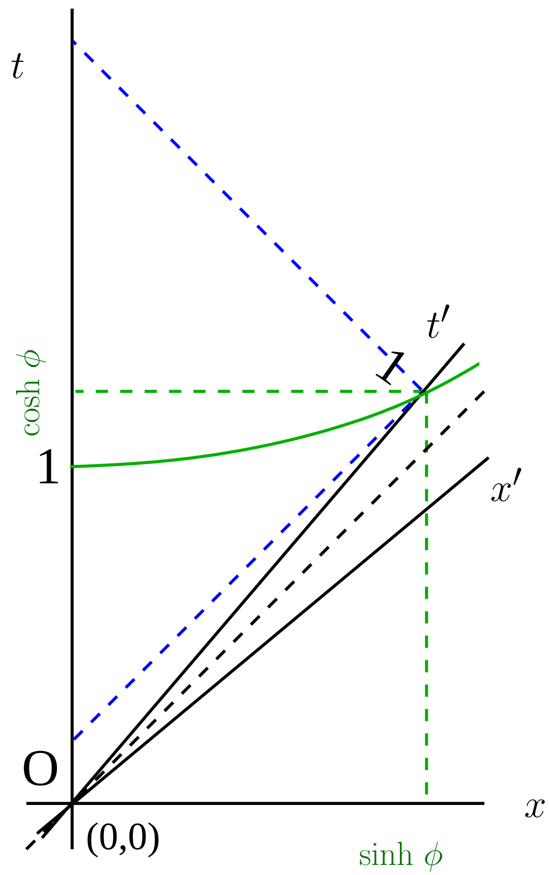
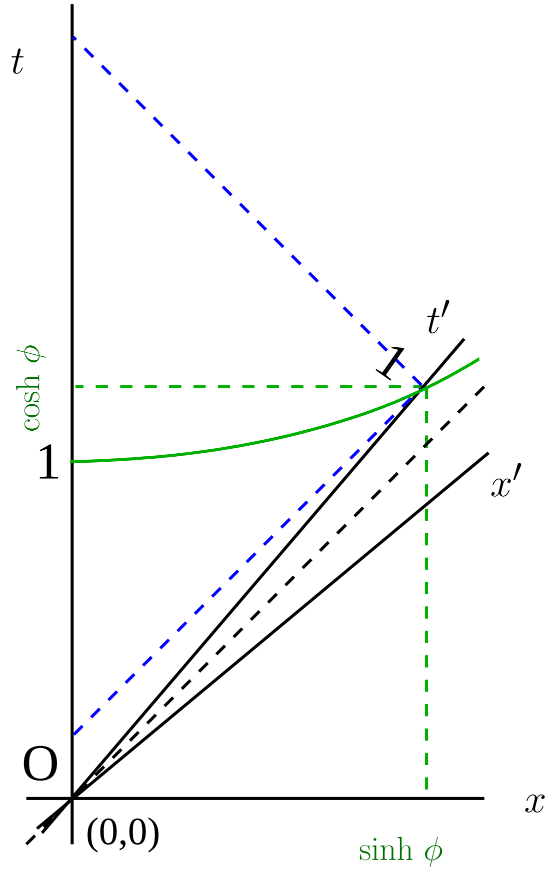


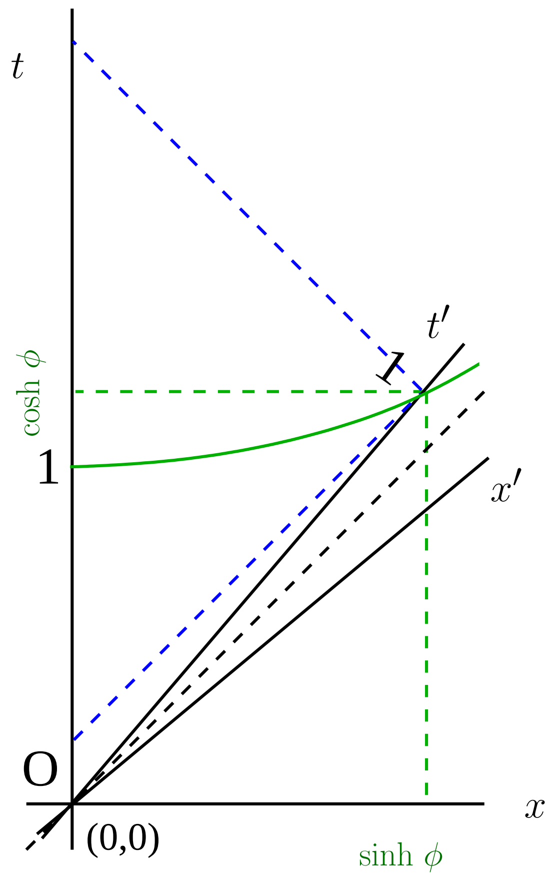
$$\cosh \phi \equiv \gamma$$

SR: worldline time dilation



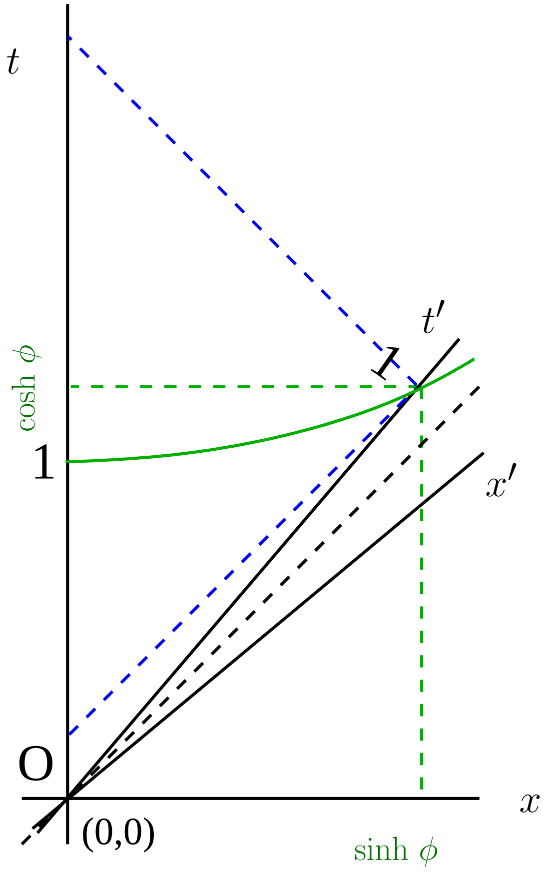
$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

SR: worldline time dilation



$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

SR: worldline time dilation



worldline "time dilation"

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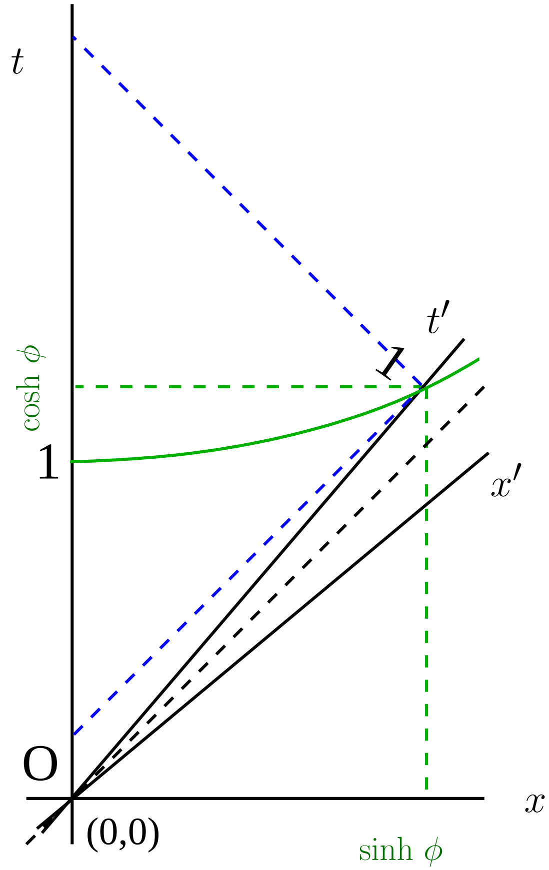

worldline “time dilation”

muons: mean lifetime 2197 ns $\ll 15$ km

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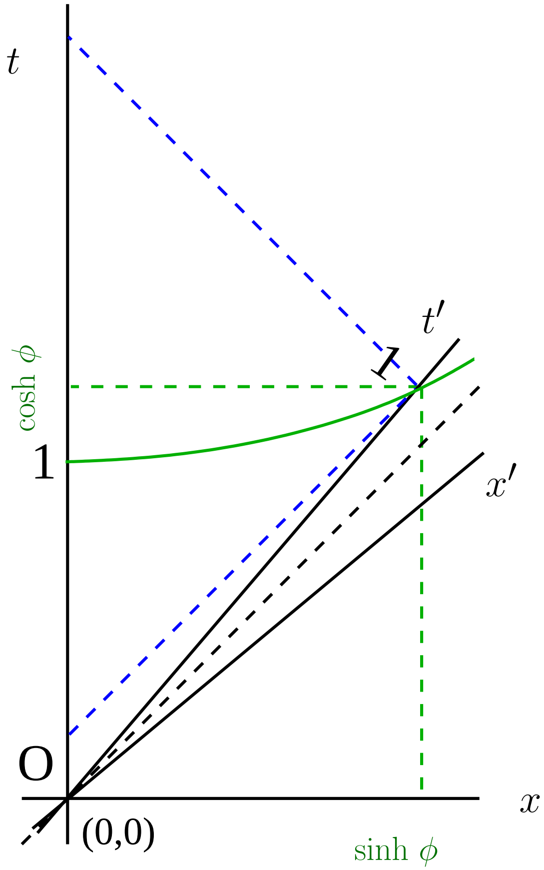


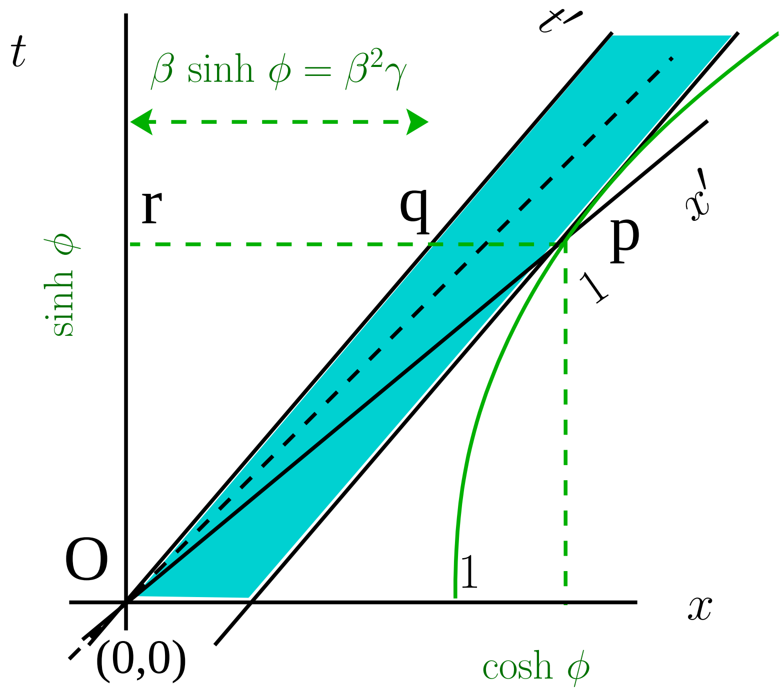
$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

worldline “time dilation”

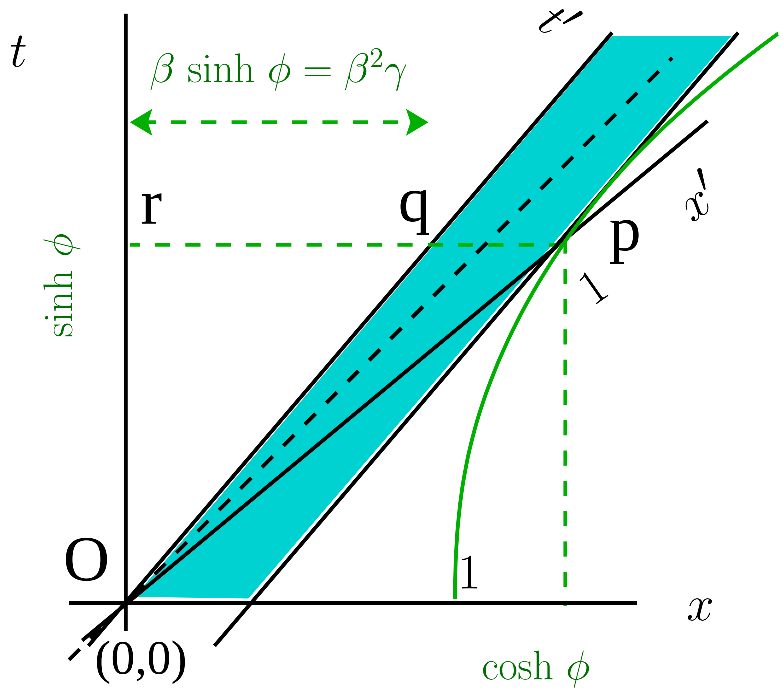
muons: mean lifetime 2197 ns $\ll 15$ km

time dilation $\Rightarrow$ muons can hit the ground

SR: worldsheet space contraction

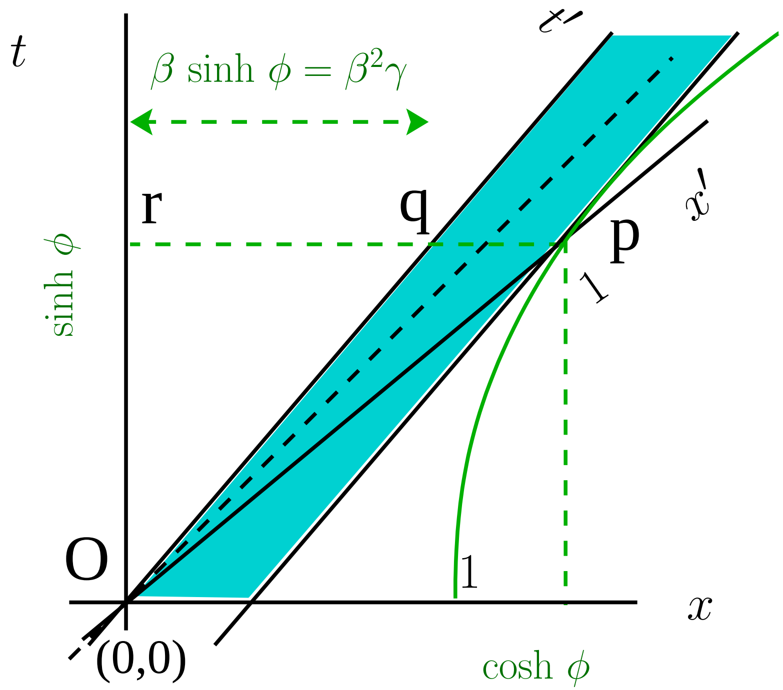


SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q}, \mathrm{p})} =$$

SR: worldsheet space contraction



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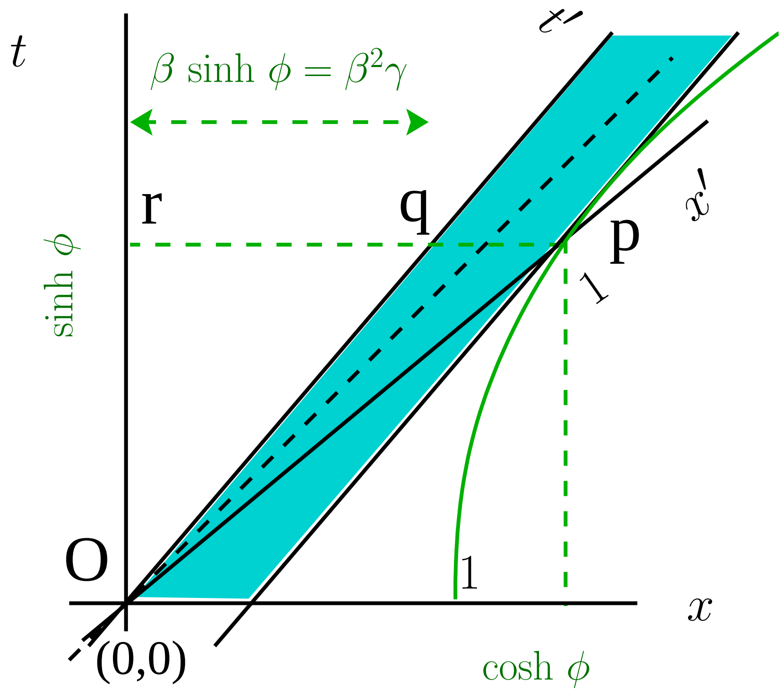

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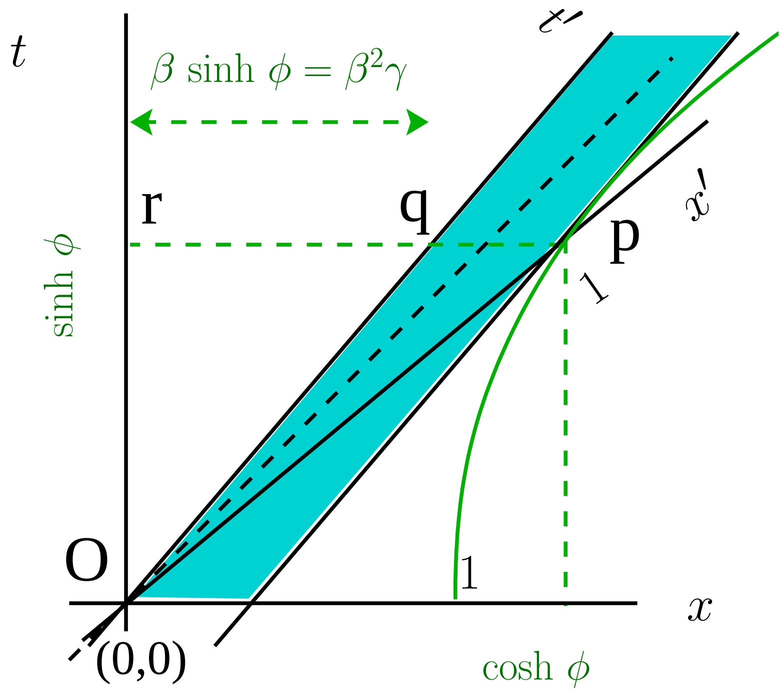
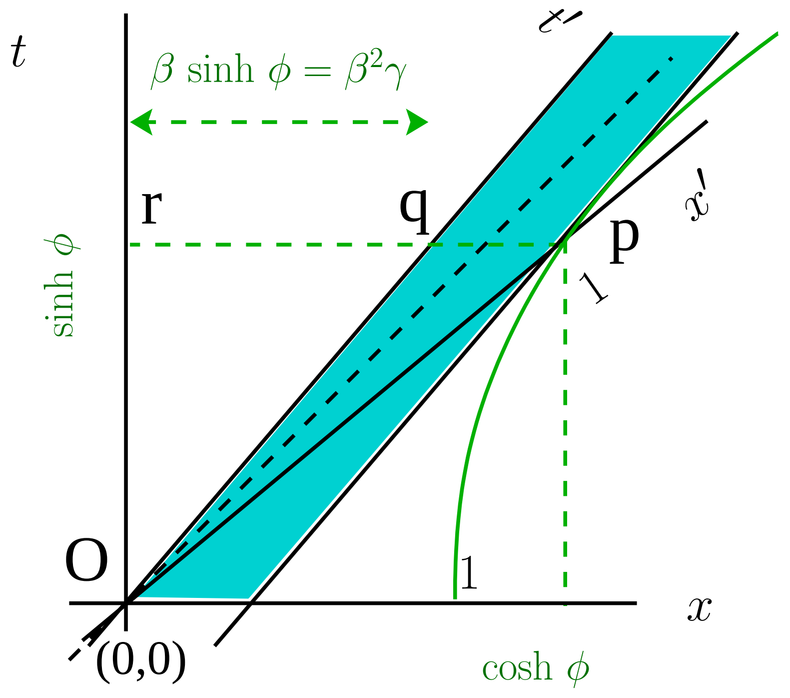
SR: worldsheet space contraction



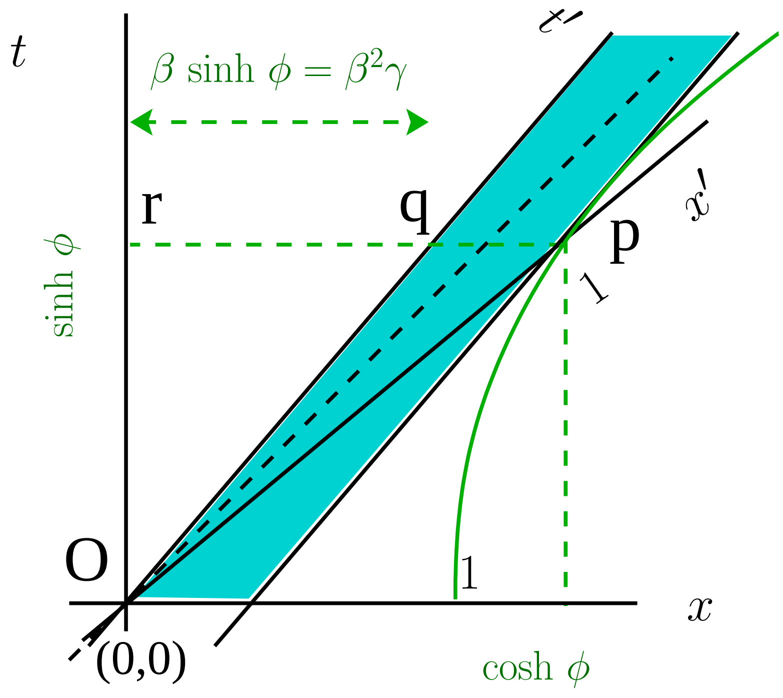
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)\gamma$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)^{1+(-1/2)}$$

SR: worldsheet space contraction



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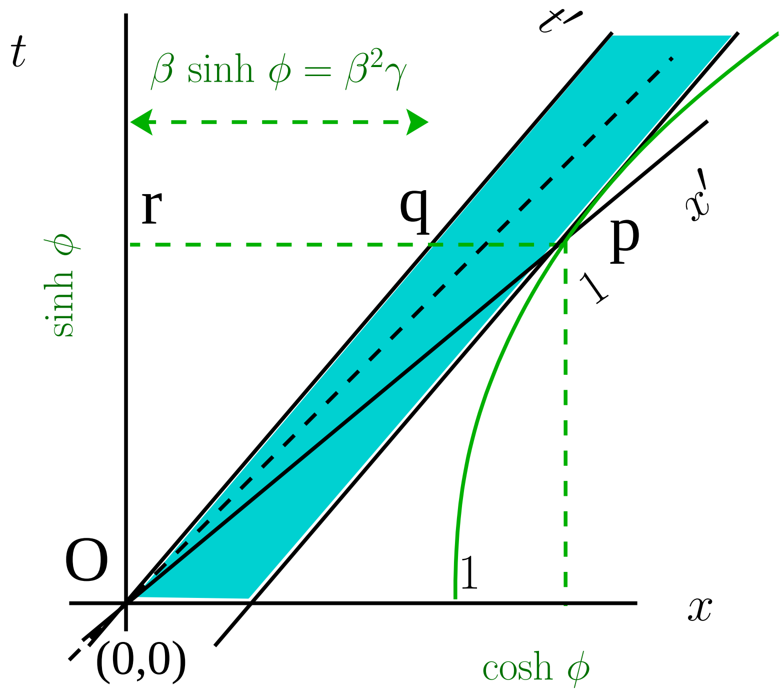
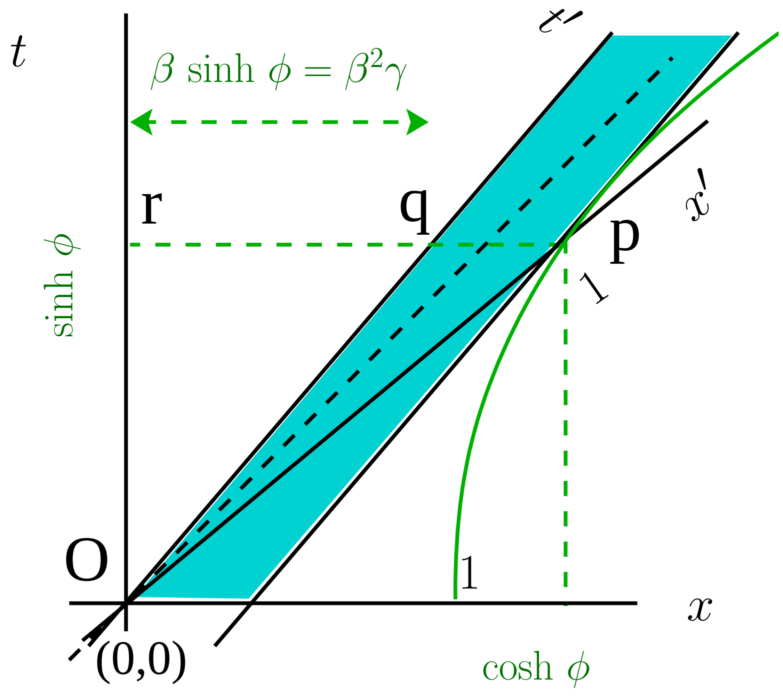


$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$

SR: worldsheet space contraction



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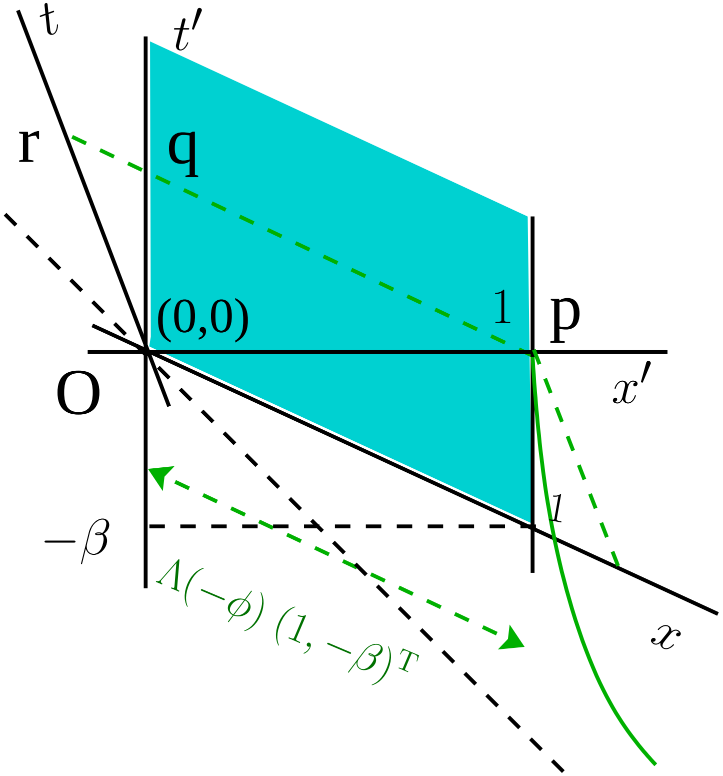
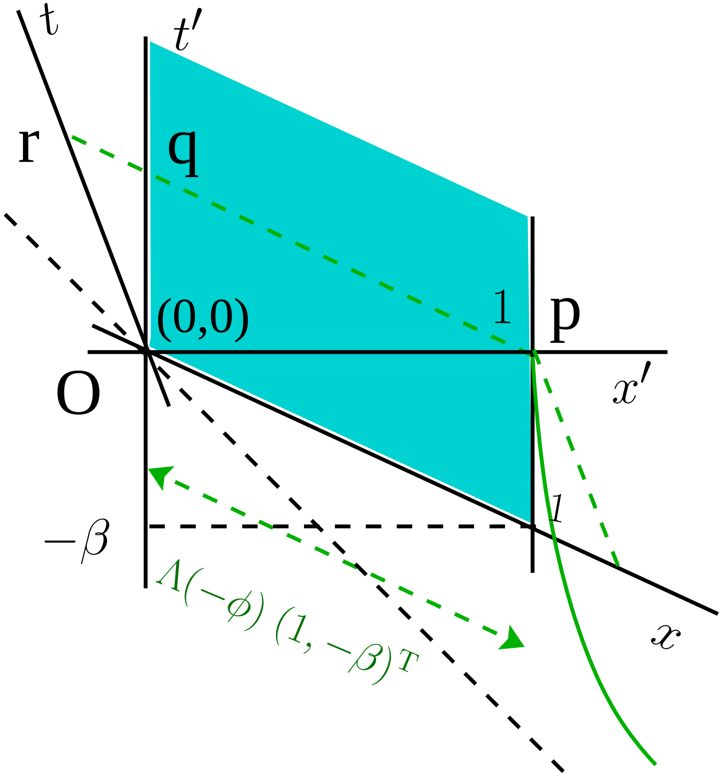
worldsheet “space contraction”

SR: worldsheet space contraction

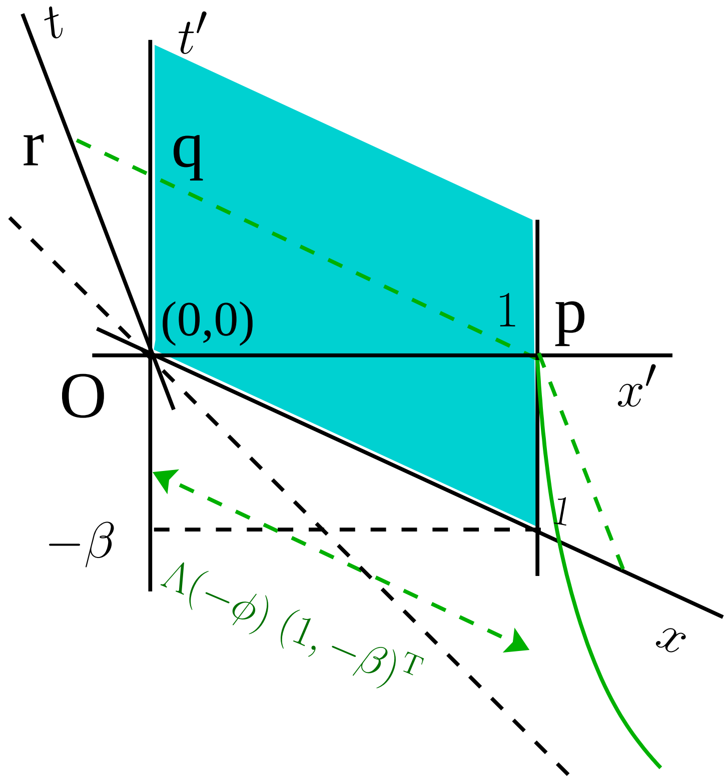


SR: worldsheet space contraction



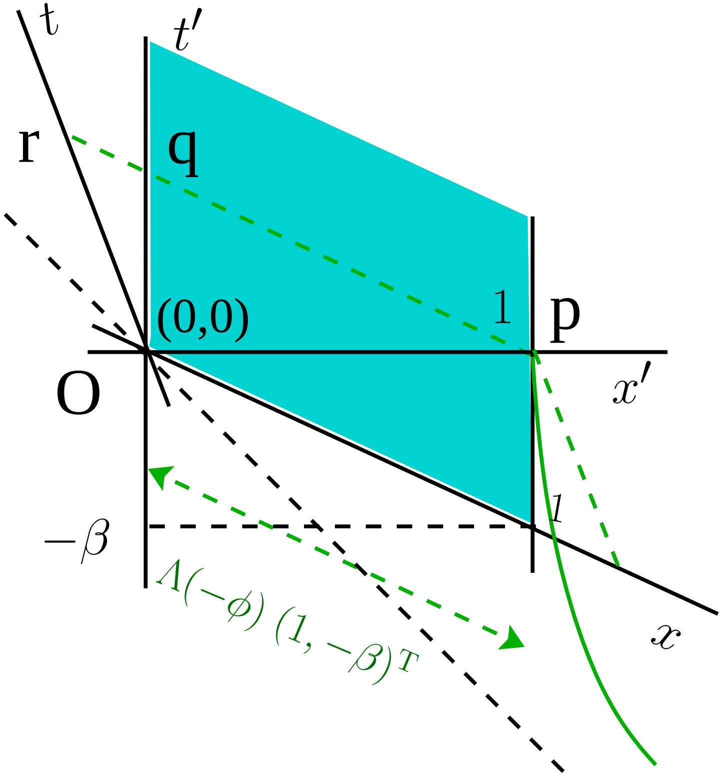
$$\Lambda^{-1}\begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \cosh\phi - \beta\sinh\phi \\ \sinh\phi - \beta\cosh\phi \end{pmatrix}$$

SR: worldsheet space contraction



$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\cosh\phi(1-\beta^2)\\ \cosh\phi(\tanh\phi-\tanh\phi)\end{pmatrix}$$

SR: worldsheet space contraction



$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\gamma(1-\beta^2)\\0\end{pmatrix}$$

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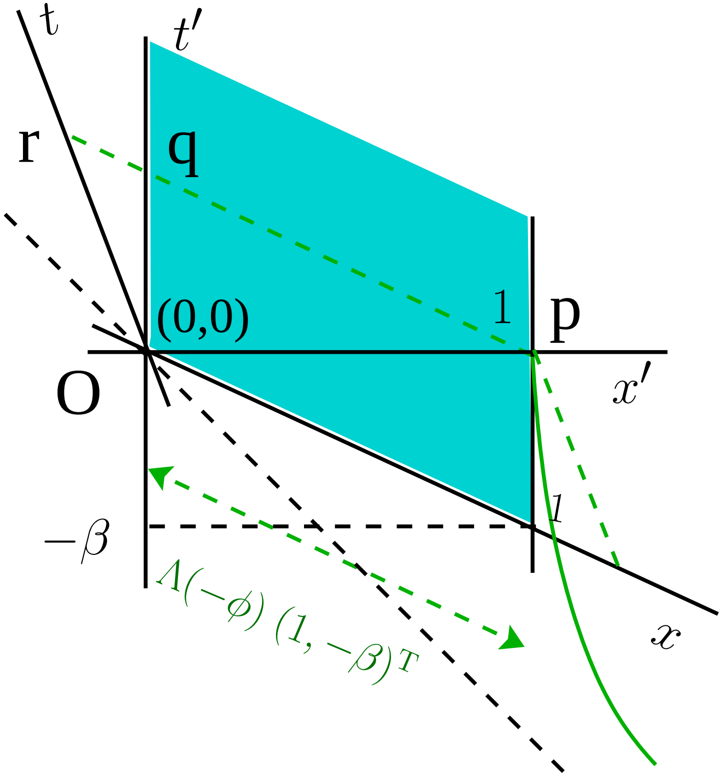

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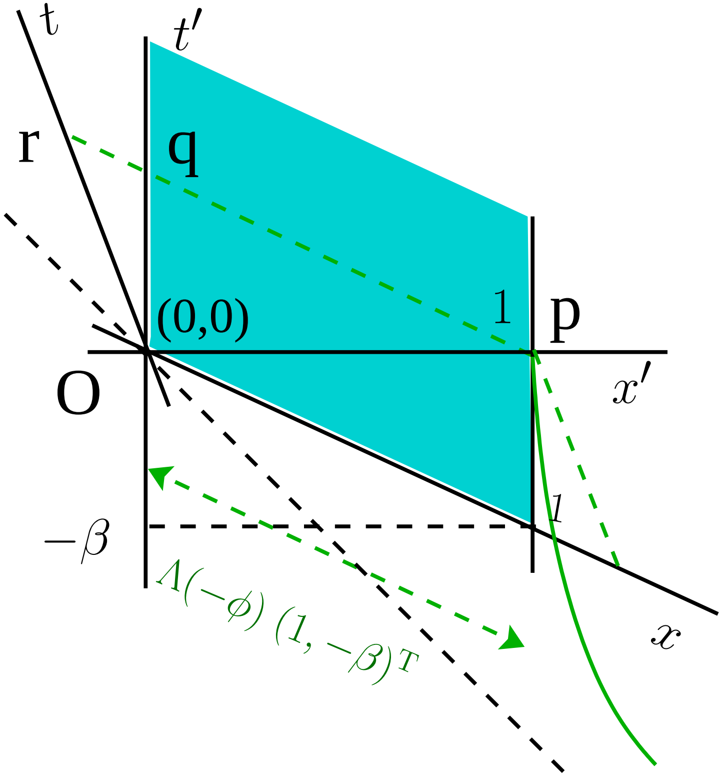

$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$

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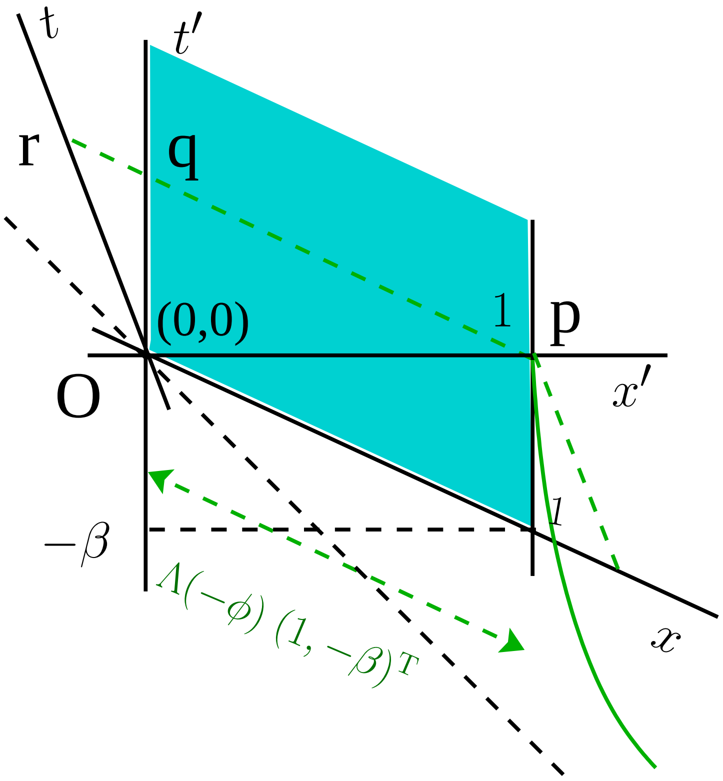
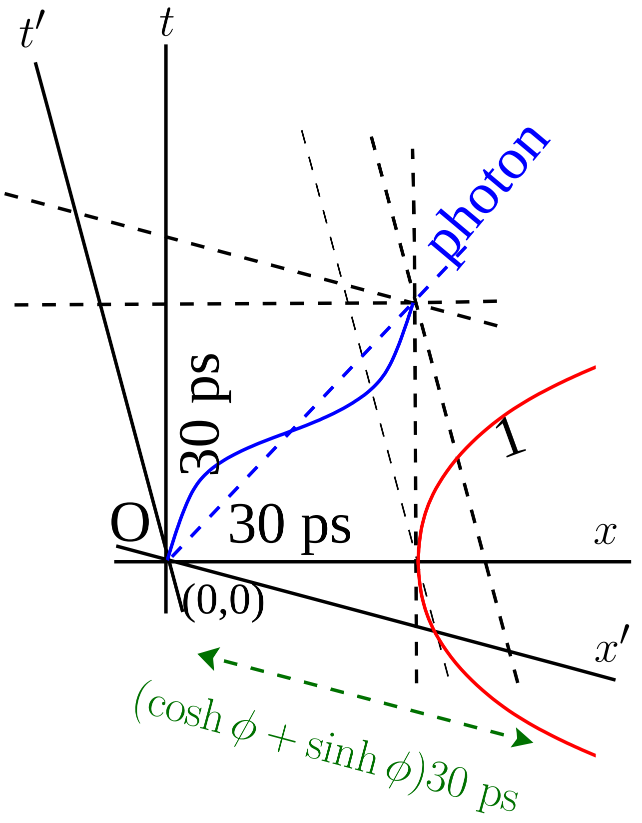


$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$ worldsheet “space contraction”

SR: Doppler shift



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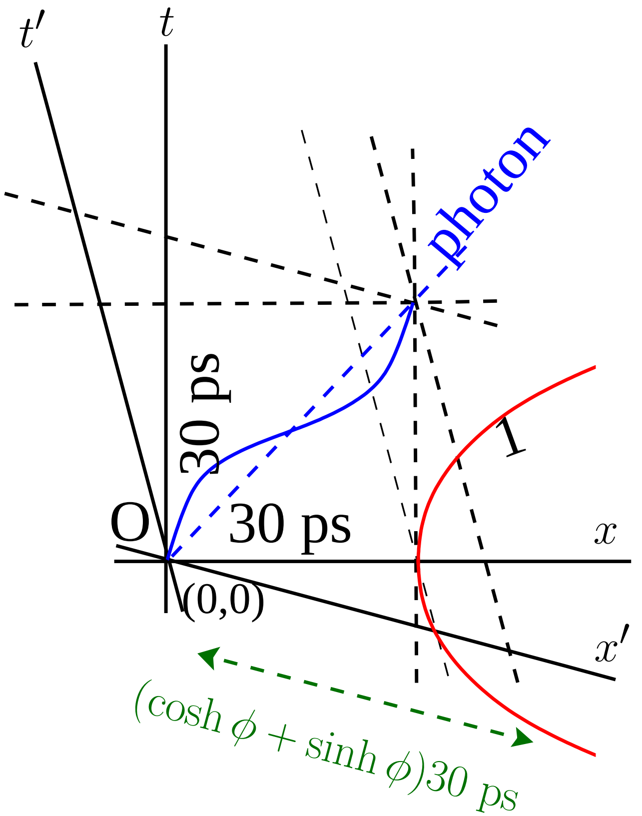
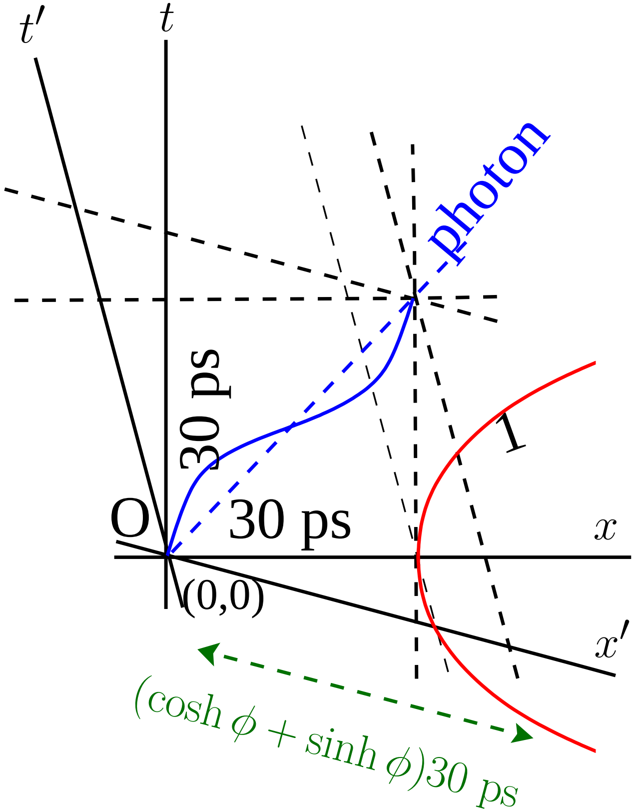


see photon worldline calculation

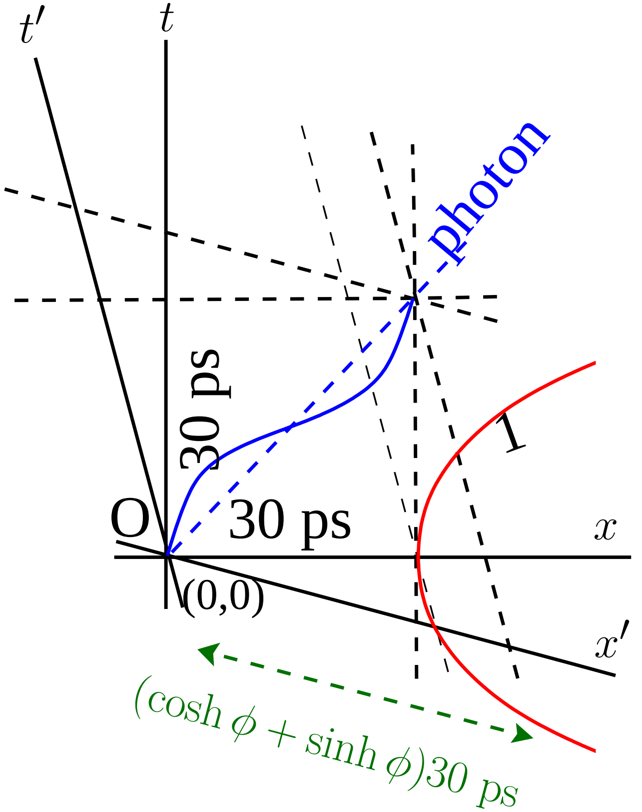
SR: Doppler shift



see photon worldline calculation

$x' = (\cosh\phi + \sinh\phi)t$

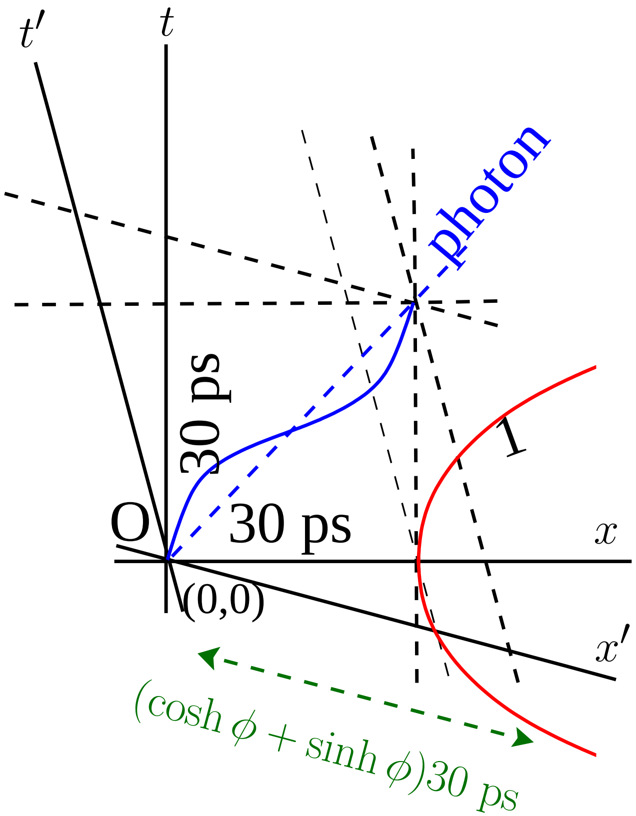
SR: Doppler shift



see photon worldline calculation

$$x' = (\cosh\phi + \sinh\phi)x$$

SR: Doppler shift



see photon worldline calculation

$$x'/x = \cosh\phi + \sinh\phi$$

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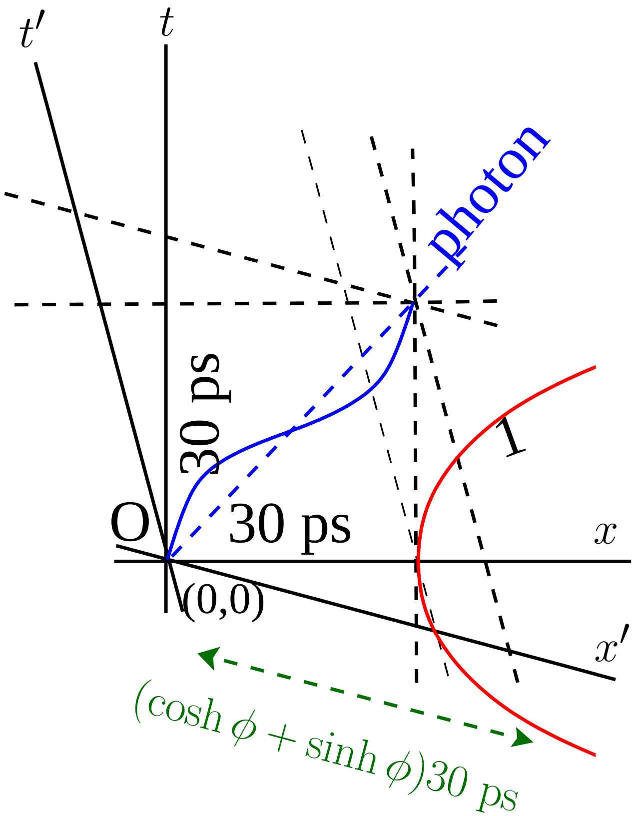
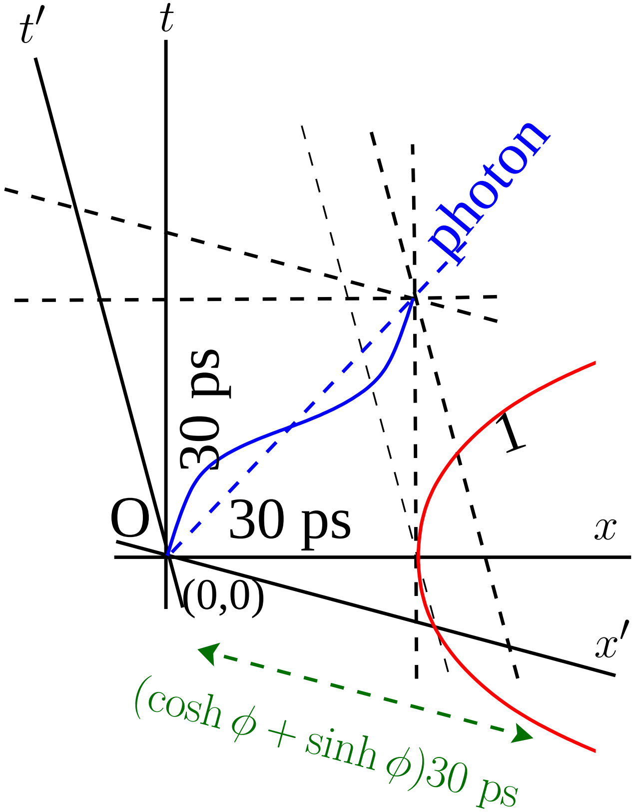


see photon worldline calculation

$x'/x = \gamma + \beta\gamma$

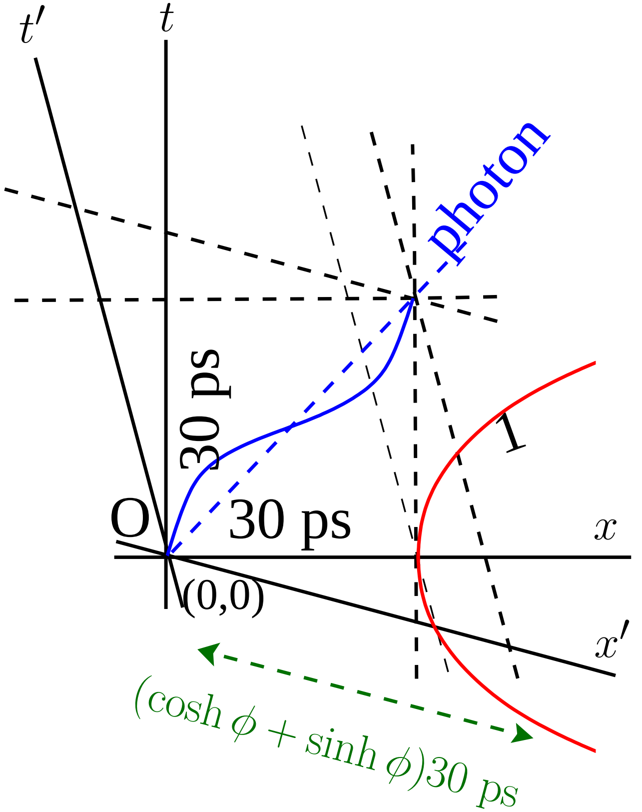
SR: Doppler shift



see photon worldline calculation

$x'/x = \gamma(1+\beta)$

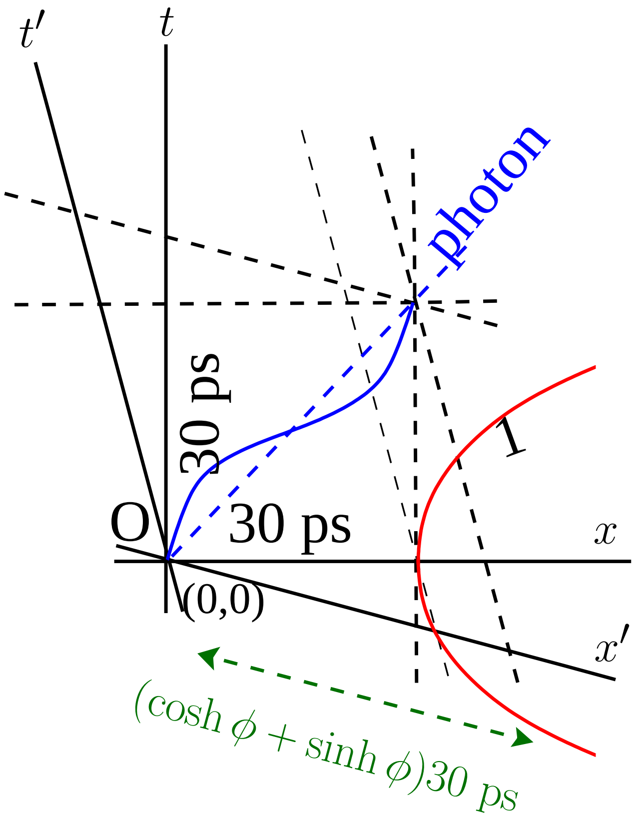
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \frac{1+\beta}{\sqrt{1-\beta^2}}$$

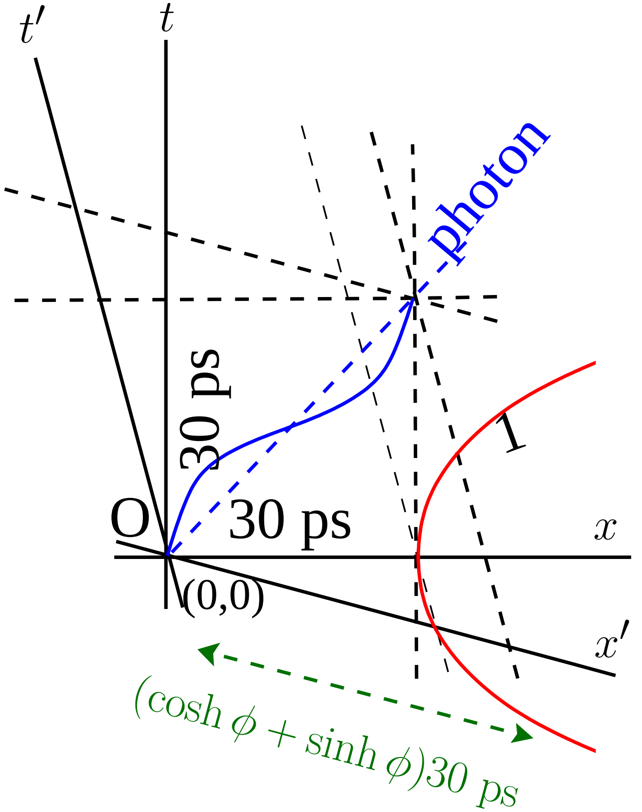
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \frac{\sqrt{(1+\beta)^2}}{\sqrt{(1-\beta)(1+\beta)}}$$

SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}}$$

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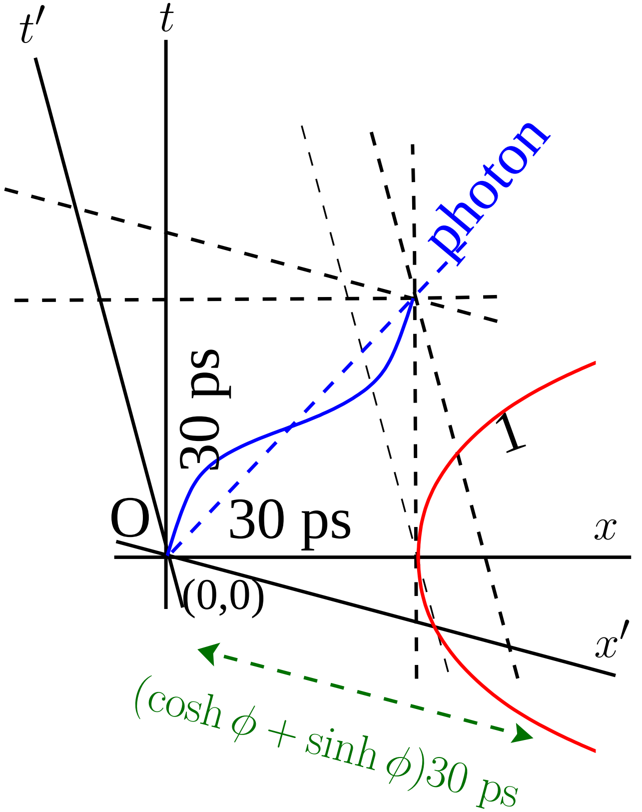
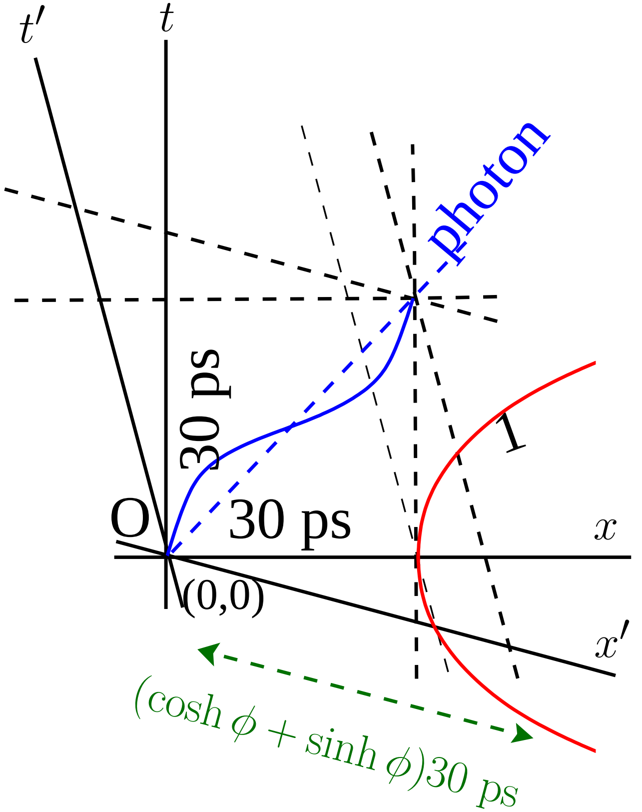


see photon worldline calculation

$$1 + z := \lambda'/\lambda = \gamma(1+\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}}$$ redshift

SR: Doppler shift

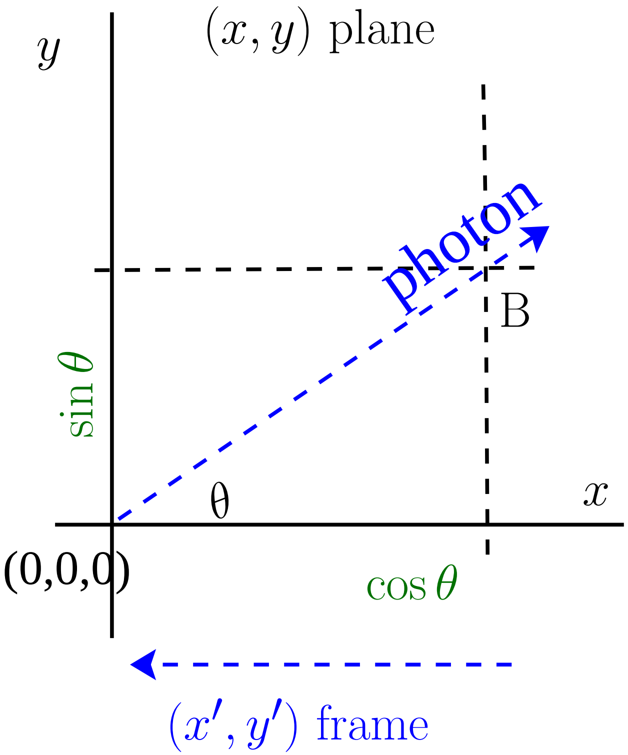


see photon worldline calculation

$1 + z := \lambda'/\lambda = \gamma(1+\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}}$ redshift

$\Rightarrow$ when $\beta \ll 1$, $z \approx \beta$

SR: relativistic aberration



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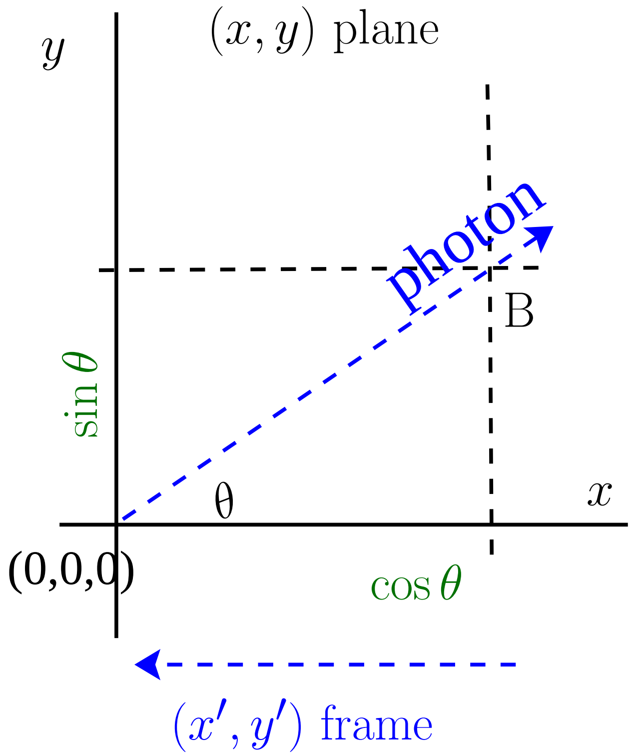

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

SR: relativistic aberration

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$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ 0 & 1 & 0 \\ \beta\gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma\cos\theta + \beta\gamma \\ \sin\theta \\ \beta\gamma\cos\theta + \gamma \end{pmatrix}$$

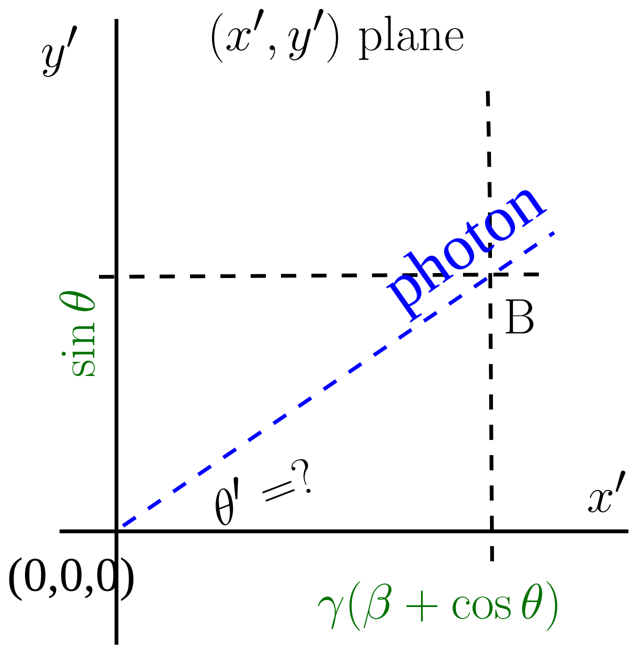
SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma(\cos\theta + \beta) \\ \sin\theta \\ \gamma(1 + \beta\cos\theta) \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$



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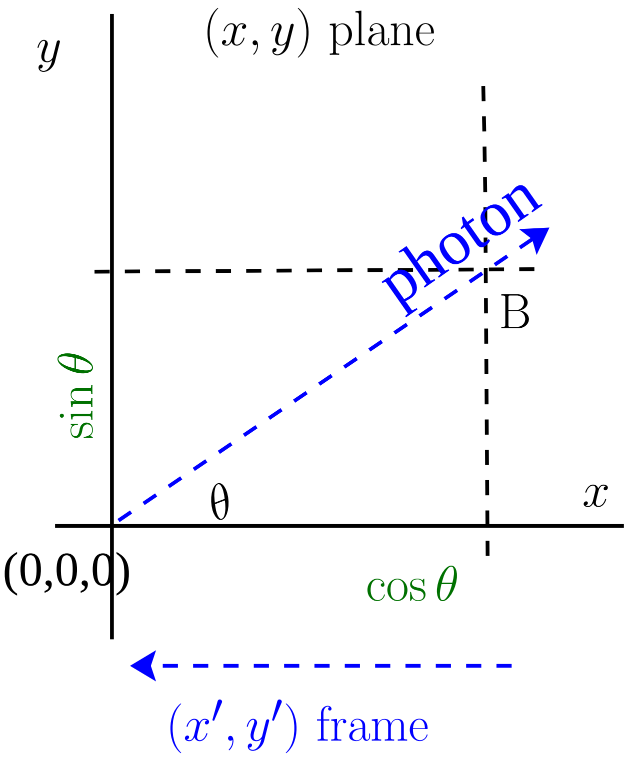

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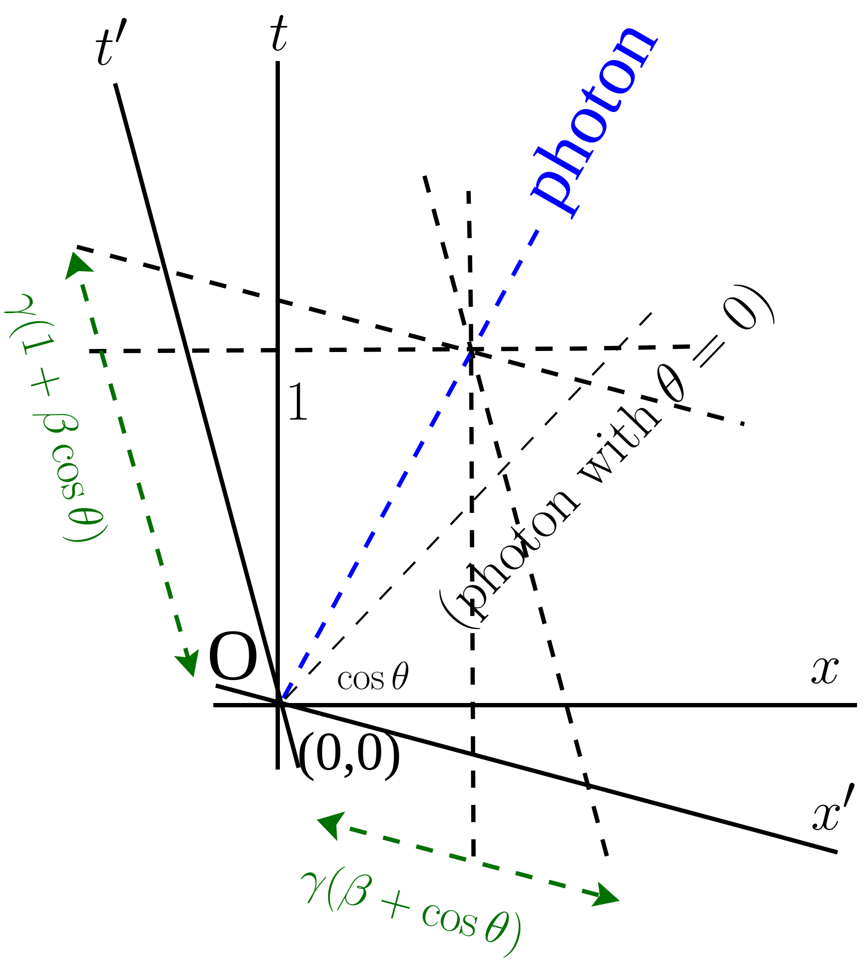

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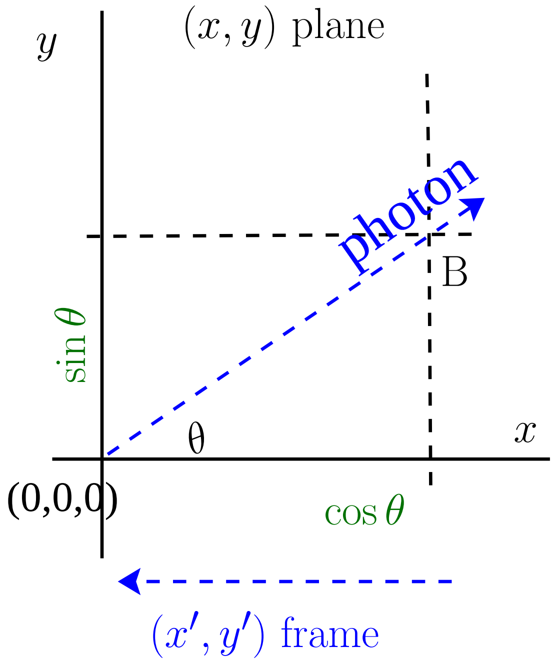
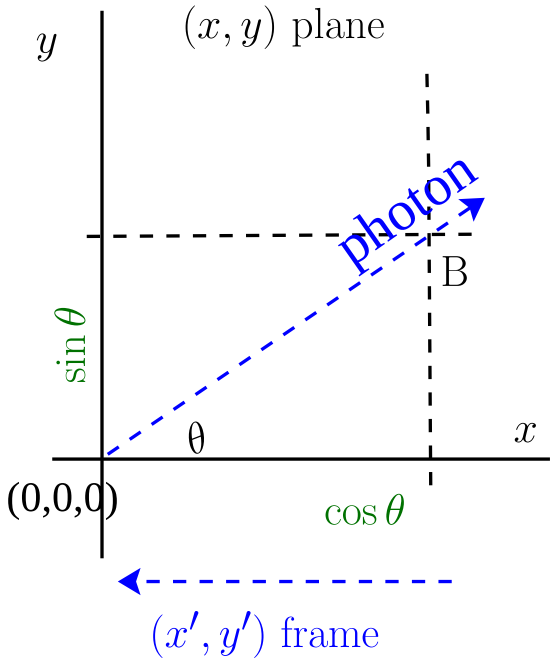


$$\tan\theta' = \frac{\sin\theta}{\gamma(\beta+\cos\theta)}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

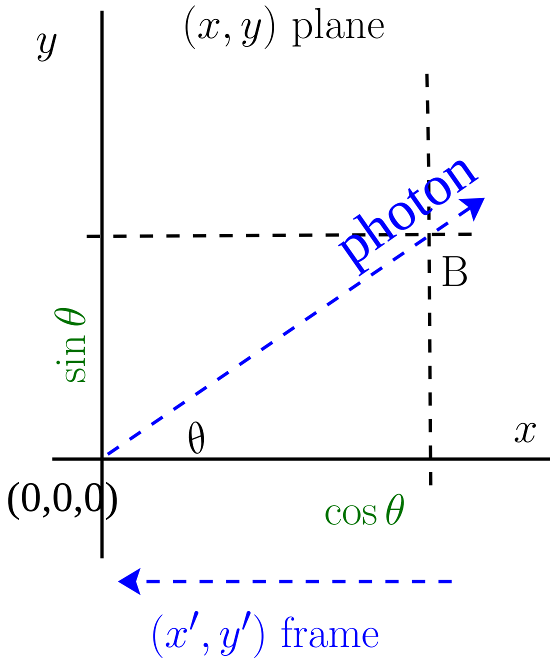


$\tan\theta' = \frac{\sin\theta}{\gamma(\beta+\cos\theta)} < \tan\theta$ if $0 < \beta < 1$

w:Relativistic aberration (2011-02-22: quality=weak)

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

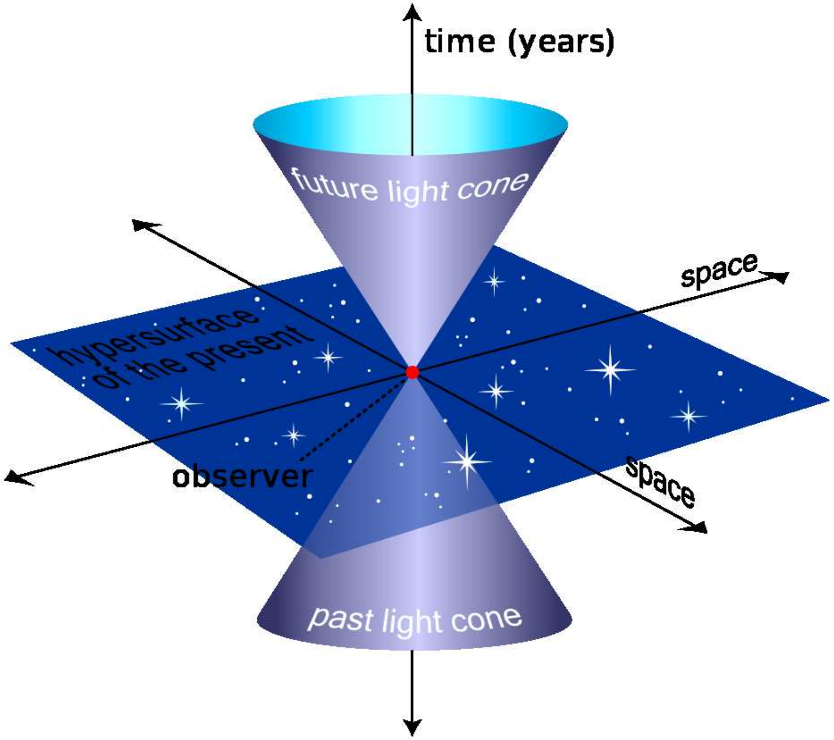


$\tan\theta' = \frac{\sin\theta}{\gamma(\beta+\cos\theta)}$ $< \tan\theta$ if $0 < \beta < 1$

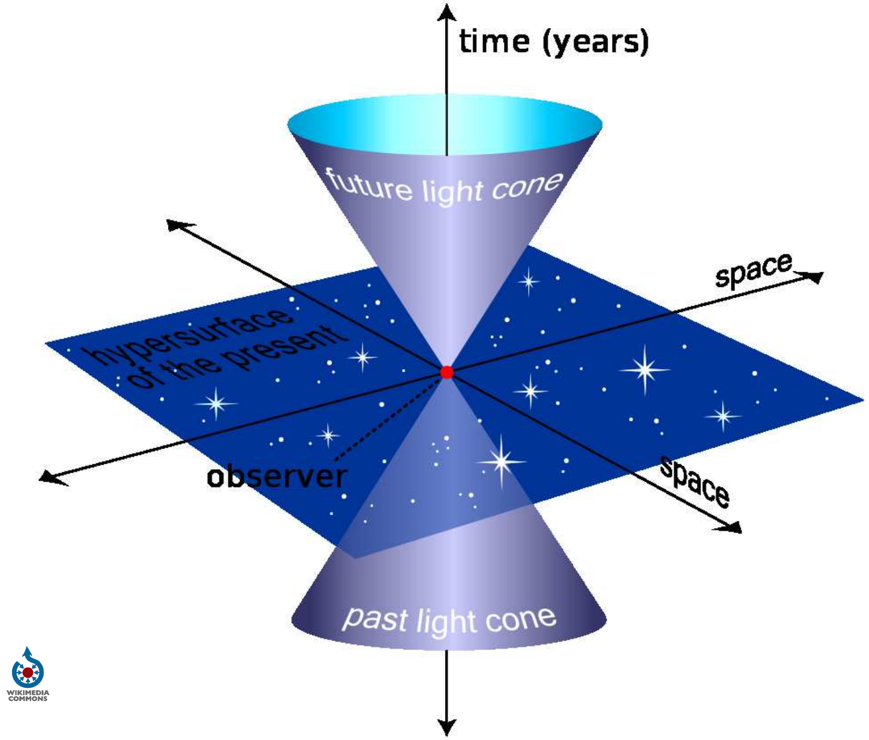
w:Relativistic aberration (2011-02-22: quality=weak)

$\Rightarrow$ relativistic beaming, e.g. AGN jets

SR: world line



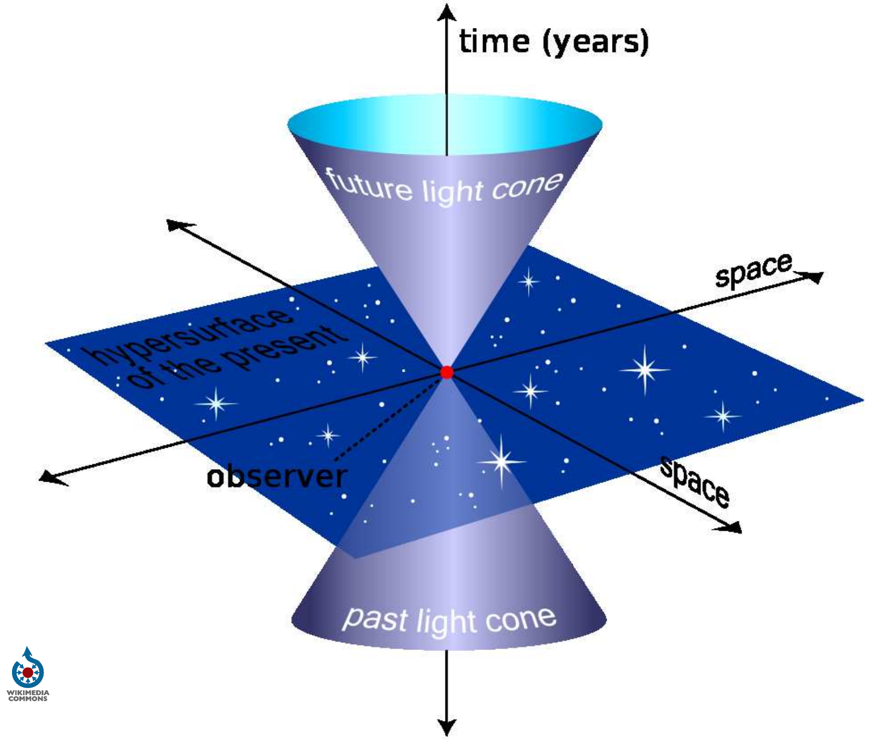
SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime =

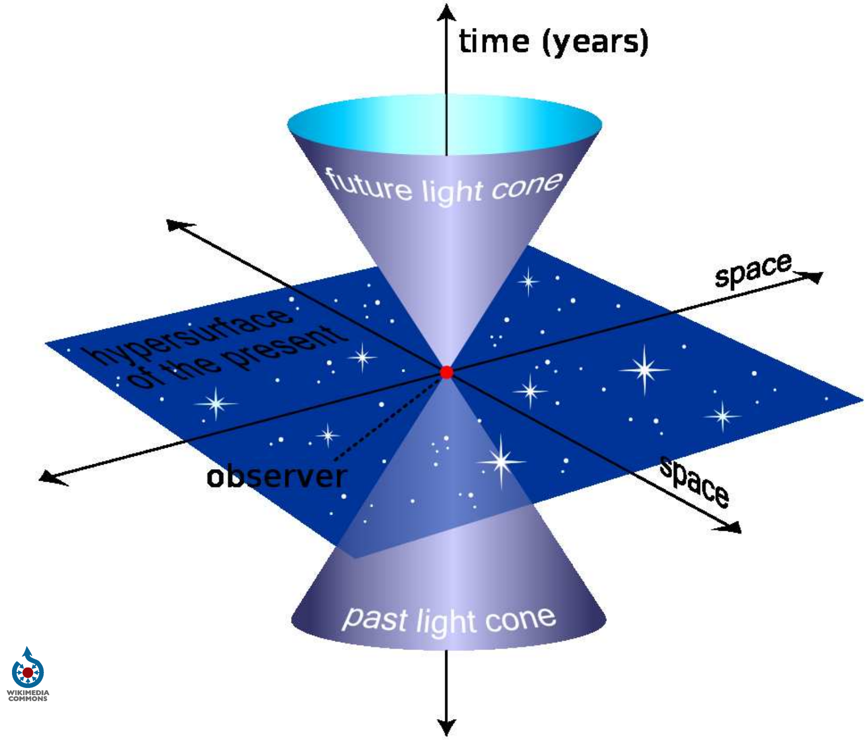
SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone

SR: world line

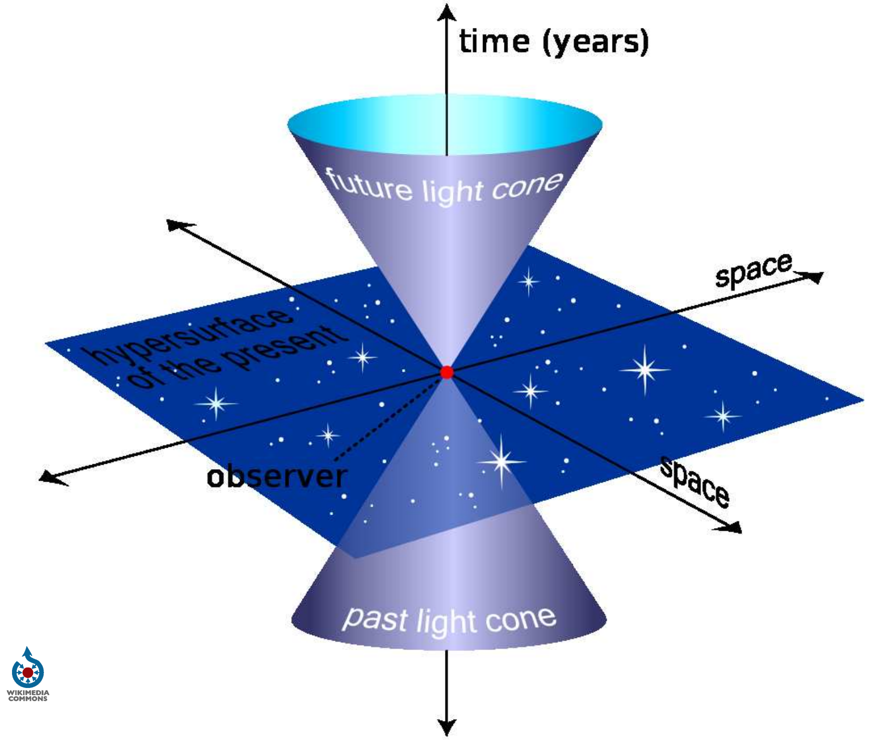


lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone

+ on future light cone + inside future light cone

SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone

+ on future light cone + inside future light cone

+ elsewhere

SR: world line

Lorentz transform of world line

SR: world line

Lorentz transform of world line

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Lorentz transform of world line

- time in spacetime model $\neq$ time in your brain (thinking)

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Lorentz transform of world line

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- $\frac{\mathrm{d}t}{\mathrm{d}t_{\mathrm{thinking}}}$ can be positive or negative

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Lorentz transform of world line

- time in spacetime model $\neq$ time in your brain (thinking)
- $\frac{\mathrm{d}t}{\mathrm{d}\lambda}$ can be positive or negative, λ arbitrary real parameter

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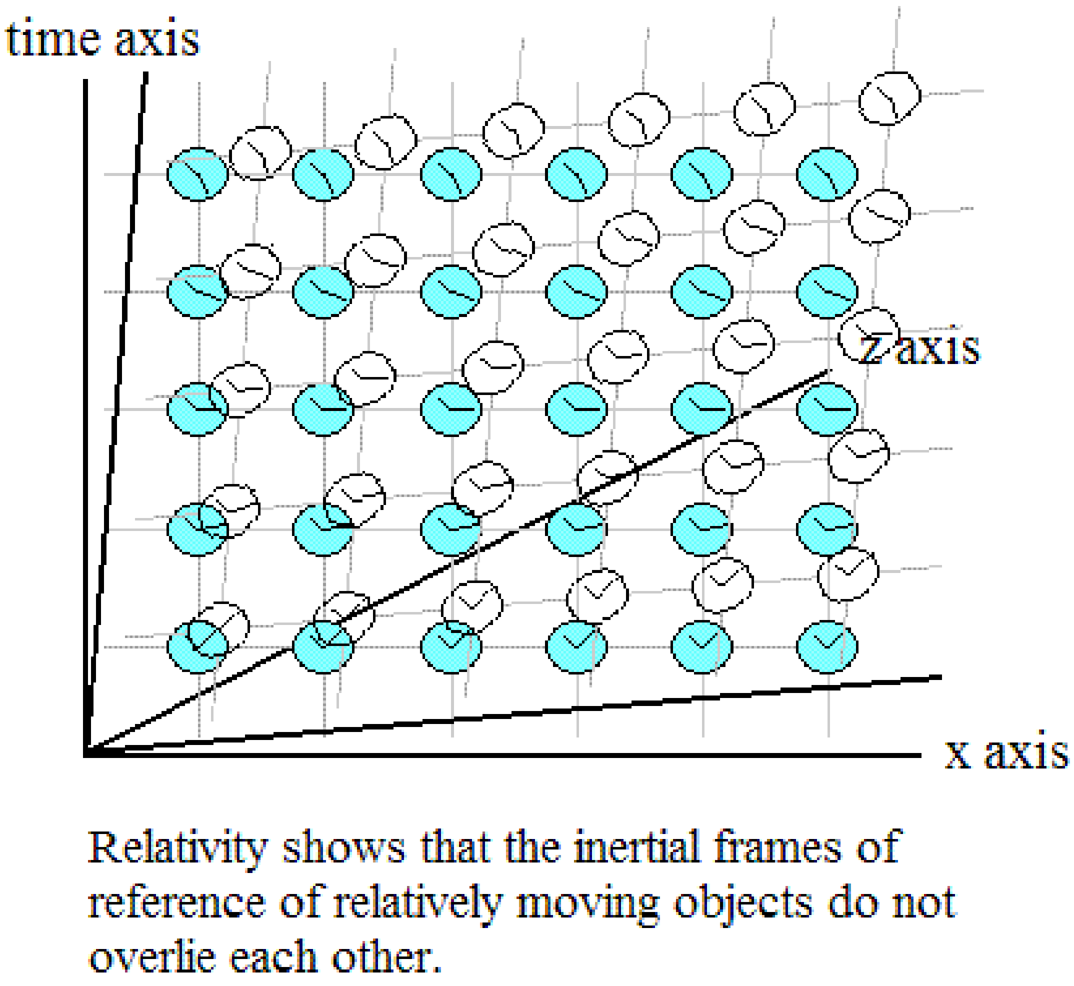
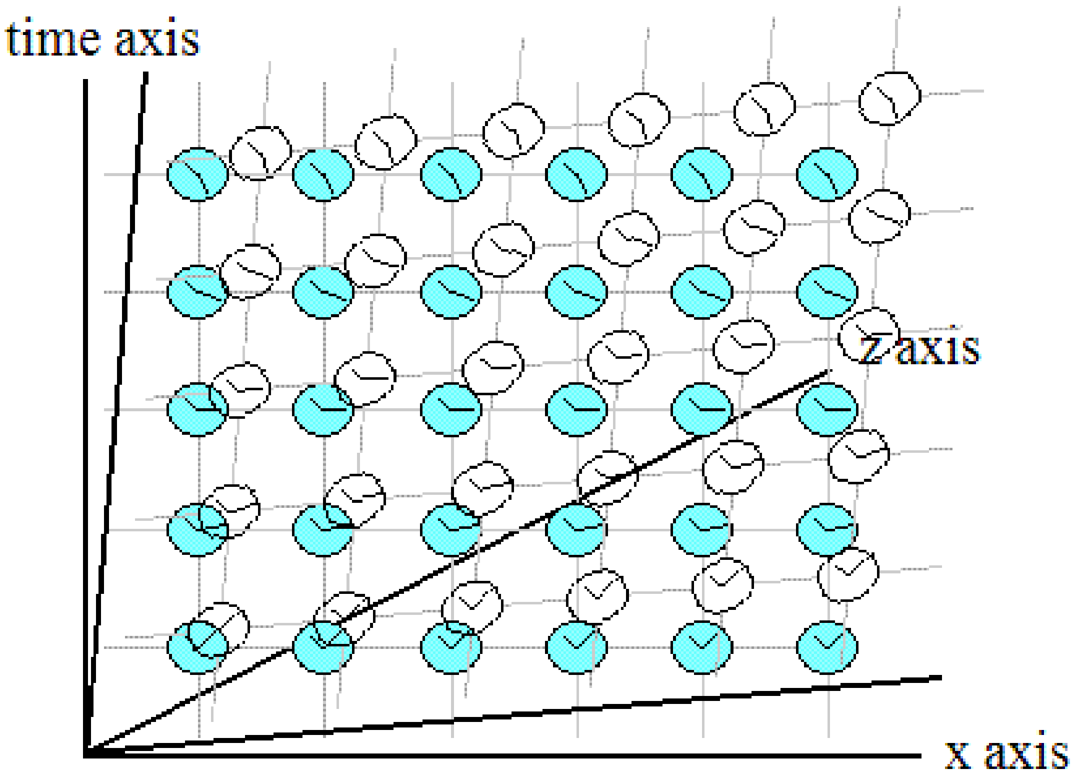
Lorentz transform of world line

- time in spacetime model $\neq$ time in your brain (thinking)
- $\frac{\mathrm{d}t}{\mathrm{d}\lambda}$ can be positive or negative, λ arbitrary real parameter
- “elsewhere” spacetime events can change from past to future even though $\frac{\mathrm{d}t}{\mathrm{d}\lambda} > 0$

SR: Rietdijk–Putnam–Penrose p.



SR: Rietdijk–Putnam–Penrose p.

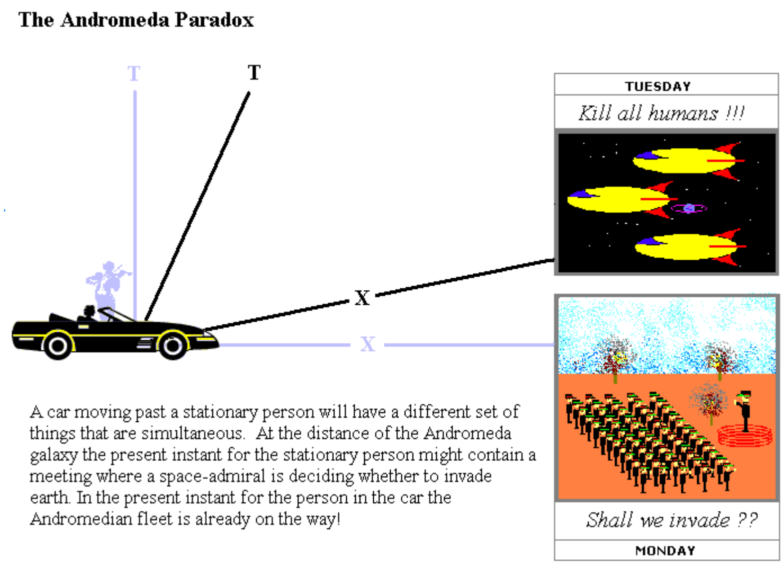


Relativity shows that the inertial frames of reference of relatively moving objects do not overlie each other.

b:Inertialoverlay.GIF

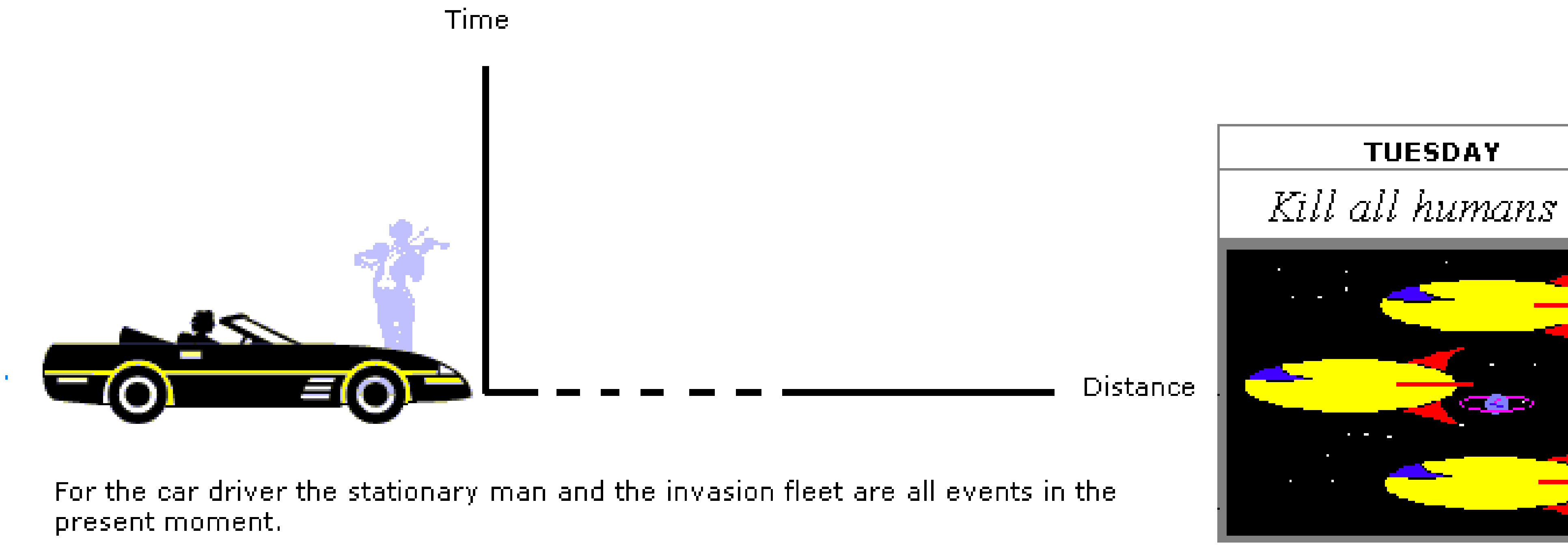
- each observer can synchronise a set of clocks and rigid rods

SR: Rietdijk–Putnam–Penrose p.



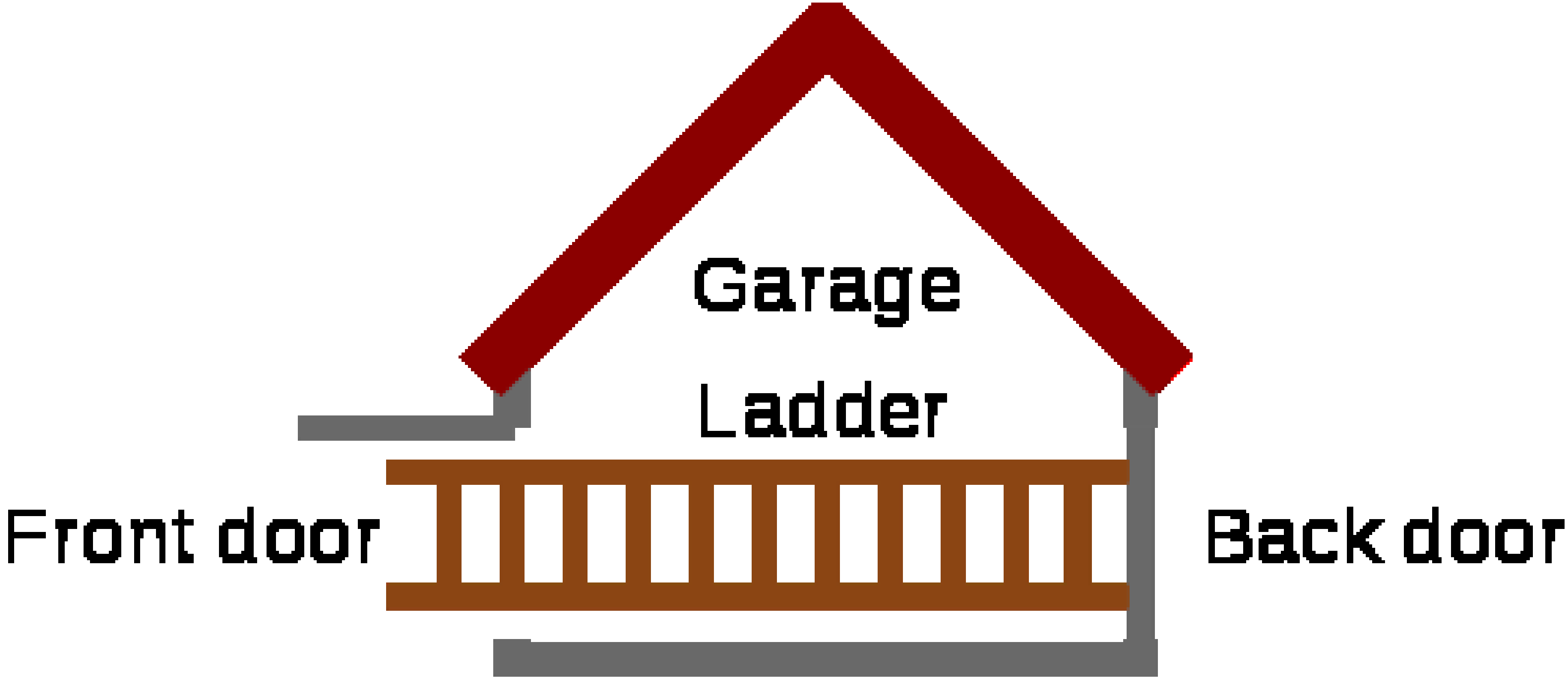
w:Rietdijk-Putnam argument b:Rel2.gif

SR: Rietdijk–Putnam–Penrose p.

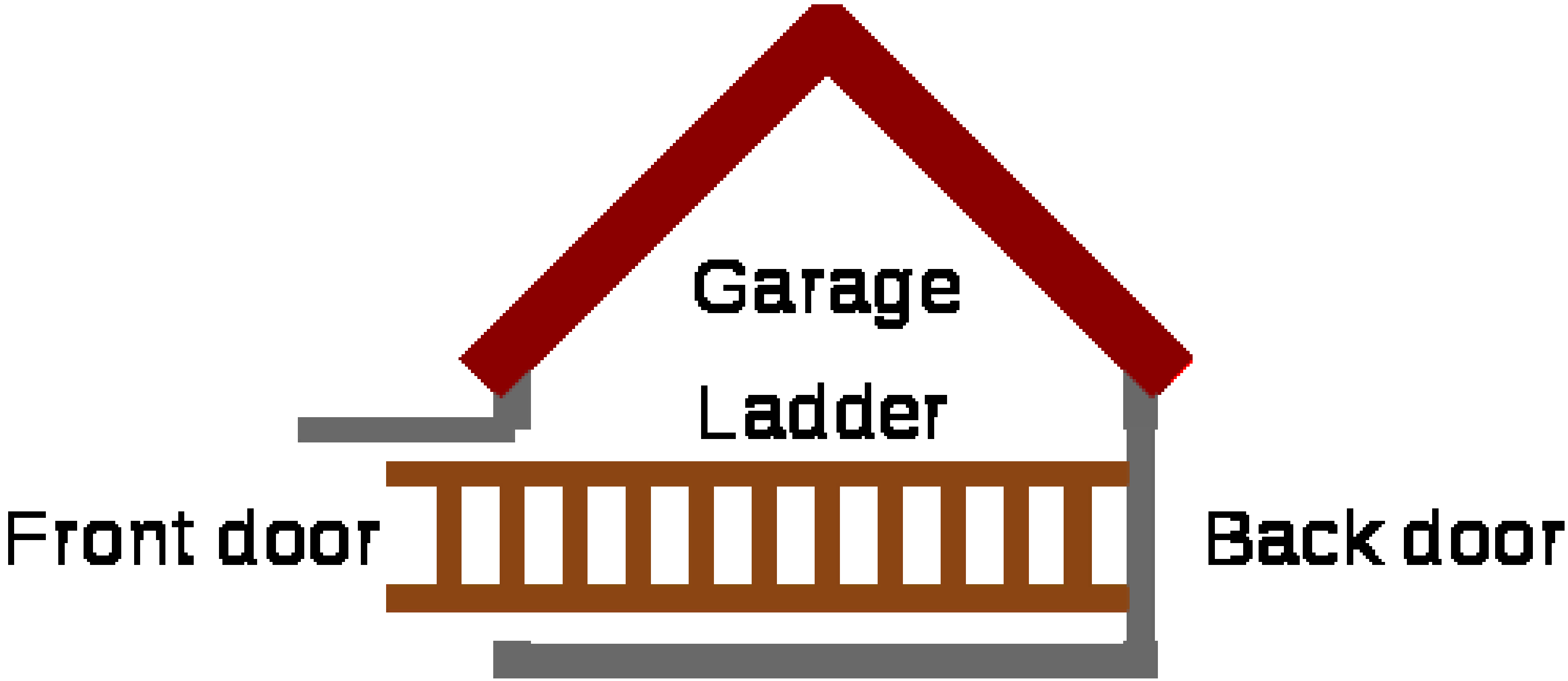


w:Rietdijk-Putnam argument b:Rel3.gif

SR: pole-barn/ladder paradox

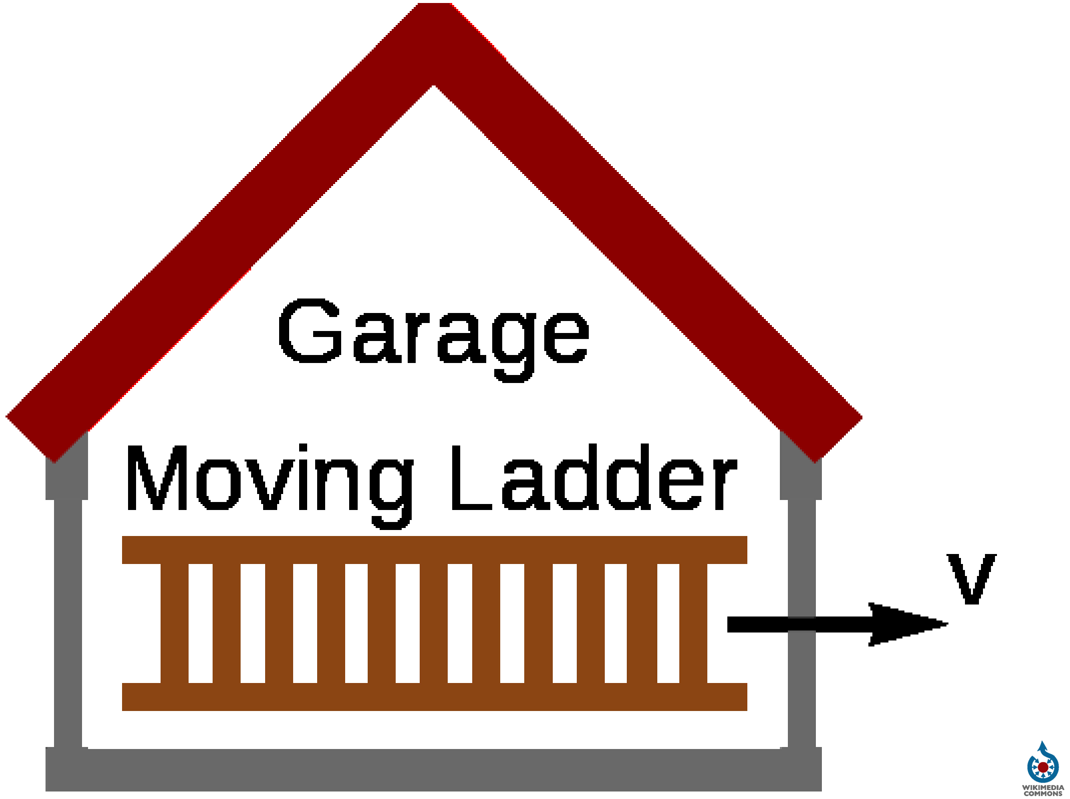


SR: pole-barn/ladder paradox



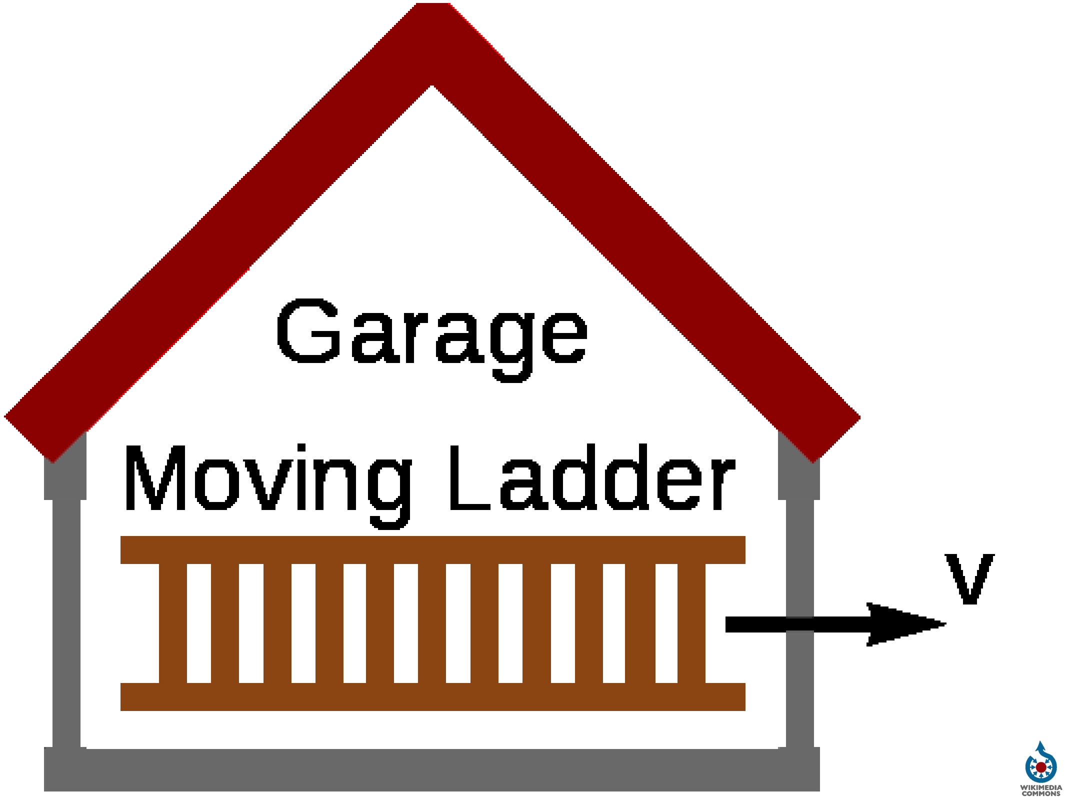
- ladder of length 29.9γ ns, garage length 30 ns

SR: pole-barn/ladder paradox



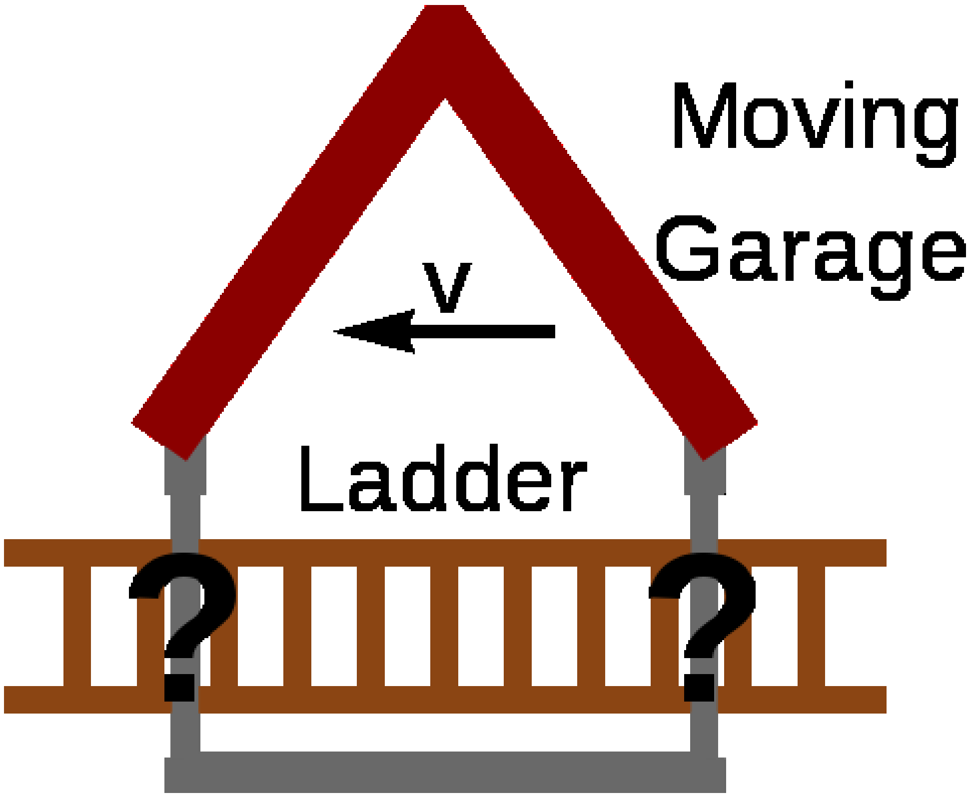
- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors

SR: pole-barn/ladder paradox



- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- 29.9γ ns $/\gamma < 30$ ns $\Rightarrow$ OK

SR: pole-barn/ladder paradox



- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- ladder frame: garage $30/\gamma$ ns long $\ll 29.9\gamma$ ns!! OK or not OK?

SR: four-velocity, ...

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GR:

\+ w:Riemannian geometry (and pseudo-Riemannian geometry)

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+ w:Intermediate treatment of tensors

GR: maxima

+ maxima - component tensor packet ctensor

GR: maxima

+ maxima - component tensor packet ctensor

GR: maxima

+ maxima - component tensor packet ctensor

GR: maxima

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+ w:Einstein field equations

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GR: an approximation method: ADM

+ w:ADM formalism

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GR: a numerical method: cactus

+ Cactus - http://cactuscode.org

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