Special and General Relativity

Boud Roukema

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SR+GR

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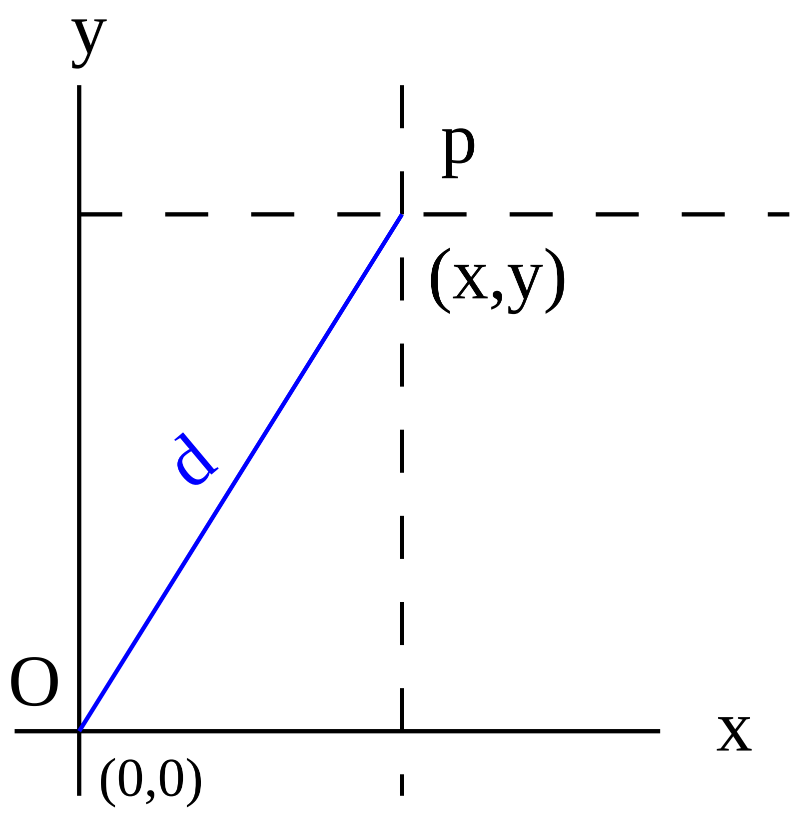
- GR: spacetime = a solution of the w:Einstein field equations

SR: Minkowski spacetime



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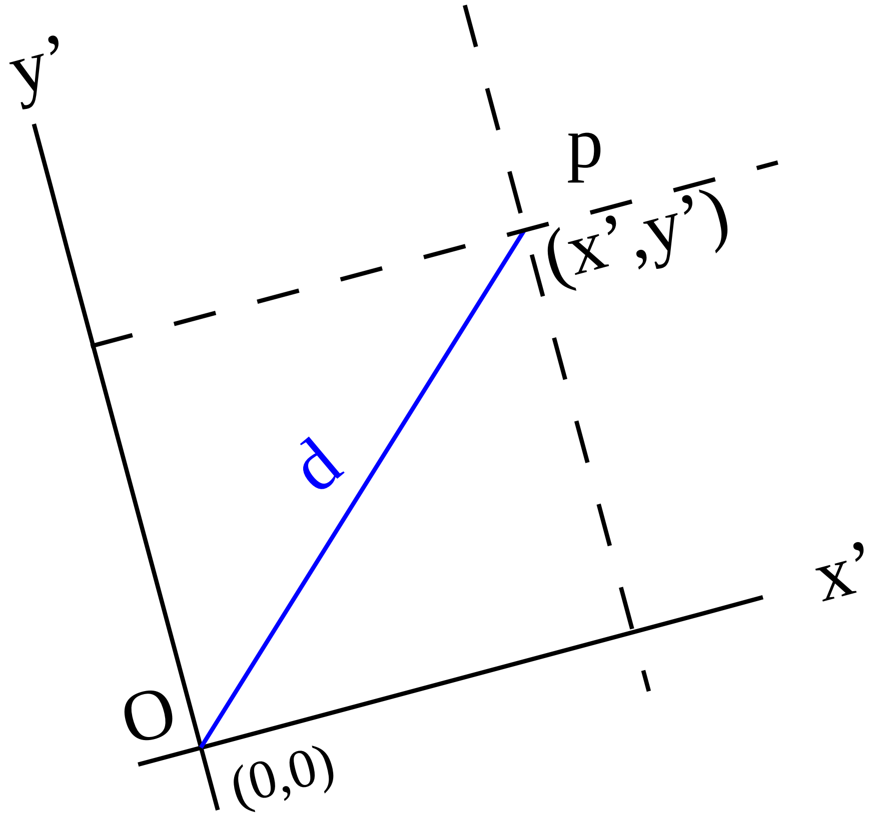

$$\begin{pmatrix} p_{x'} \\ p_{y'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

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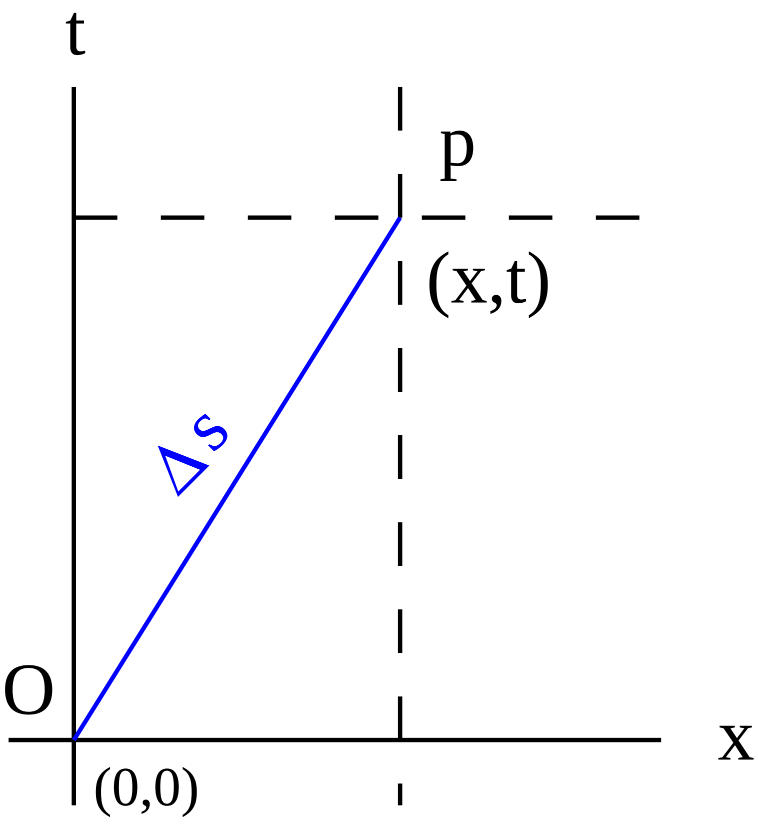

p at (x,t), w:invariant interval from observer at O is Δs where $(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$

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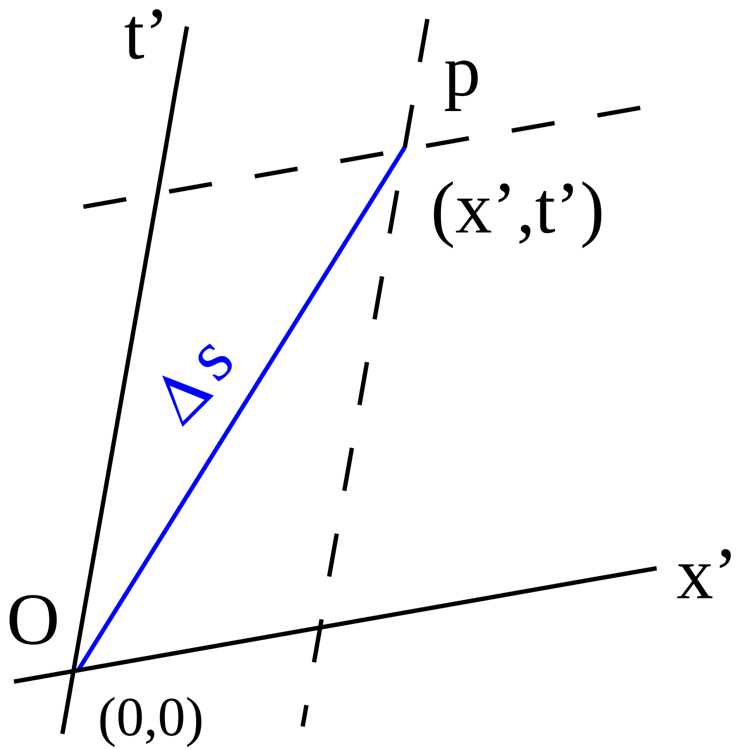

$$\begin{pmatrix} p_{x'} \\ p_{t'} \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} p_x \\ p_t \end{pmatrix}$$

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SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

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$= \frac{1 \text{ s}}{1 \text{ s}} = 1$ (dimensionless)

SR: rapidity ϕ vs velocity β

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in A's coordinate system, B's worldline is:

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$$\Rightarrow x = t' \sinh \phi = t \tanh \phi = \beta t$$

where velocity $\beta := v/c \equiv v = \tanh \phi$

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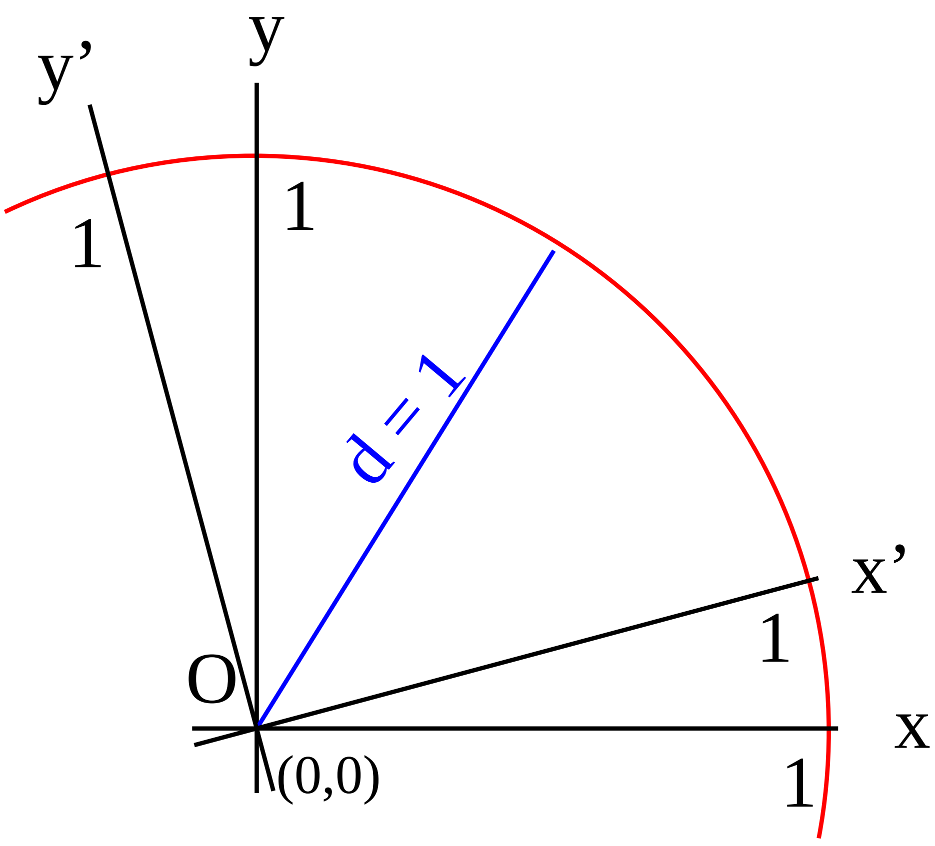

constant distance $d = 1$ from $(0, 0)$

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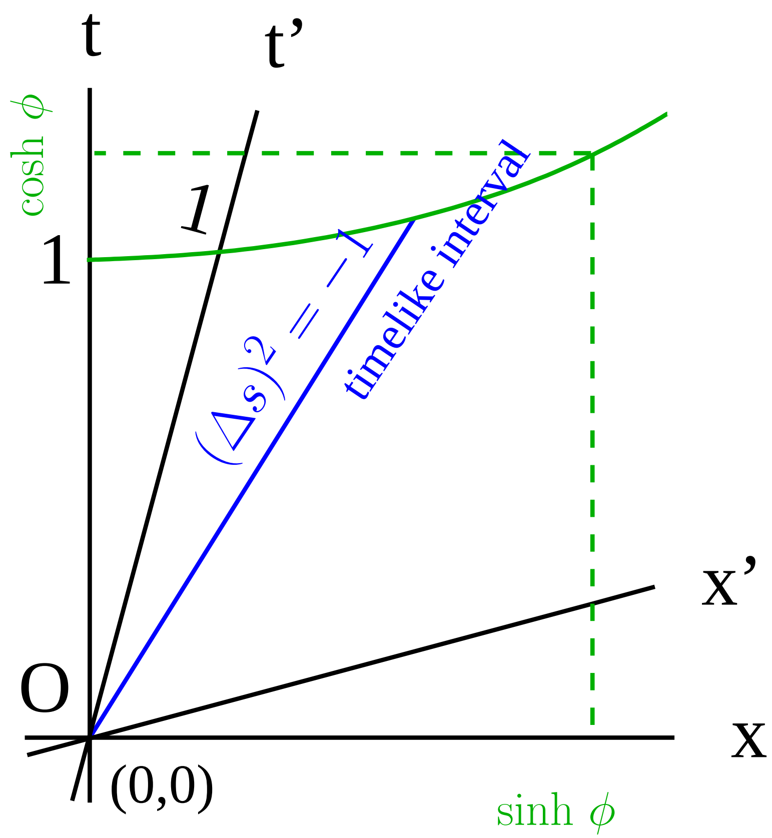

constant interval
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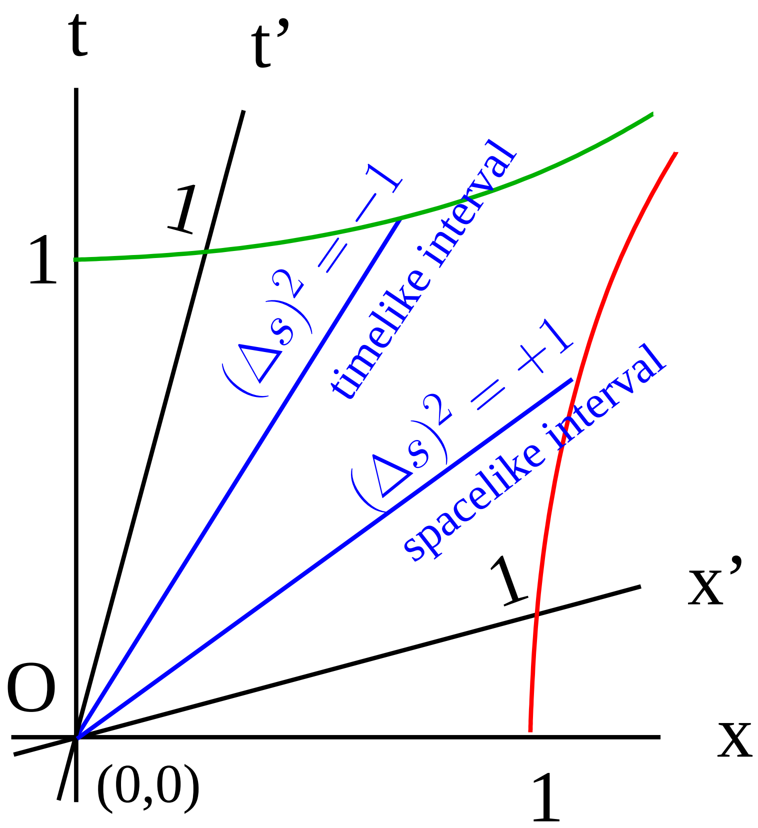

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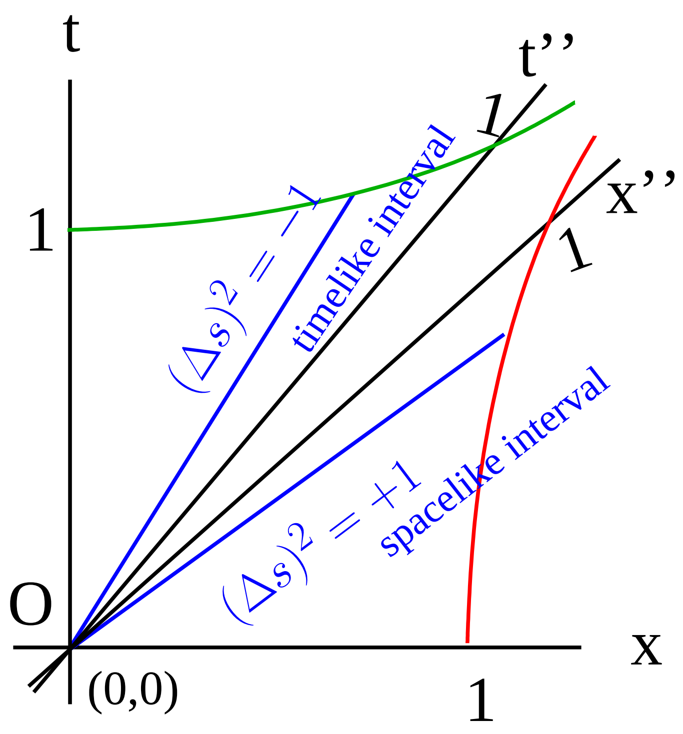

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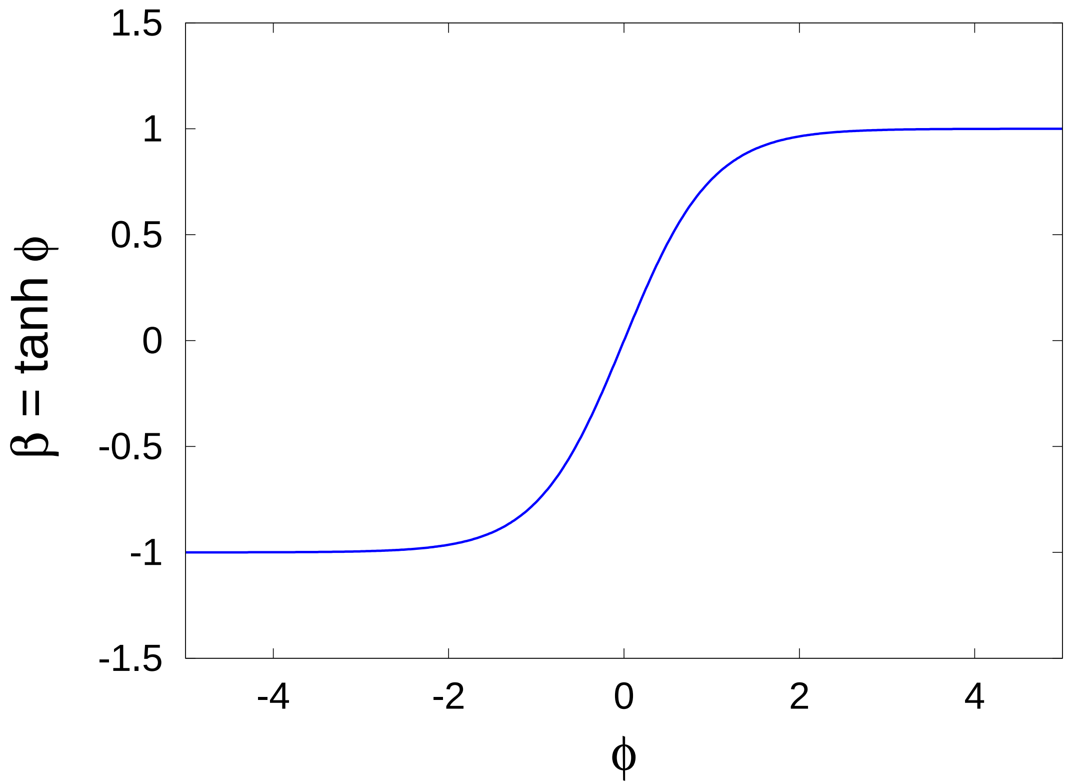

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- w:Michelson-Morley experiment (1887)

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interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?

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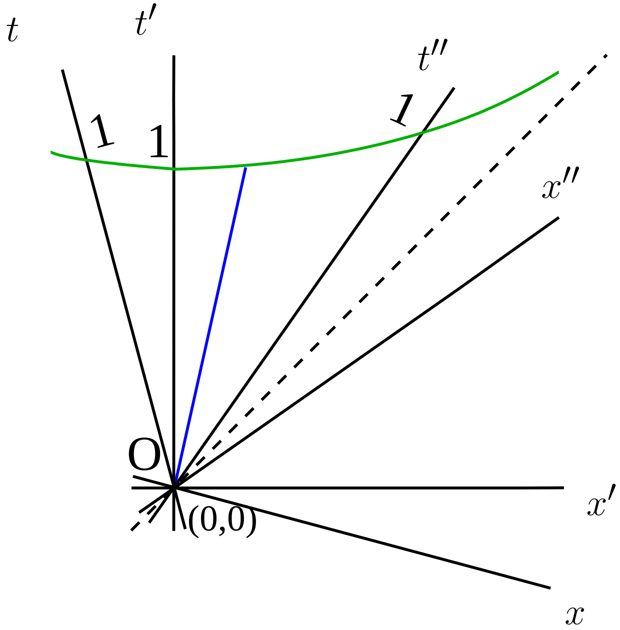

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where $\tanh\phi_1 = \beta_1 = 0.1$

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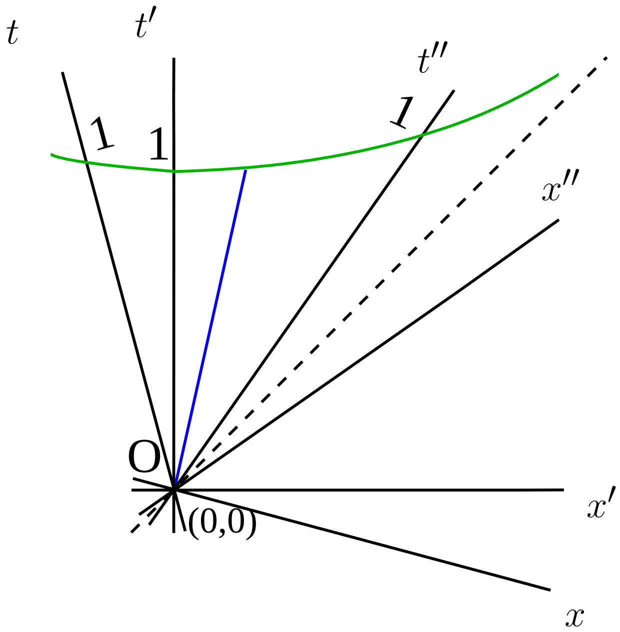

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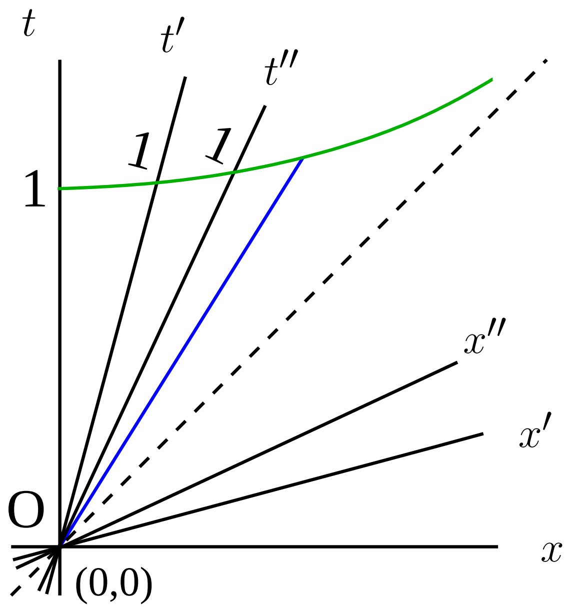

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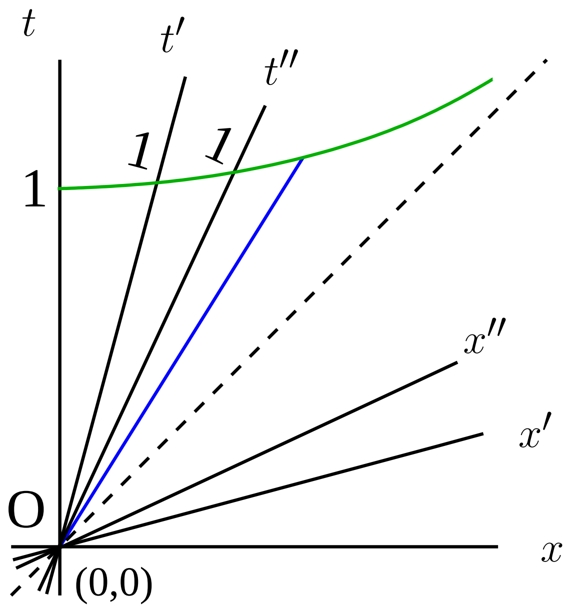

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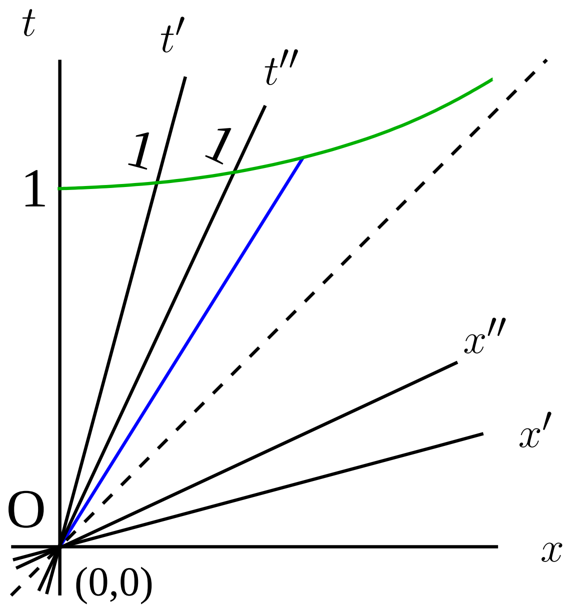

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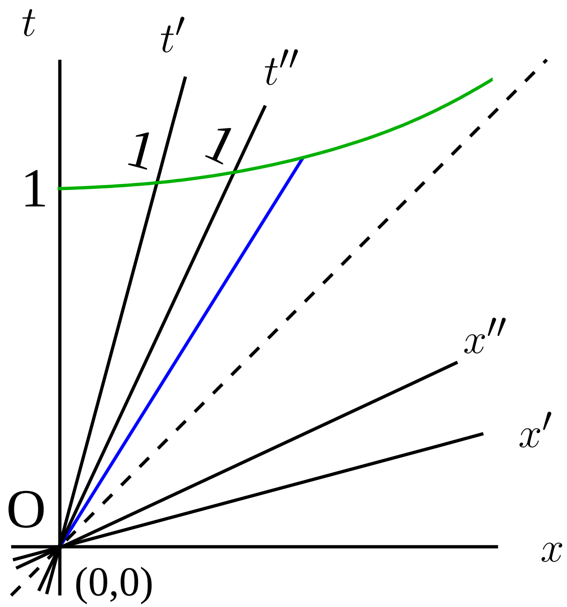

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cf. rotation θ_1 "plus" rotation θ_2 = rotation $\theta_1 + \theta_2$

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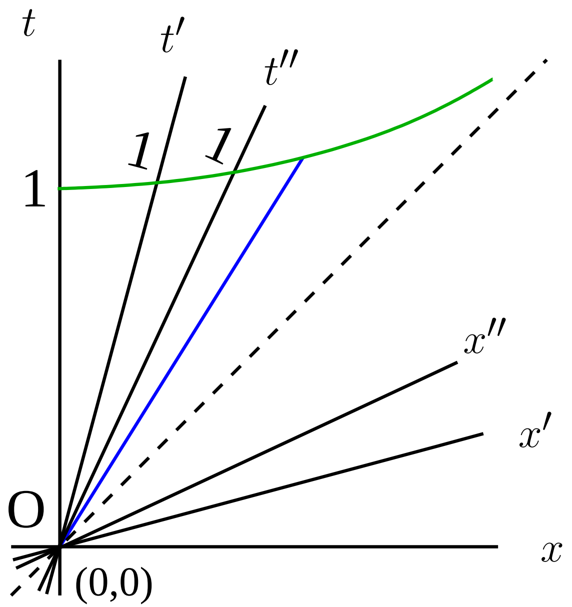

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so $\beta_3 = \tanh(\phi_1 + \phi_2)$

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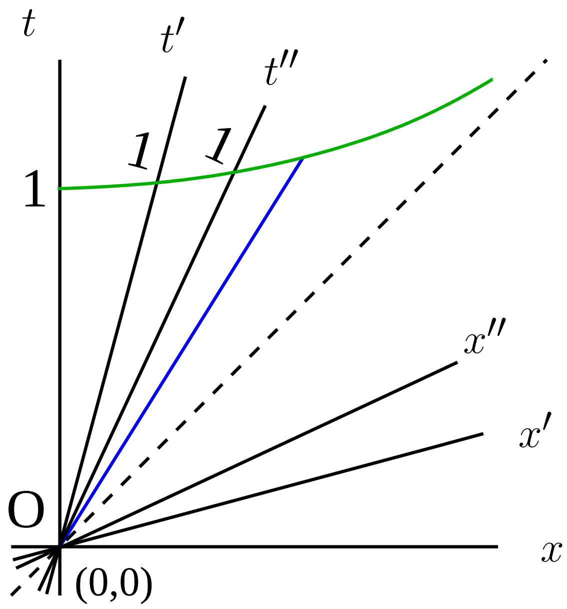

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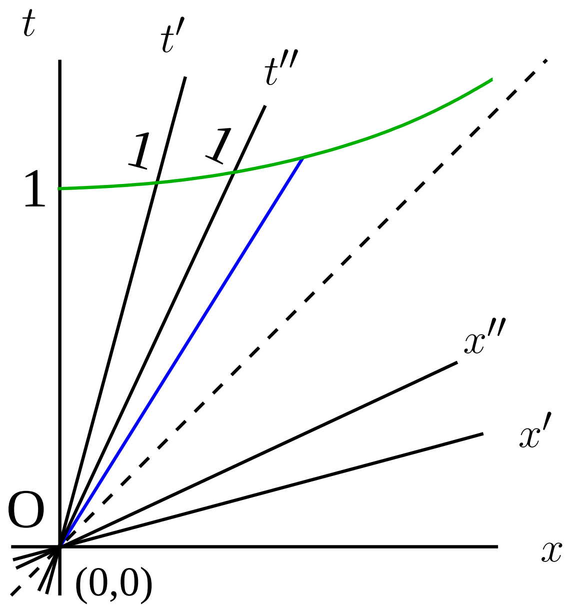
so $\beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\tanh\phi_1 + \tanh\phi_2}{1 + \tanh\phi_1 \tanh\phi_2}$

SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



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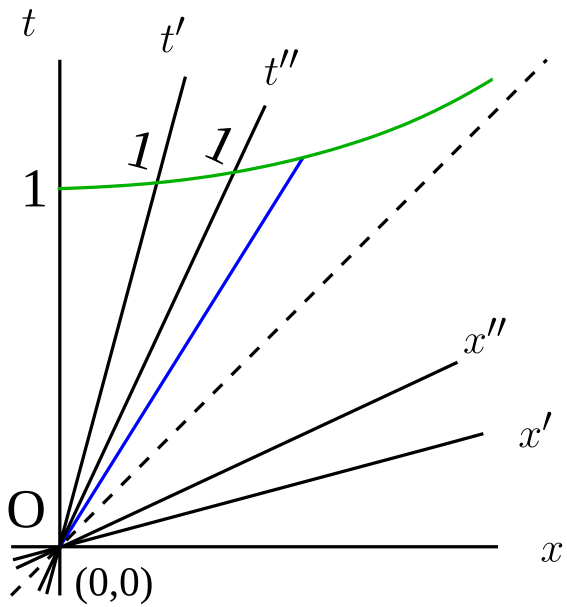

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so $\beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2} = \frac{0.1+0.5}{1+0.1\times0.5} \approx 0.57$

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

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$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

$$\cosh\phi := \frac{e^{\phi}+e^{-\phi}}{2}$$

$$\sinh\phi := \frac{e^{\phi}-e^{-\phi}}{2}$$

w:hyperbolic function

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

$$\Lambda(\beta) := \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}$$

$\beta = \tanh\phi$

$\gamma := (1 - \beta^2)^{-1/2}$ = Lorentz factor

$\gamma = \cosh\phi$

$\beta\gamma = \sinh\phi$

SR: model summary

Minkowski spacetime: draw a correct diagram

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Lorentz transformation (boost) $\Lambda(\phi)$ or $\Lambda(\beta)$

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Minkowski spacetime: draw a correct diagram

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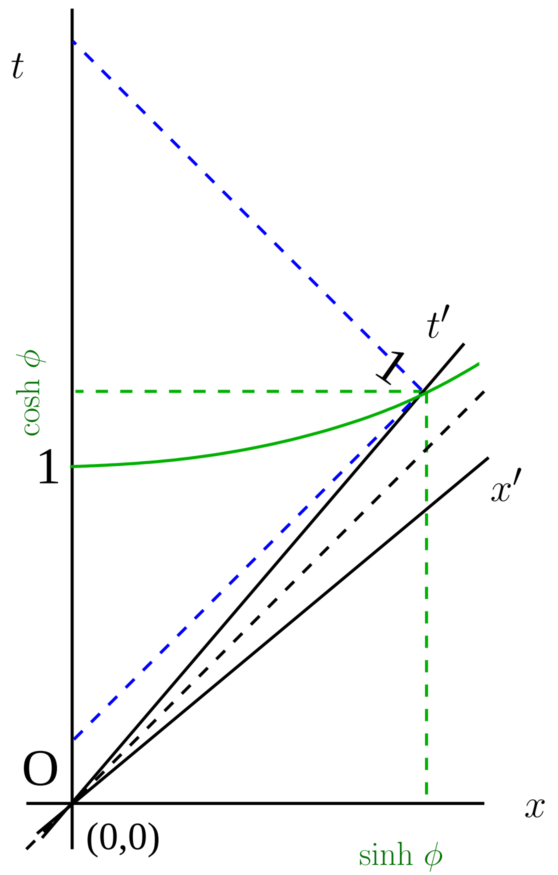
refuse the assumption of absolute simultaneity (time)

SR: worldline time dilation



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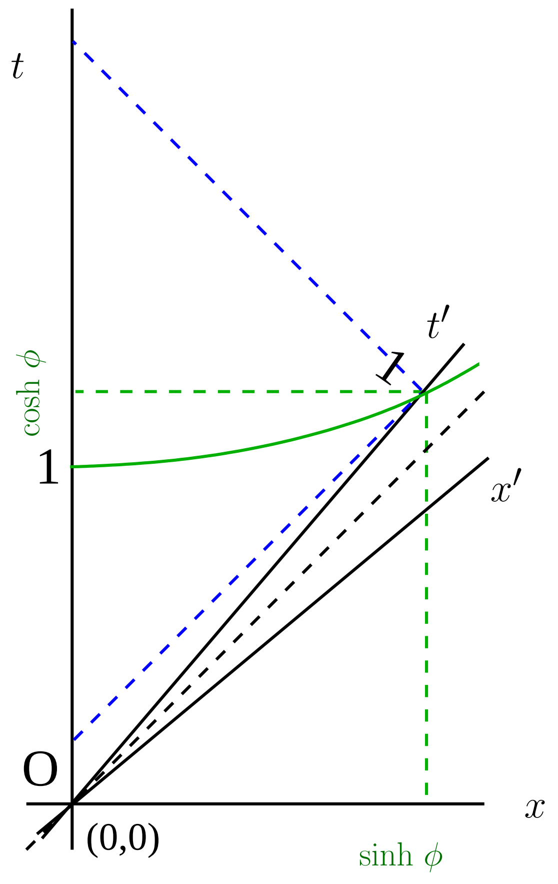
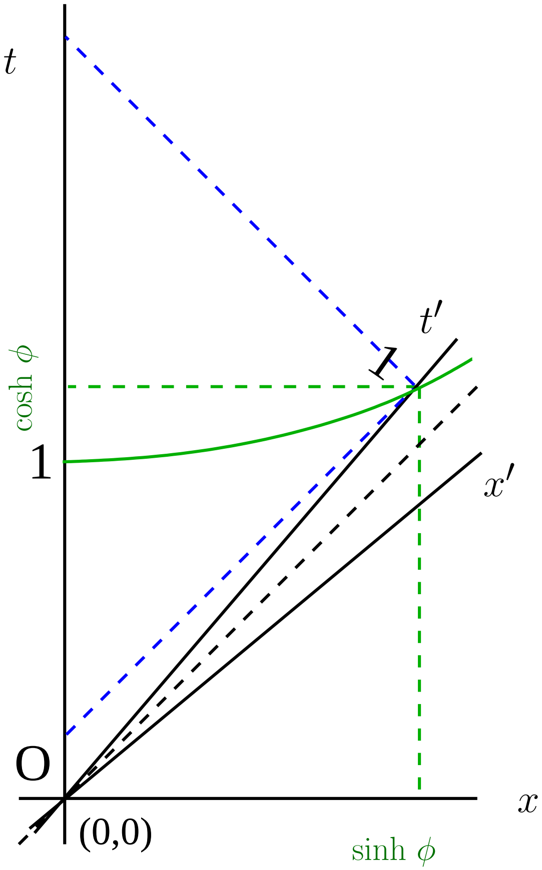


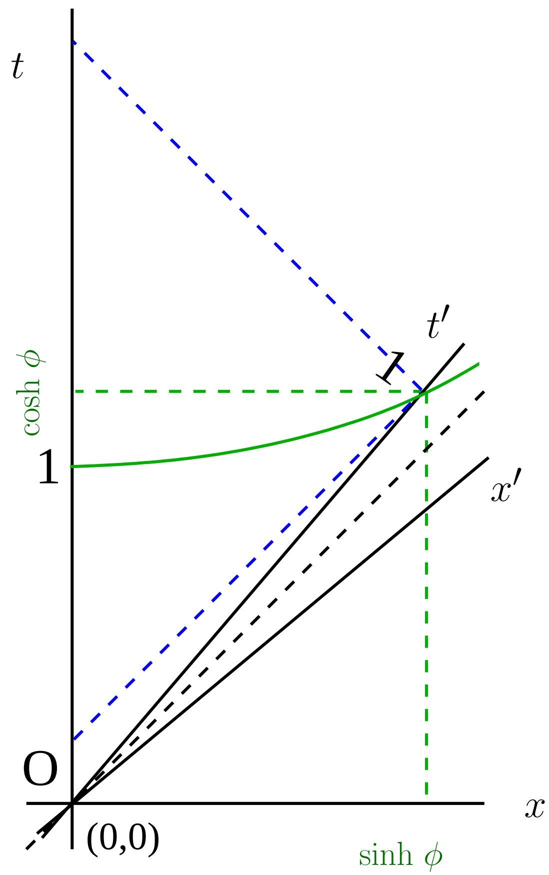
$$\cosh \phi \equiv \gamma$$

SR: worldline time dilation



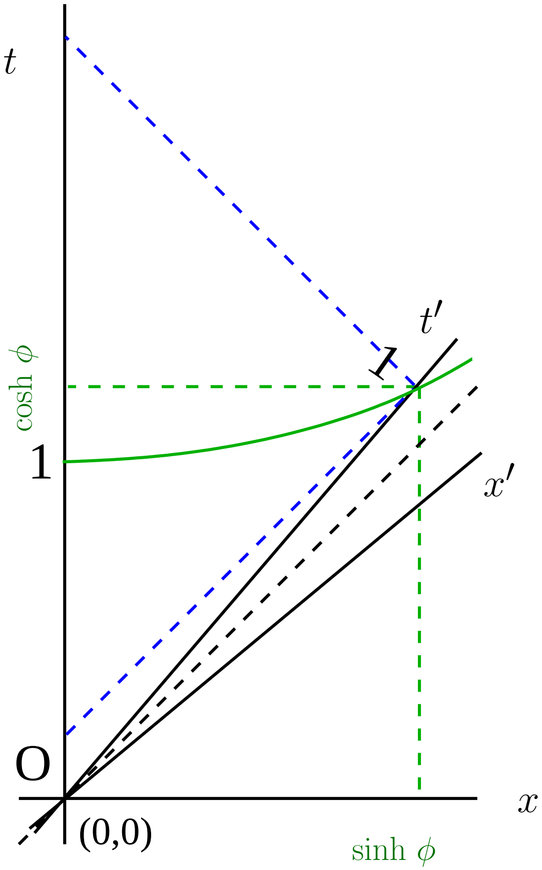
$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

SR: worldline time dilation



$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

SR: worldline time dilation



worldline “time dilation”

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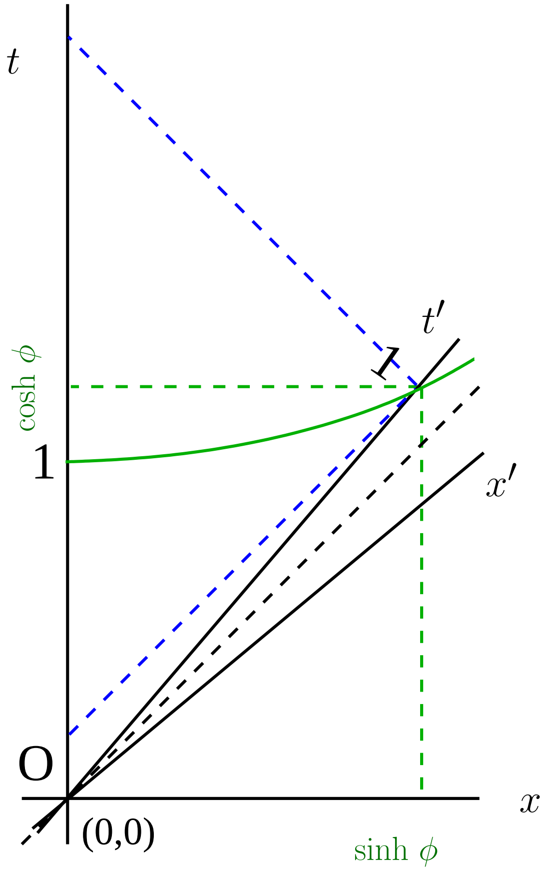

worldline “time dilation”

muons: mean lifetime 2197 ns $\ll 15$ km

$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

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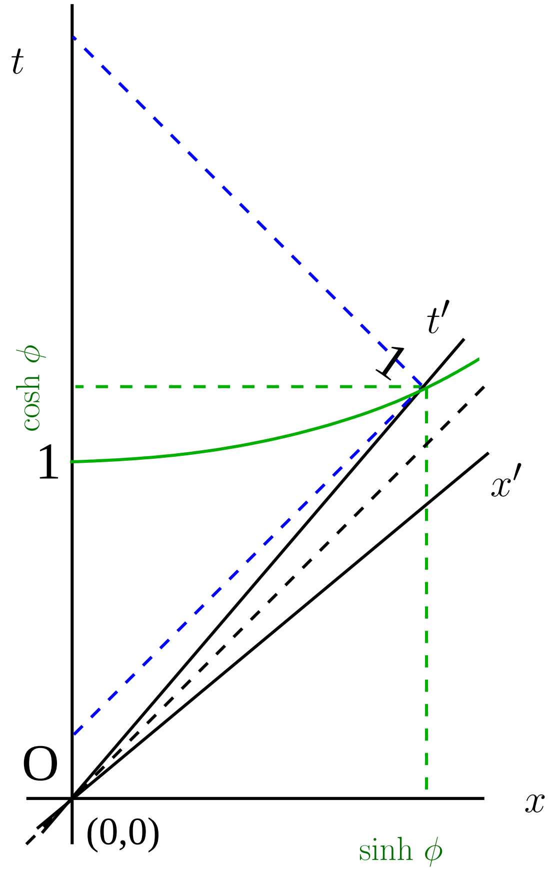


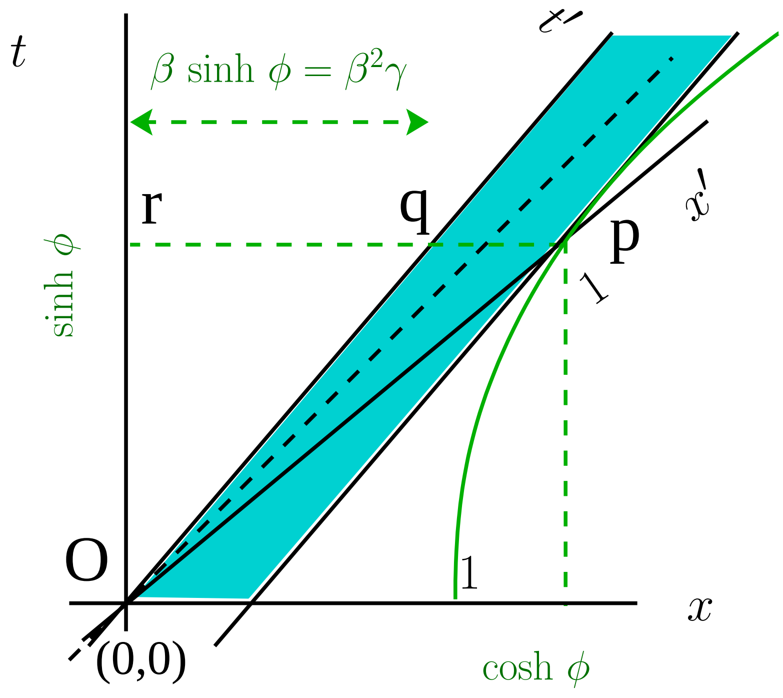
$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

worldline “time dilation”

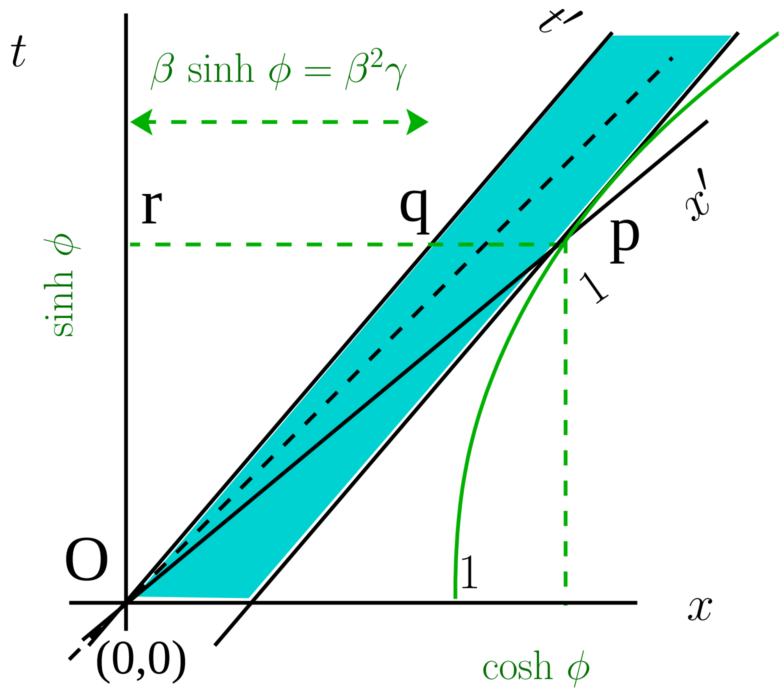
muons: mean lifetime 2197 ns $\ll 15$ km

time dilation $\Rightarrow$ muons can hit the ground

SR: worldsheet space contraction

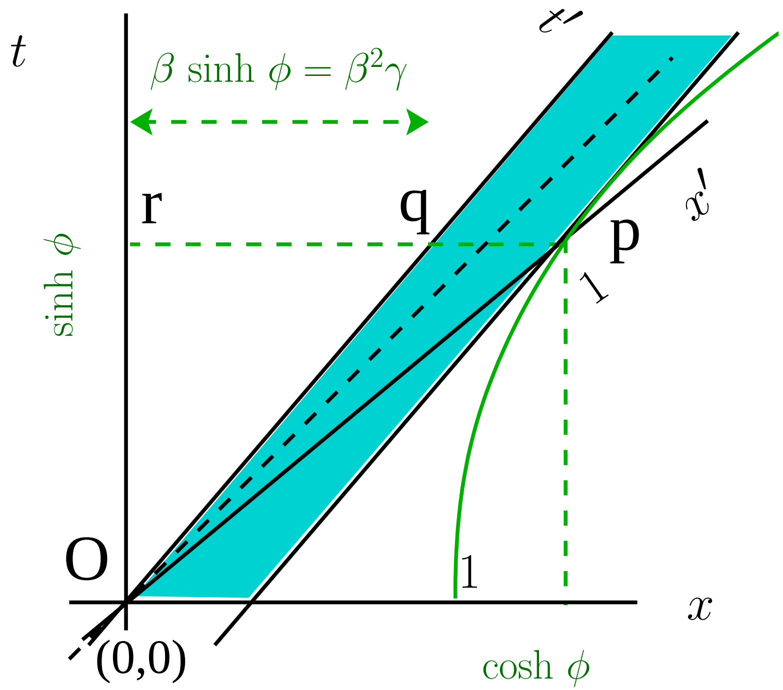


SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} =$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \cosh\phi - \beta\sinh\phi$$

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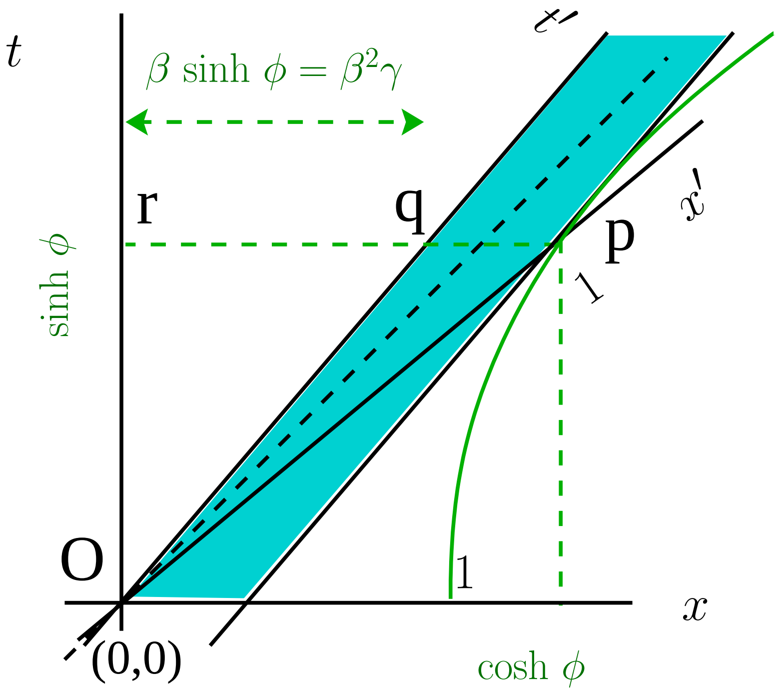

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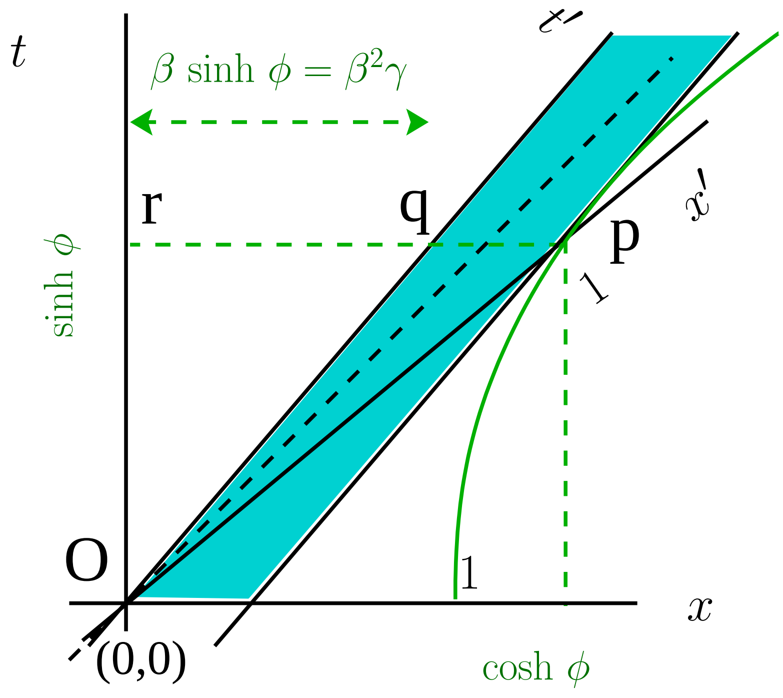
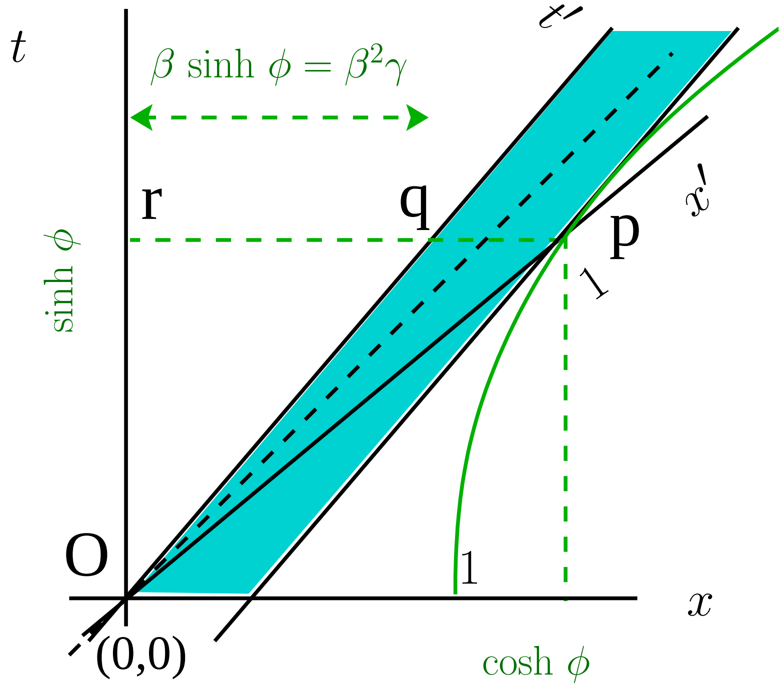
SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)\gamma$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)^{1+(-1/2)}$$

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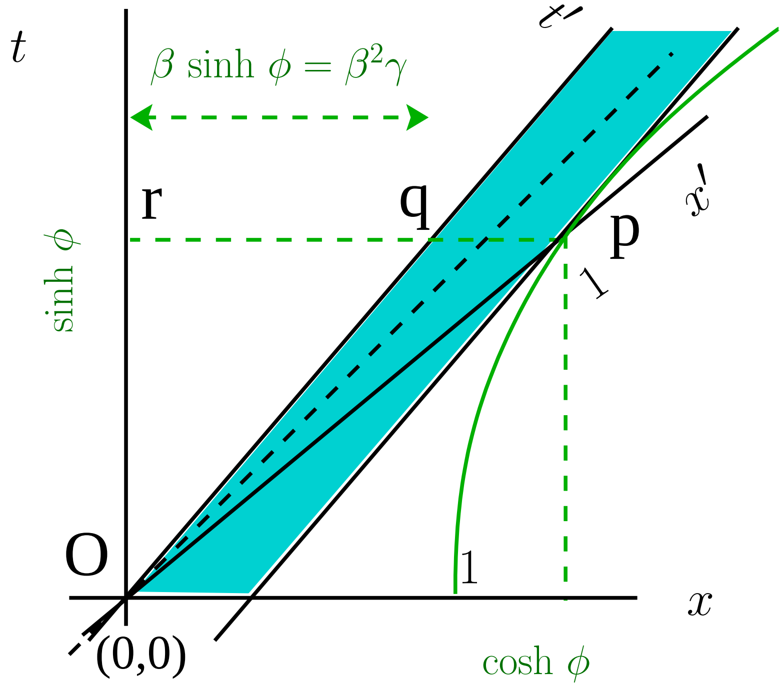
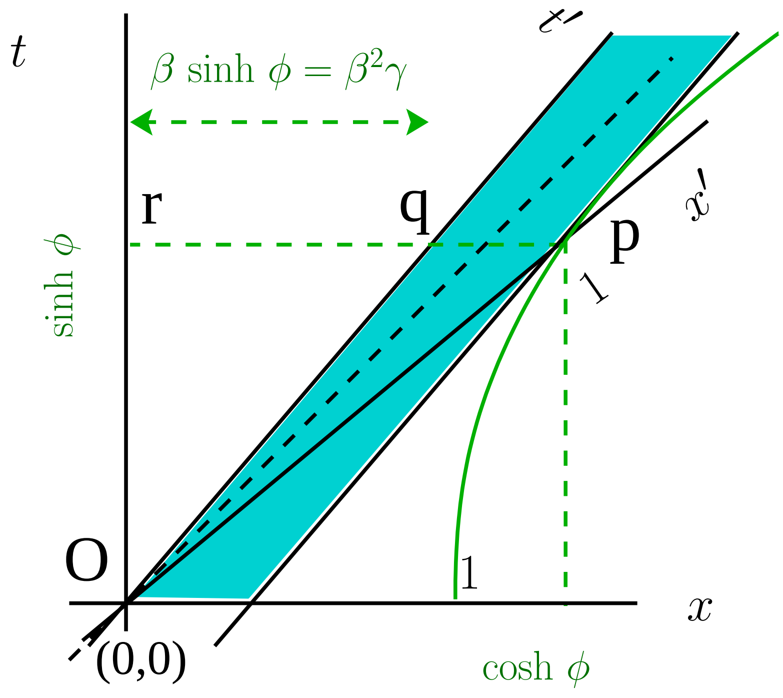


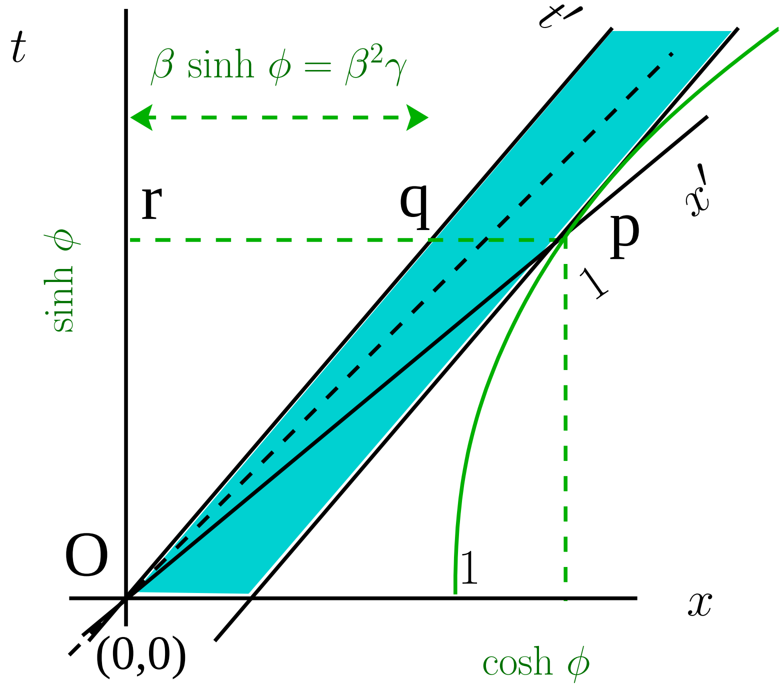
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)^{1/2}$$

SR: worldsheet space contraction



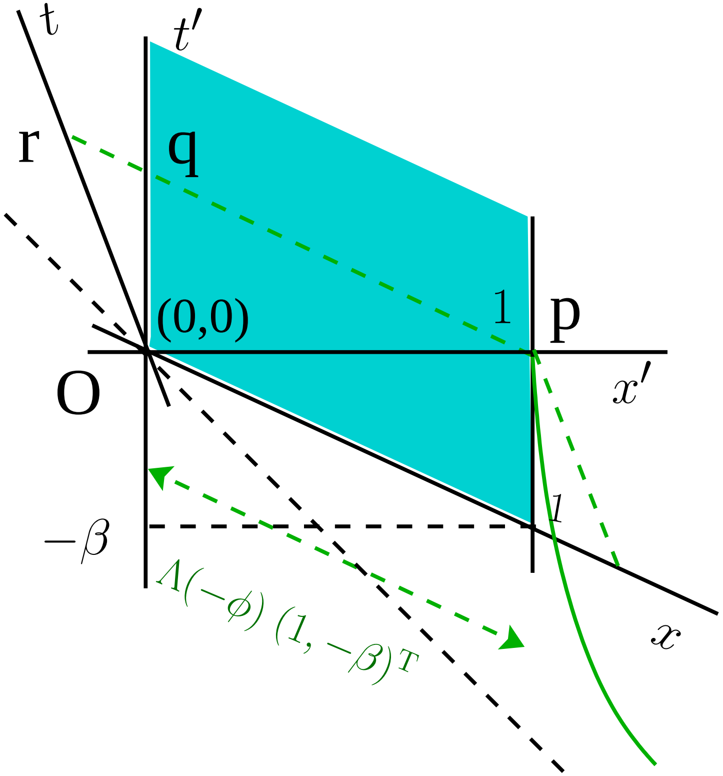
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$ worldsheet “space contraction”

SR: worldsheet space contraction



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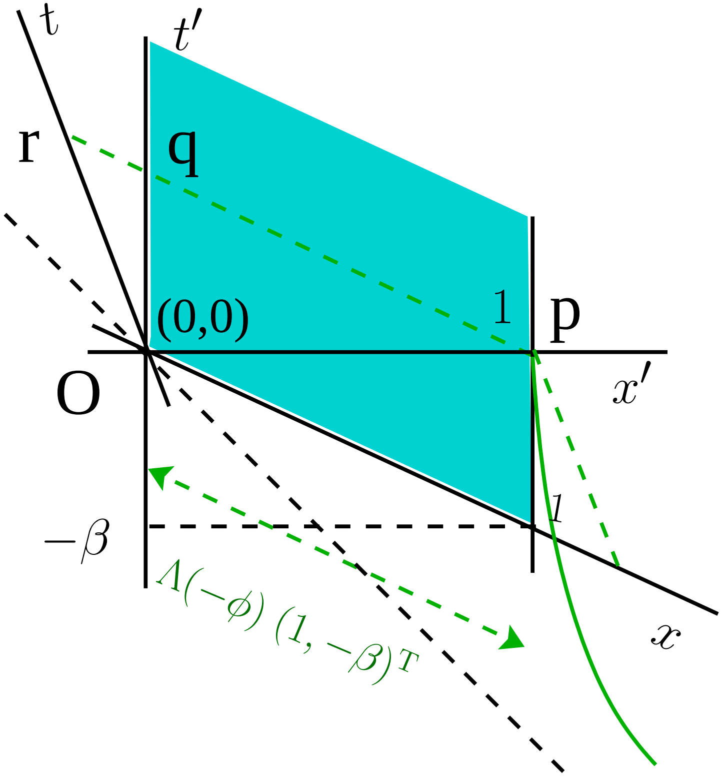
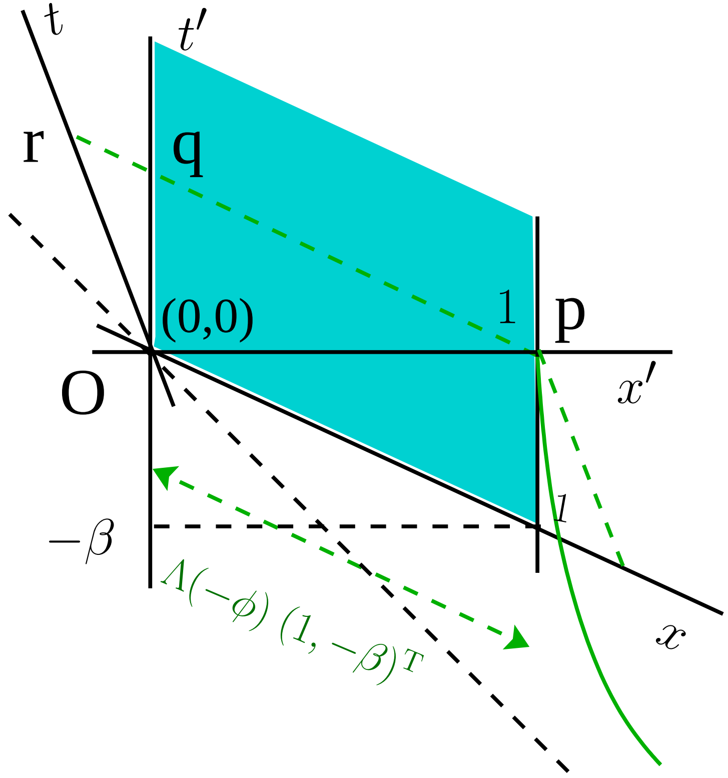


$$\Lambda^{-1} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \cosh\phi - \beta\sinh\phi \\ \sinh\phi - \beta\cosh\phi \end{pmatrix}$$

SR: worldsheet space contraction



$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\cosh\phi(1-\beta^2)\\\cosh\phi(\tanh\phi-\tanh\phi)\end{pmatrix}$$

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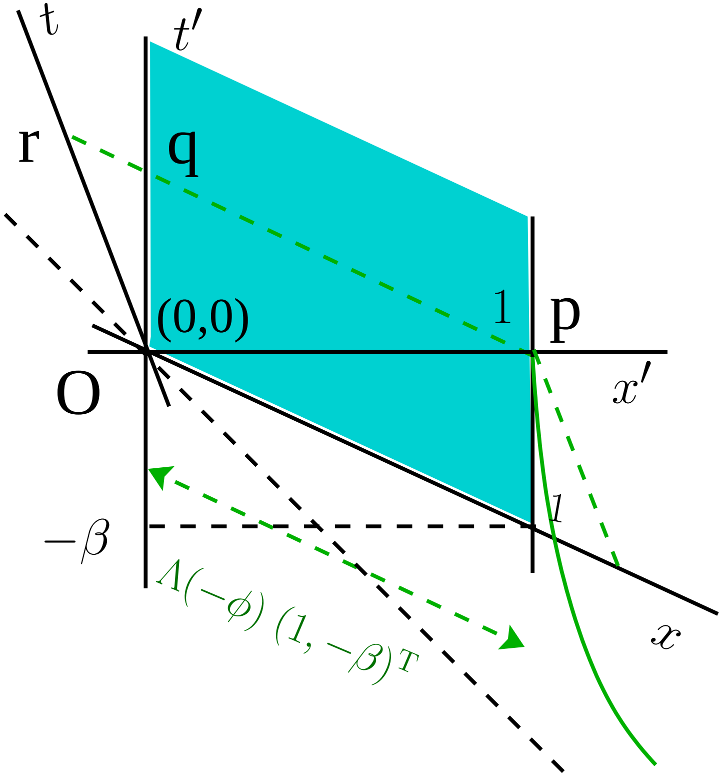

$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\gamma(1-\beta^2)\\0\end{pmatrix}$$

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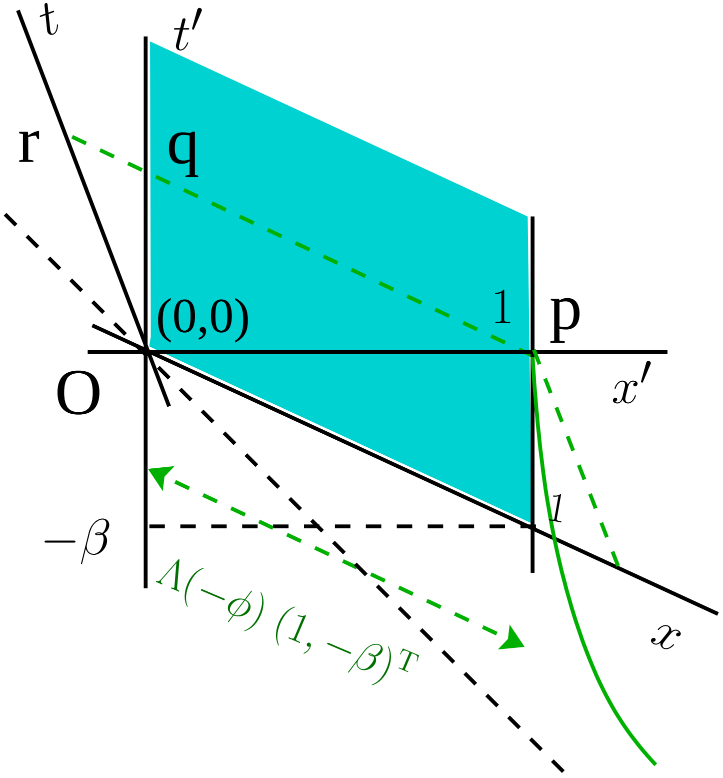

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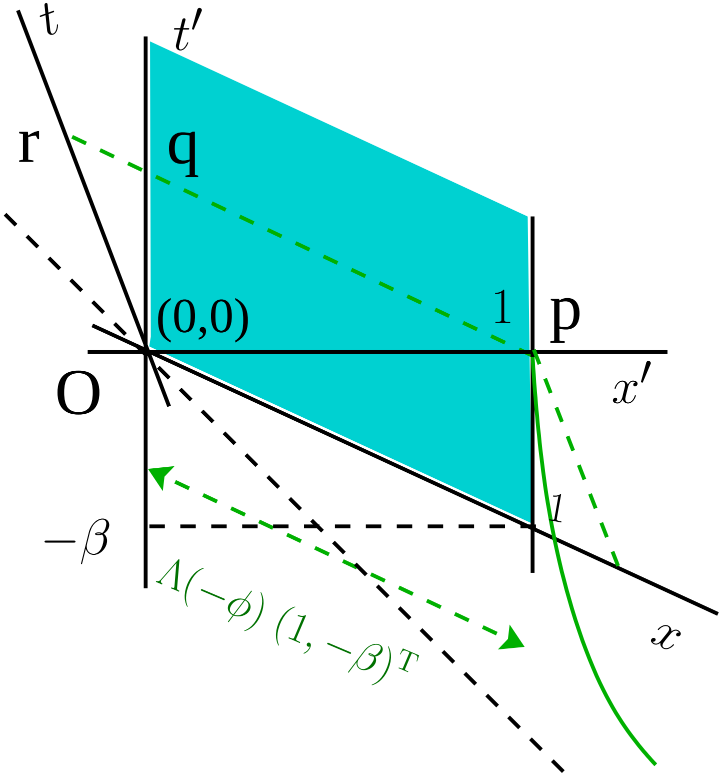

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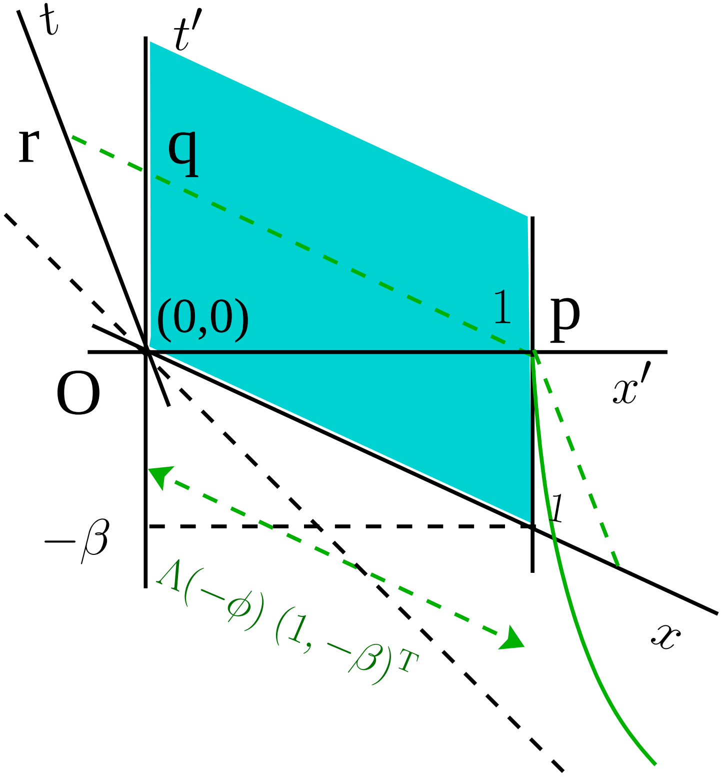
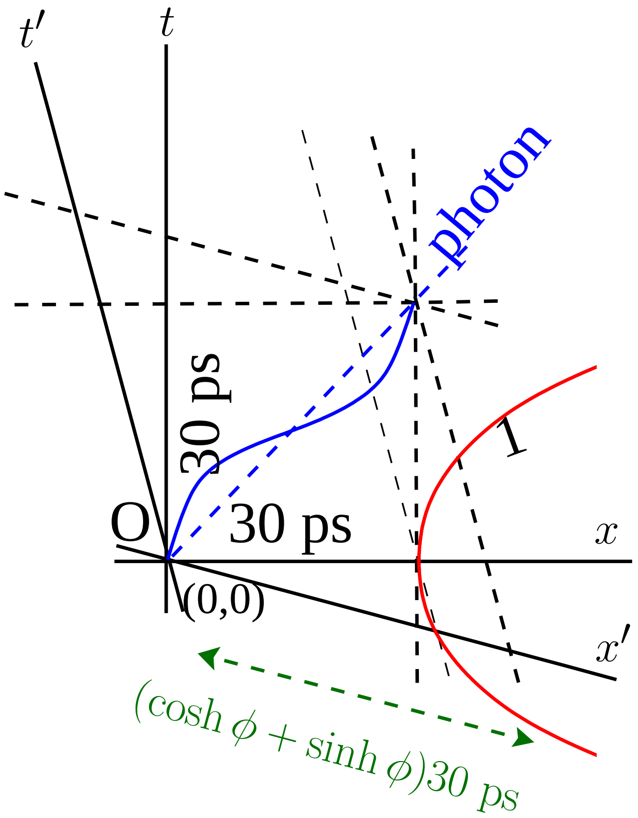


$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$ worldsheet “space contraction”

SR: Doppler shift



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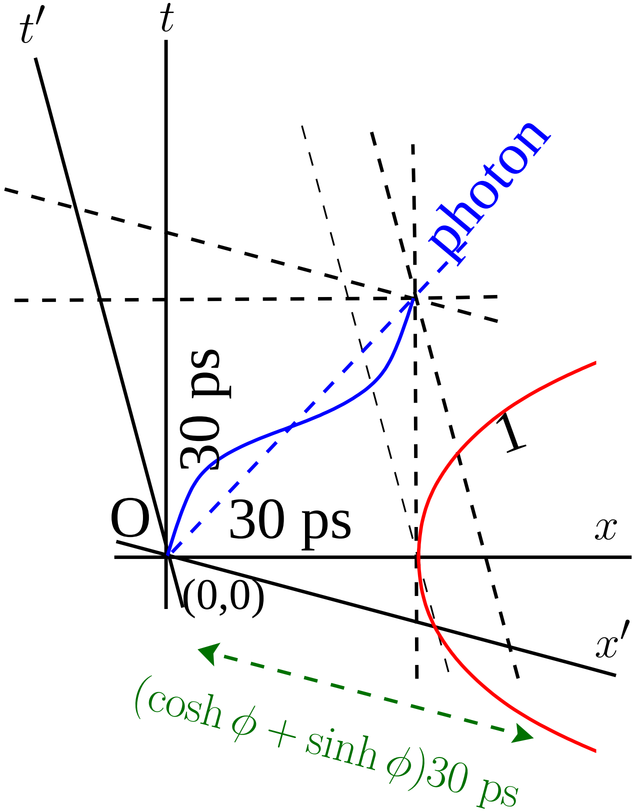
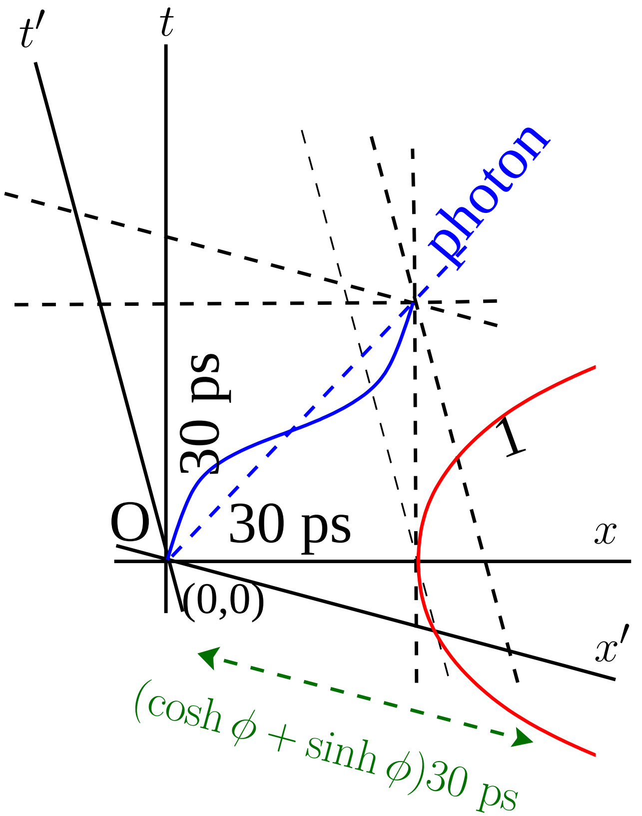


see photon worldline calculation

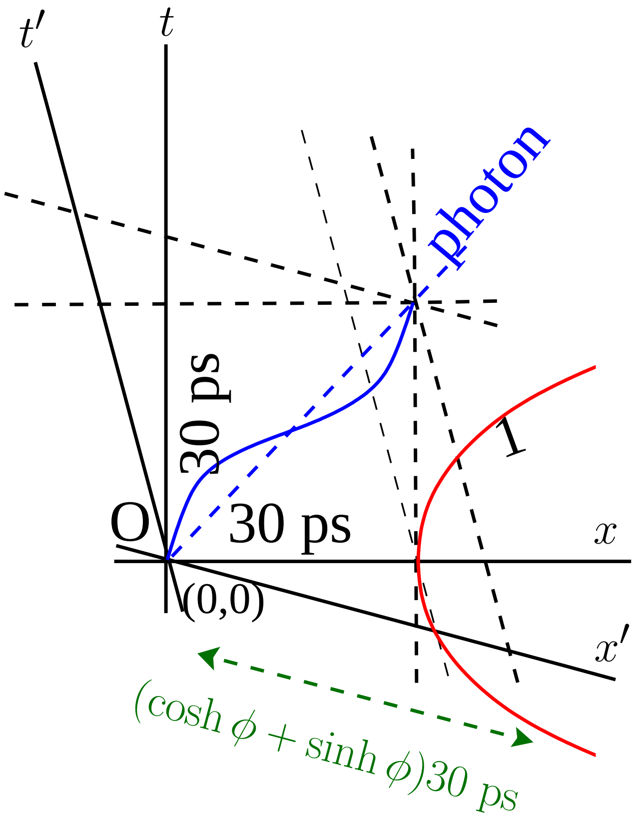
SR: Doppler shift



see photon worldline calculation

$$x' = (\cosh\phi + \sinh\phi)t$$

SR: Doppler shift



see photon worldline calculation

$$x' = (\cosh\phi + \sinh\phi)x$$

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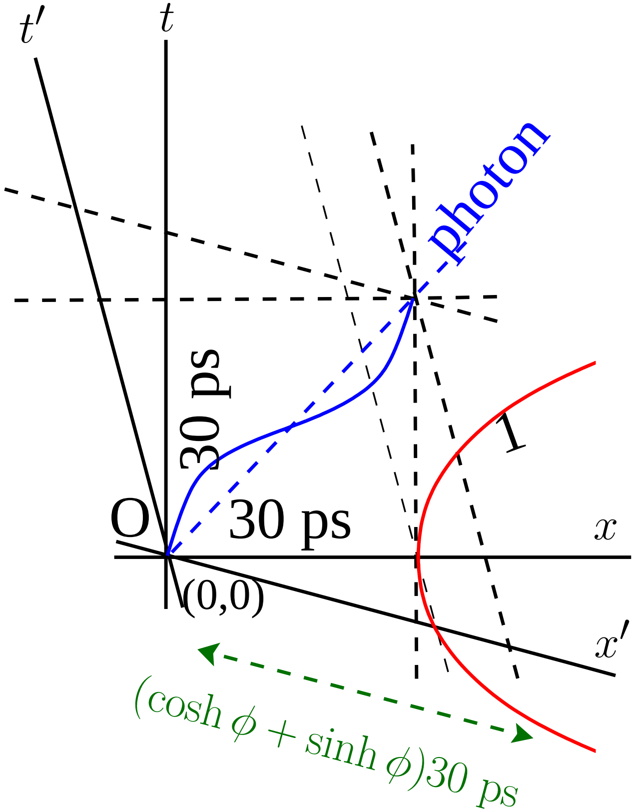
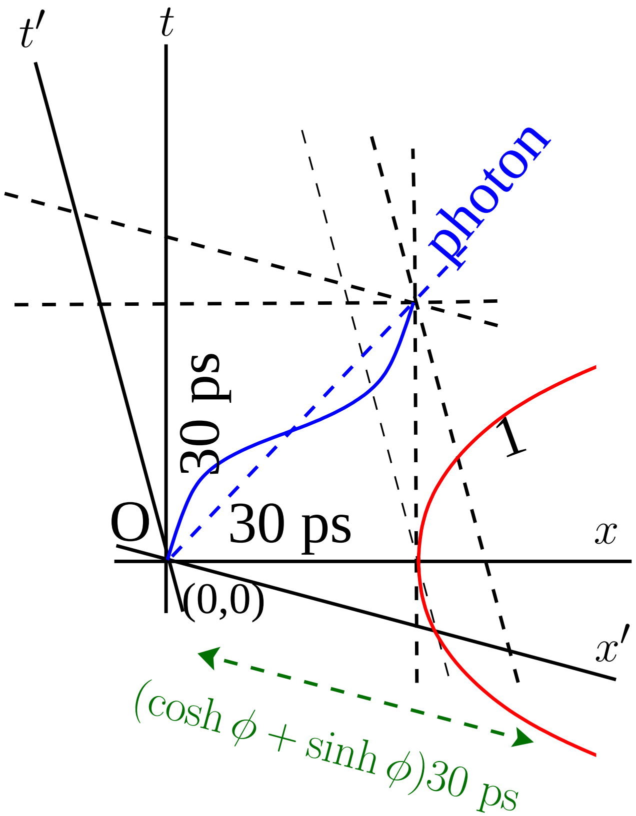


see photon worldline calculation

$$x'/x = \cosh\phi + \sinh\phi$$

SR: Doppler shift



see photon worldline calculation

$x'/x = \gamma + \beta\gamma$

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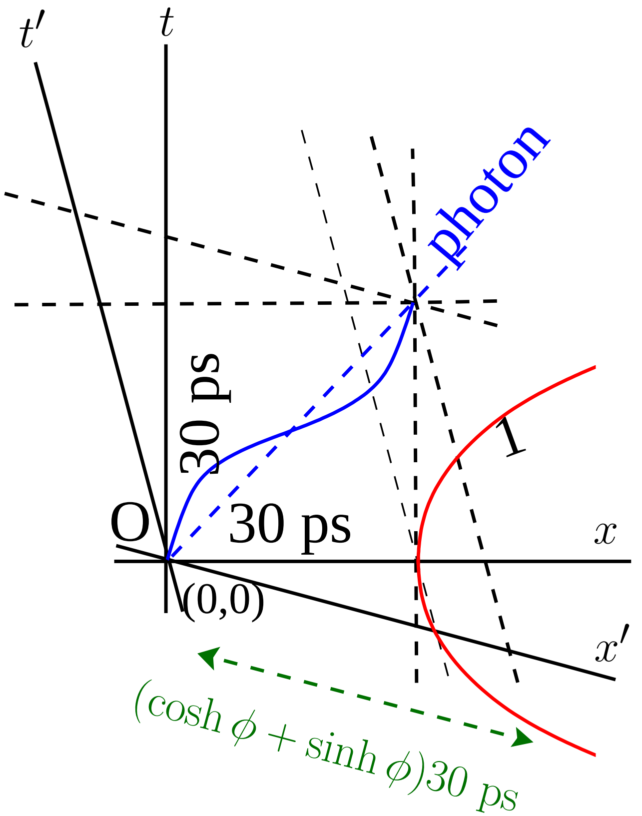
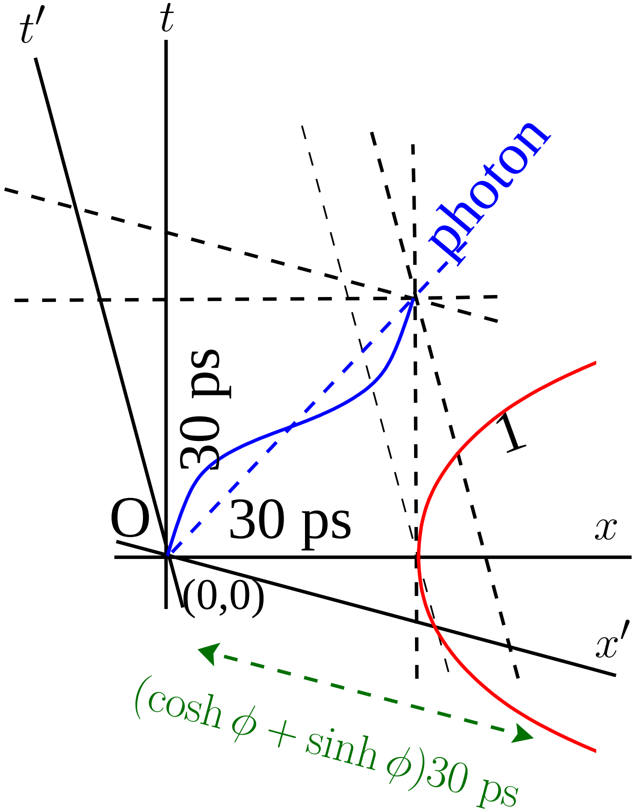


see photon worldline calculation

$x'/x = \gamma(1+\beta)$

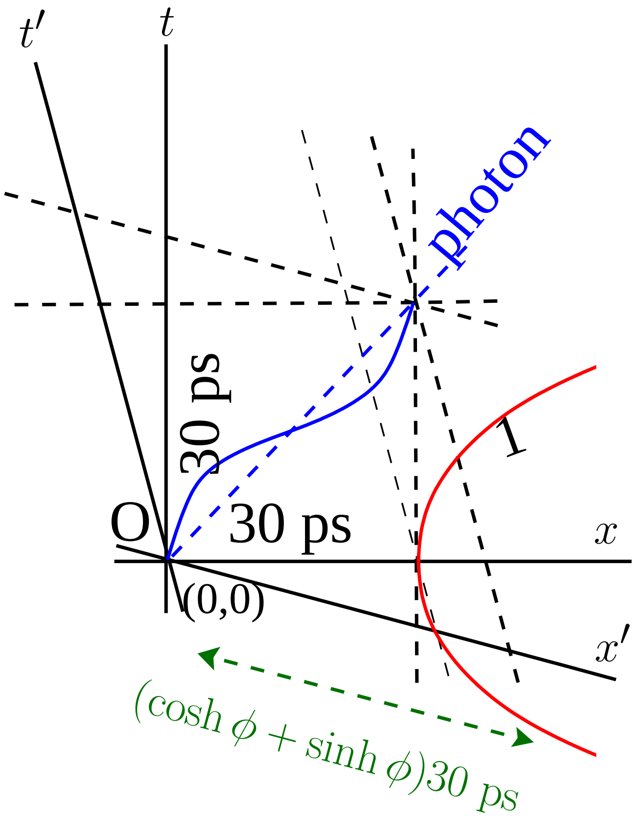
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \frac{1+\beta}{\sqrt{1-\beta^2}}$$

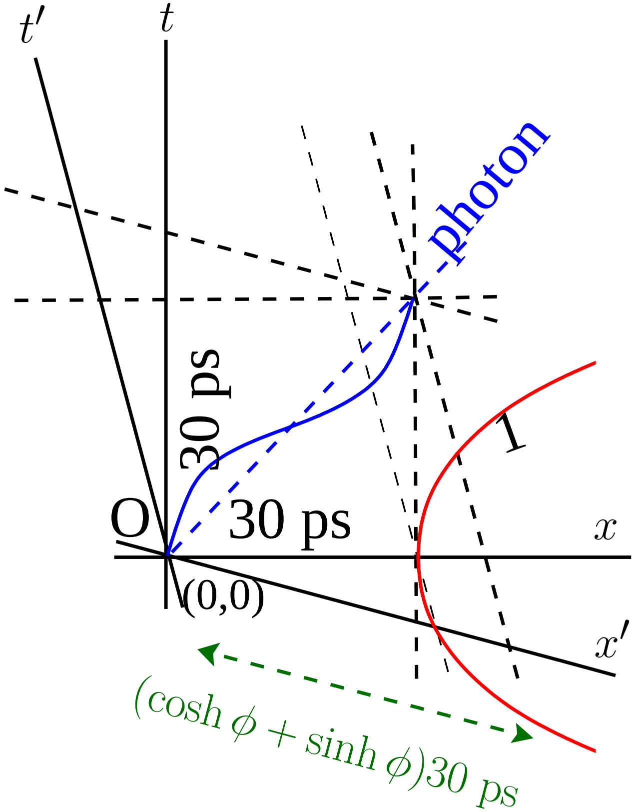
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \frac{\sqrt{(1+\beta)^2}}{\sqrt{(1-\beta)(1+\beta)}}$$

SR: Doppler shift



see photon worldline calculation

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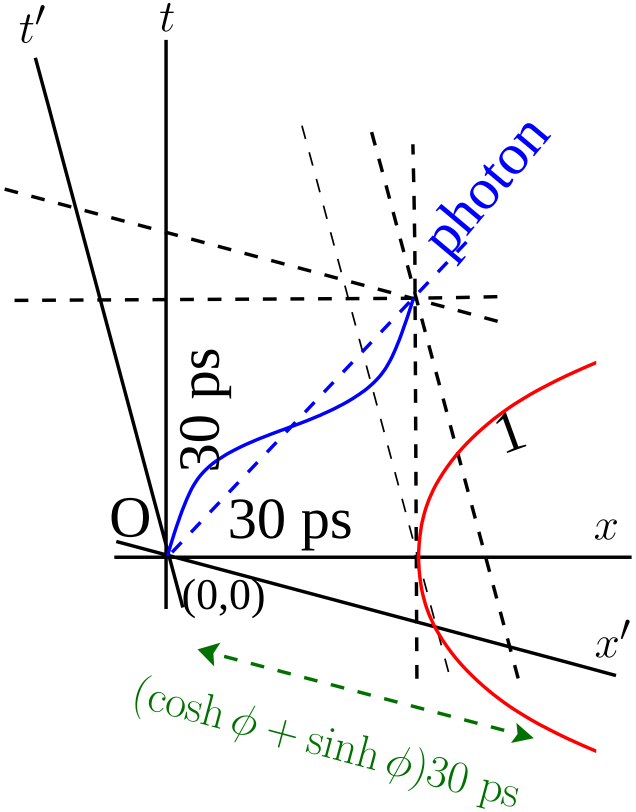
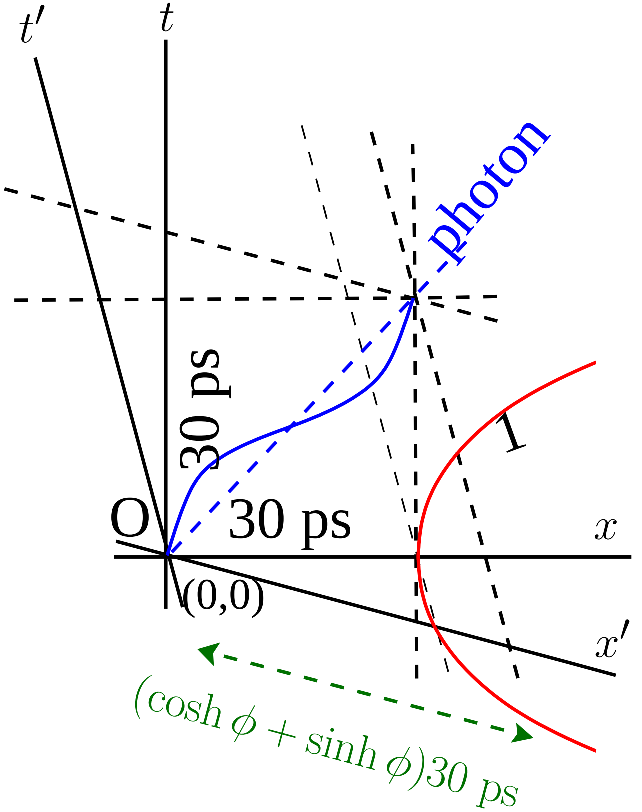


see photon worldline calculation

$$1 + z := \lambda'/\lambda = \gamma(1+\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}}$$ redshift

SR: Doppler shift

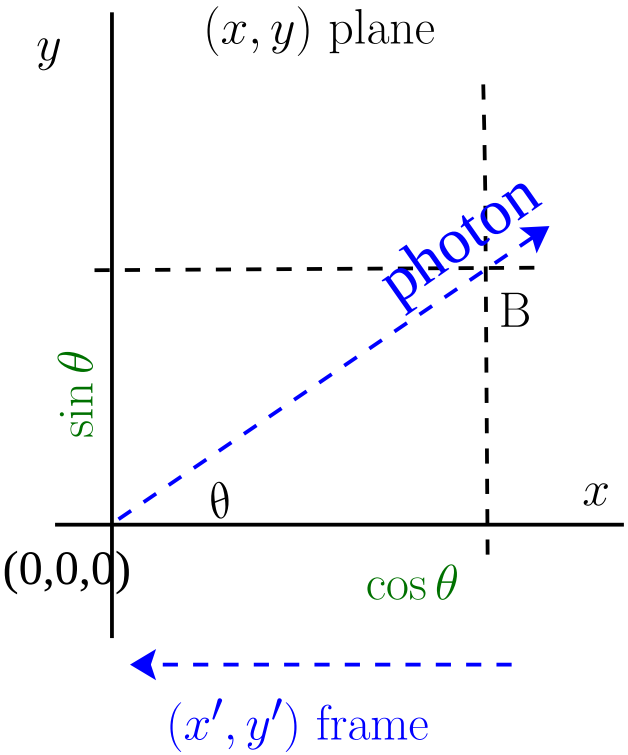


see photon worldline calculation

$$1 + z := \lambda'/\lambda = \gamma(1+\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}}$$ redshift

$\Rightarrow$ when $\beta \ll 1$, $z \approx \beta$

SR: relativistic aberration



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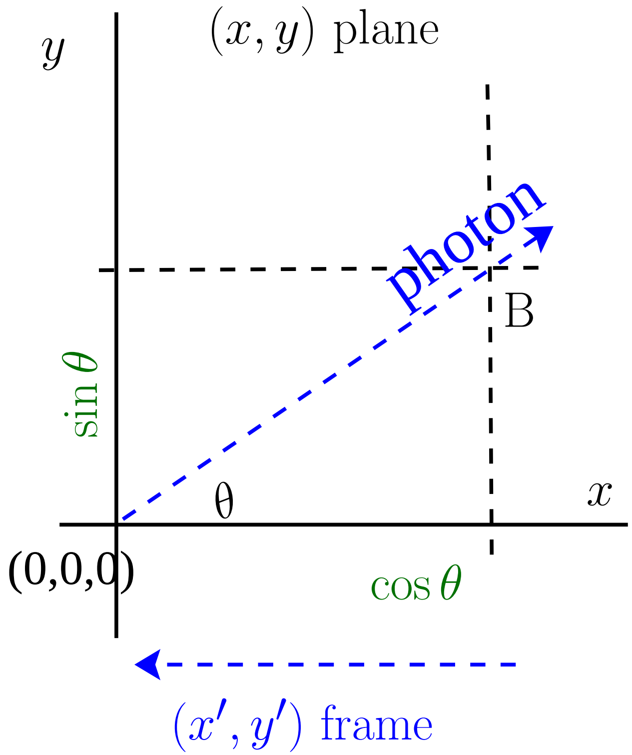

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

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$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

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event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ 0 & 1 & 0 \\ \beta\gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma\cos\theta + \beta\gamma \\ \sin\theta \\ \beta\gamma\cos\theta + \gamma \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma(\cos\theta + \beta) \\ \sin\theta \\ \gamma(1 + \beta\cos\theta) \end{pmatrix}$$

SR: relativistic aberration

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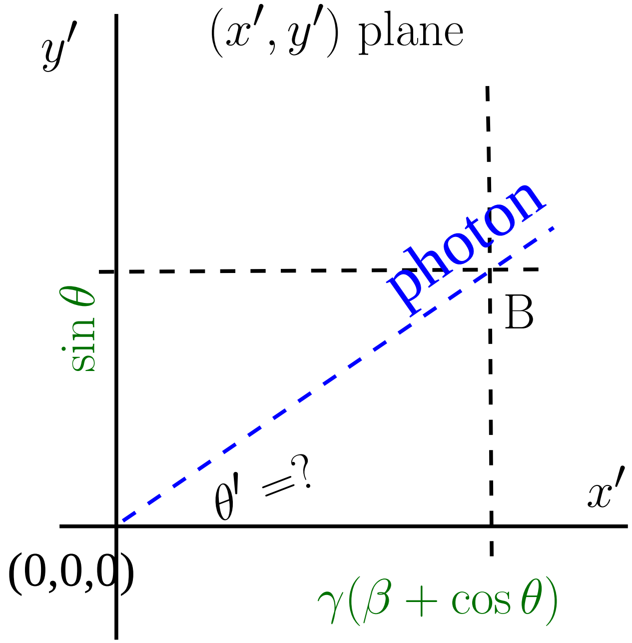

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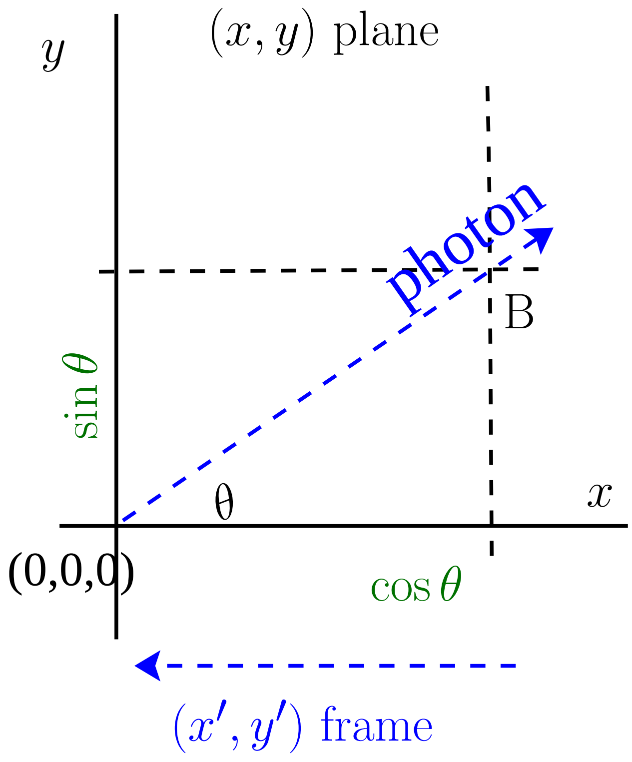

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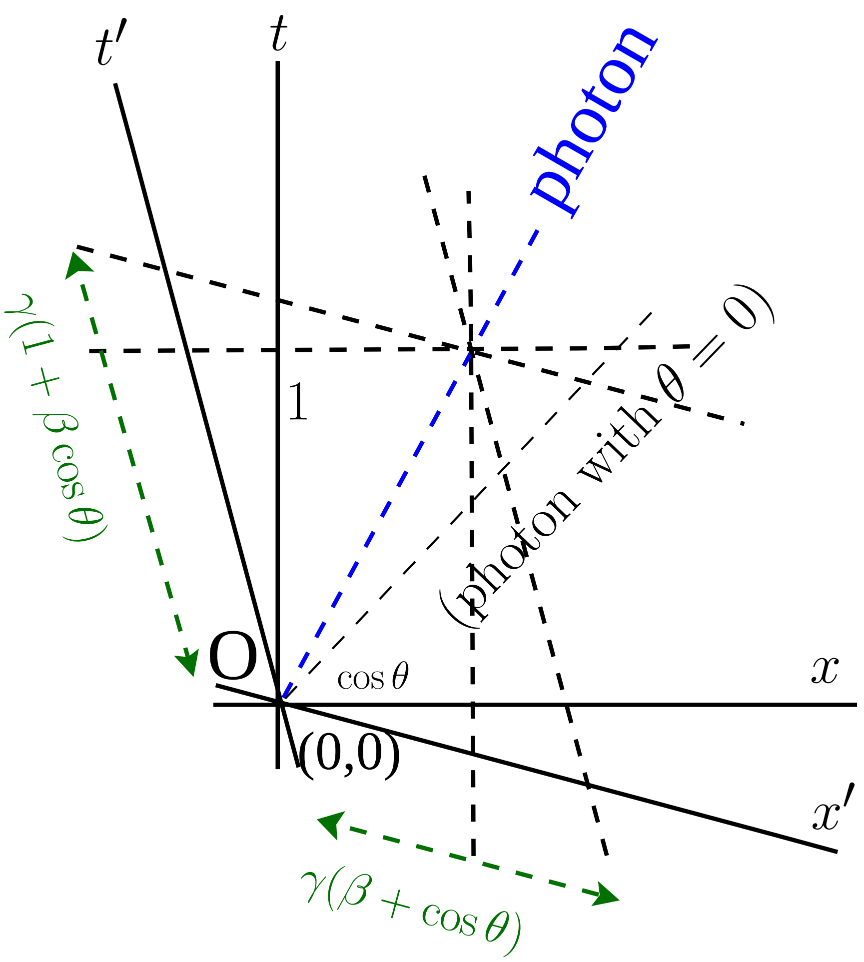

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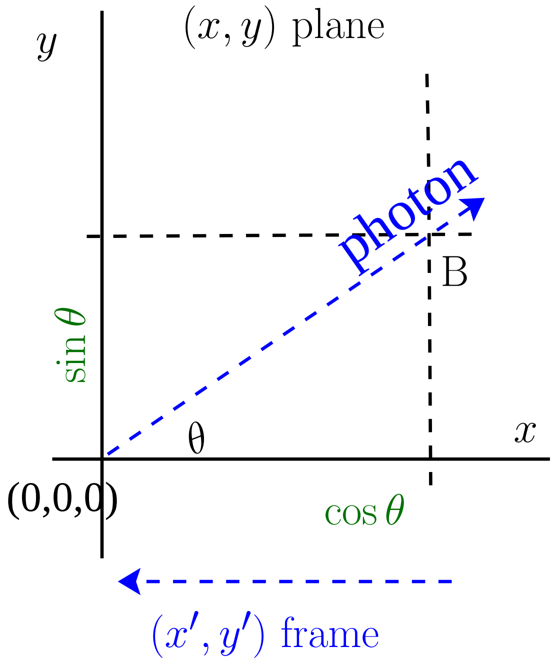

$$\tan\theta' = \frac{\sin\theta}{\gamma(\beta + \cos\theta)}$$

SR: relativistic aberration

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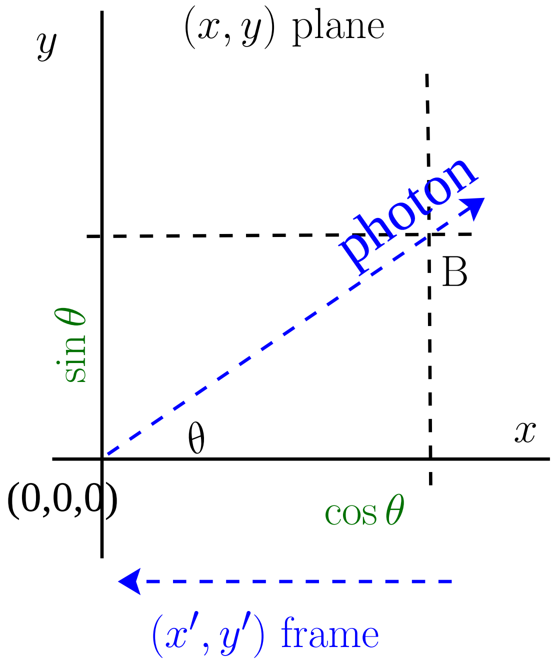

$$\tan\theta' = \frac{\sin\theta}{\gamma(\beta + \cos\theta)}$$

$\Rightarrow$ relativistic beaming, e.g. AGN jets

SR: four-velocity, ...

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GR:

+ w:Riemannian geometry (and pseudo-Riemannian geometry)

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GR: an approximation method: ADM

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GR: a numerical method: cactus

+ Cactus - http://cactuscode.org

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